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OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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JOURNAL

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The Life Assurance Companies Act, 1870.

AFTER much discussion this Act has at last been passed, requiring all Life Assurance Companies doing business in the United Kingdom to make annual returns in a fixed form of their receipts and expenditure, assets and liabilities, and periodically to make such further detailed returns as will admit of the liabilities of each Company under its contracts being very approximately estimated.

The Act is not in all respects what we could have desired, but we anticipate that it will produce on the whole very beneficial effects upon the business of Life Insurance. So far as it extends to promote greater publicity of accounts, its operation will, we believe, be one of unmixed good; and if the returns required to be made by Companies may appear to some persons needlessly minute, still we believe we may say that the Companies will cheerfully comply with the Act, being satisfied that whatever increases public confidence must result advantageously for the Companies themselves.

That there have been great evils in the state of Life Assurance business, is admitted on all hands; and the Act seems well adapted to cure those evils. It will no longer be possible for a Company that is hopelessly insolvent to claim the public confidence and obtain large amounts of new business, thus deferring the evil day,

but only with the result of largely increasing the magnitude of the loss caused by its failure, and the number of persons injured thereby; and it will be no longer possible for the managers of an unsuccessful Company to dispose of its business to an insolvent Company, receiving themselves a handsome bonus for the transaction. Nor is it matter for regret that the mystery with which many flourishing Companies have enveloped their affairs, will now be dissipated. Companies that ask the public to place the confidence in them that is implied in entering into a lifelong contract for the benefit of wife and children, should not be slow in proving themselves worthy of that confidence. There should be nothing in the conduct of a Life Insurance Company that the managers should not be prepared, if necessary, to proclaim to all the world and justify. In future, when an actuary is asked his opinion of the solvency of an Office, he will no longer be compelled in many cases to say that he knows nothing of it for certain, and can give no information except from general repute. He will have materials at command, from which to form his own estimate of the solvency and stability of any Office in reference to which he may be consulted; and the opinions thus formed on trustworthy materials will be gradually diffused through the length and breadth of the country. The inevitable result must be that public confidence and support will be rapidly withdrawn from those Offices—few in number, we trust they will be found—that do not show themselves worthy of that confidence and support.

The only provision which seems to us open to serious objection is that contained in *Section* 3:—that every new Company shall deposit £20,000 with the Accountant-General of the Court of Chancery. This is quite contrary to the principles of free trade which have of late years been so fully accepted by the Legislature, and there seems no good reason why the business of Life Assurance and Granting of Annuities should be restricted to persons or Companies who are prepared to make the above deposit. In particular, the provision seems likely to interfere materially with the future establishment of Mutual Insurance Companies. So also, the Act will prevent the formation of small local Societies for Life and Fire Insurance, and for the mutual guarantee of Survivorship Annuities, such as were at one time very common. It is true that these Societies have almost always proved unsuccessful, but that does not seem a sufficient reason for forbidding them in future; and in no other department of trade has the Legislature thought it necessary to restrain the public from investing their monies in ventures likely to prove unsuccessful. In a word, the requiring

of this deposit is a measure of protection to the existing Assurance Companies, and must therefore, we are satisfied, be prejudicial to the interests of the public.

Section 4. Several of the provisions of the Act have our unqualified approval. It is undoubtedly only right that the Life Assurance Fund of a Company transacting other business besides that of Life Assurance, should not be liable to pay the losses under fire or marine policies; and this no doubt will ultimately be the effect of the 4th Section, although it is not very clearly expressed.

Section 7 provides that an investigation into the financial condition of every Life Assurance Company shall be periodically made "by an actuary"; but there is no interpretation contained in the Act as to who is to be considered an actuary. This we hope will be supplied at some future time, and we trust the day is not far distant when the profession will be fully recognized and have a legal standing. It is a great step in the right direction that the existence of professional actuaries is recognized by the Law; and the next step, that of preventing incompetent persons from assuming the title, surely cannot be long delayed.

The provisions of Sections 12 and 13, that a list of the names and addresses of the shareholders and a printed copy of the deed of settlement, shall be furnished to every shareholder or policy-holder requiring them, are extremely proper.

Sections 14, 15 relate to amalgamations, and are intended to prevent any amalgamations of Insurance Companies being carried out except with the authority of the Court of Chancery. Experience only will show whether they will be efficacious for that purpose. It is notorious that most of the amalgamations hitherto effected have been carried out without proper powers, and might have been prevented if called in question within a sufficient time. There is, however, no doubt that the Act will place difficulties in the way of those legitimate amalgamations which are beneficial to all parties interested.

Section 21 relates to the winding up of an insolvent Insurance Company. This is a question of extreme difficulty. How is the Court to be satisfied that a Life Insurance Company is insolvent, taking into account its liabilities under policies and other contracts? What rule is the Court to adopt for estimating that liability? And in the very probable case of the Court having the opinions of

B 2

several skilled actuaries laid before it, some saying that in their opinion the Company is insolvent, and others that it is solvent, how is the Court to decide between them? Assuming this difficulty to be got over, Section 22 is admirable. There can be no doubt that Life Assurance Companies proved to be insolvent should not be wound up as other Companies,—the contracts being considered as put an end to, and the policyholders admitted to prove for damages in consequence of the breach of contract;—but that the justice of the case requires that the contracts should be considered as continuing in force for a reduced amount.

The Act is as follows:—

33 & 34 Vict. Chap. 61. a.d. 1870.

An Act to amend the law relating to Life Assurance Companies. [9th August, 1870.]

BE it enacted by the Queen's most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows:

Short title.

1. This Act may be cited as "The Life Assurance Companies Act, 1870."

2. In this Act—

Interpretation of terms.

The term "company" means any person or persons, corporate or unincorporate, not being registered under the Acts relating to friendly societies, who issue or are liable under policies of assurance upon human life within the United Kingdom, or who grant annuities upon human life within the United Kingdom:

The term "chairman" means the person for the time being presiding over the court or board of directors of the company:

The term "policy holder" means the person who for the time being is the legal holder of the policy for securing the life assurance, endowment, annuity, or other contract with the company:

The term "financial year" means each period of twelve months at the end of which the balance of the accounts of the company is struck, or if no such balance is struck, then each period of twelve months ending with the thirty-first day of December:

The term "Court" means, in the case of a company registered or having its head office in England, the High Court of Chancery; in the case of a company registered or having its head office in Ireland, the Court of Chancery in Ireland; in all cases of companies registered or having its head office in Scotland, the Court of Session, in either division thereof:

The term "registrar" means the Registrar of Joint Stock Companies in England and Scotland, and the Assistant Registrar of Joint Stock Companies in Ireland.

3. Every company established after the passing of this Act within the United Kingdom, and every company established or to be established out of the United Kingdom which shall after the passing of this Act commence to carry on the business of life assurance within the United Kingdom, shall be required to deposit the sum of twenty thousand pounds with the Accountant General of the Court of Chancery, to be invested by him in one of the securities usually accepted by the Court for the investment of funds placed from time to time under its administration, the company electing the particular security and receiving the income therefrom, and the registrar shall not issue a certificate of incorporation unless such deposit shall have been made, and the Accountant General shall return such deposit to the company so soon as its life assurance fund accumulated out of the premiums shall have amounted to forty thousand pounds. **Deposit.**

4. In the case of a company established after the passing of this Act transacting other business besides that of life assurance, a separate account shall be kept of all receipts in respect of the life assurance and annuity contracts of the company, and the said receipts shall be carried to and form a separate fund to be called the life assurance fund of the company, and such fund shall be as absolutely the security of the life policy and annuity holders as though it belonged to a company carrying on no other business than that of life assurance, and shall not be liable for any contracts of the company for which it would not have been liable had the business of the company been only that of life assurance; and in respect to all existing companies, the exemption of the life assurance fund from liability for other obligations than to its life policy holders shall have reference only to the contracts entered into after the passing of this Act, unless by the constitution of the company such exemption already exists: Provided always, that this section shall not apply to any contracts made by any existing company by the terms of whose deed of settlement the whole of the profits of all the business are paid exclusively to the life policy holders, and on the face of which contracts the liability of the assured distinctly appears. **Life funds separate.**

5. From and after the passing of this Act every company shall, at the expiration of each financial year of such company, prepare a statement of its revenue account for such year, and of its balance sheet at the close of such year, in the forms respectively contained in the first and second schedules to this Act. **Statements to be made by companies.**

6. Every company which, concurrently with the granting of policies of assurance or annuities on human life, transacts any other kind of assurance or other business shall, at the expiration of each such financial year as aforesaid, prepare statements of its revenue account for such year, and of its balance sheet at the close of such year, in the forms respectively contained in the third and fourth schedules of this Act. **Statements by company doing other than life business.**

7. Every company shall, once in every five years if established after the passing of this Act, and once every ten years if established before the passing of this Act, or at such shorter **Actuarial report and abstract.**

intervals as may be prescribed by the instrument constituting the company, or by its regulations or byelaws, cause an investigation to be made into its financial condition by an actuary, and shall cause an abstract of the report of such actuary to be made in the form prescribed in the fifth schedule to this Act.

Statement of life and annuity business.

8. Every company shall, on or before the thirty-first day of December one thousand eight hundred and seventy-two, and thereafter within nine months after the date of each such investigation as aforesaid into its financial condition, prepare a statement of its life assurance and annuity business in the form contained in the sixth schedule to this Act, each of such statements to be made up as at the date of the last investigation, whether such investigation be made previously or subsequently to the passing of this Act: Provided as follows:

(1.) If the next financial investigation after the passing of this Act of any company fall during the year one thousand eight hundred and seventy-three, the said statement of such company shall be prepared within nine months after the date of such investigation, instead of on or before the thirty-first day of December one thousand eight hundred and seventy-two:

(2.) If such investigation be made annually by any company, such company may prepare such statement at any time, so that it be made at least once in every three years.

The expression date of each such investigation in this section shall mean the date to which the accounts of each company are made up for the purposes of each such investigation.

Forms may be altered.

9. The Board of Trade, upon the applications of or with the consent of a company, may alter the forms contained in the schedules to this Act, for the purpose of adapting them to the circumstances of such company, or of better carrying into effect the objects of this Act.

Statements, &c. to be signed and printed and deposited with Board of Trade.

10. Every statement or abstract herein-before required to be made shall be signed by the chairman and two directors of the company and by the principal officer managing the life assurance business, and, if the company has a managing director, by such managing director, and shall be printed; and the original, so signed as aforesaid, together with three printed copies thereof, shall be deposited at the Board of Trade within nine months of the dates respectively herein-before prescribed as the dates at which the same are to be prepared. And every annual statement so deposited after the next investigation shall be accompanied by a printed copy of the abstract required to be made by section seven.

Copies of statements to be given to shareholders, &c.

11. A printed copy of the last deposited statement, abstract, or other document by this Act required to be printed shall be forwarded by the company, by post or otherwise, on application, to every shareholder and policy holder of the company.

List of shareholders.

12. Every company which is not registered under "The Companies Act, 1862," and which has not incorporated in its deed of settlement section ten of "The Companies Clauses

Consolidation Act, 1845," shall keep a "Shareholders Address Book," in accordance with the provisions of that section, and shall furnish, on application, to every shareholder and policy holder of the company a copy of such book, on payment of a sum not exceeding sixpence for every hundred words required to be copied for such purpose.

13. Every company which is not registered under "The Companies Act, 1862," shall cause a sufficient number of copies of its deed of settlement to be printed, and shall furnish, on application, to every shareholder and policy holder of the company a copy of such deed of settlement on payment of a sum not exceeding two shillings and sixpence. **Deed of settlement to be printed.**

14. Where it is intended to amalgamate two or more companies, or to transfer the life assurance business of one company to another, the directors of any one or more of such companies may apply to the Court, by petition, to sanction the proposed arrangement, notice of such application being published in the Gazette, and the Court, after hearing the directors and other persons whom it considers entitled to be heard upon the petition, may confirm the same if it is satisfied that no sufficient objection to the arrangement has been established. **Amalgamation or transfer.**

Before any such application is made to the Court a statement of the nature of the amalgamation or transfer, as the case may be, together with an abstract containing the material facts embodied in the agreement or deed under which such amalgamation or transfer is proposed to be effected, and copies of the actuarial or other reports upon which such agreement or deed is founded, shall be forwarded to each policy holder of both companies in case of amalgamation, or to each policy holder of the transferred company in case of transfer, by the same being transmitted in manner provided by section one hundred and thirty-six of The Companies Clauses Consolidation Act, 1845, for the transmission to shareholders of notices not requiring to be served personally; and the agreement or deed under which such amalgamation or transfer is effected shall be open for the inspection of the policy holders and shareholders at the office or offices of the company or companies for a period of fifteen days after the issuing of the abstract herein provided.

The Court shall not sanction any amalgamation or transfer in any case in which it appears to the Court that policy holders representing one tenth or more of the total amount assured in any company which it is proposed to amalgamate, or in any company the business of which it is proposed to transfer, dissent from such amalgamation or transfer.

No company shall amalgamate with another, or transfer its business to another, unless such amalgamation or transfer is confirmed by the Court in accordance with this section.

Provided always, that this section shall not apply in any case in which the business of any company which is sought to be amalgamated or transferred does not comprise the business of life assurance.

Statements in case of amalgamation or transfer.

15. When an amalgamation takes place between any companies, or when the business of one company is transferred to another company, the combined company or the purchasing company, as the case may be, shall, within ten days from the date of the completion of the amalgamation or transfer, deposit with the Board of Trade certified copies of statements of the assets and liabilities of the companies concerned in such amalgamation or transfer, together with a statement of the nature and terms of the amalgamation or transfer, and a certified copy of the agreement or deed under which such amalgamation or transfer is effected, and certified copies of the actuarial or other reports upon which such agreement or deed is founded; and the statement and agreement or deed of amalgamation or transfer shall be accompanied by a declaration under the hand of the chairman of each company and the principal managing officer of each company, that to the best of their belief every payment made or to be made to any person whatsoever on account of the said amalgamation or transfer is therein fully set forth, and that no other payments beyond those set forth have been made or are to be made either in money, policies, bonds, valuable securities, or other property by or with the knowledge of any parties to the said amalgamation or transfer.

Documents may be transferred from Board of Trade to registry of joint stock companies.

16. The Board of Trade may direct any printed or other documents required by this Act, or certified copies threeof, to be kept by the registrar of Joint Stock Companies or other officer of the Board of Trade; and any person may, on payment of such fees as the Board of Trade may direct, inspect the same at his office, and procure copies thereof.

Documents to be received in evidence.

17. Every statement, abstract, or other document deposited with the Board of Trade or with the registrar of Joint Stock Companies under this Act shall be receivable in evidence; and every document purporting to be certified by one of the secretaries or assistant secretaries of the Board of Trade, or by the said registrar, to be such deposited document, and every document purporting to be similarly certified to be a copy of such deposited document, shall, if produced out of the custody of the Board of Trade or of the said registrar, be deemed to be such deposited document as aforesaid, or a copy thereof, and shall be received in evidence as if it were the original document, unless some variation between it and the original document shall be proved.

Penalty for non-compliance with Act.

18. Every company which makes default in complying with the requirements of this Act shall be liable to a penalty not exceeding fifty pounds for every day during which the default continues; and if default continue for a period of three months after notice of default by the Board of Trade, which notice shall be published in one or more newspapers as the Board of Trade may direct, and after such publication the Court may order the winding up of the company, in accordance with The Companies Act, 1862, upon the application of one or more policy holders or shareholders.

19. If any statement, abstract, or other document required by this Act is false in any particular to the knowledge of any person who signs the same, such person shall be liable on conviction thereof on indictment to fine and imprisonment, or on summary conviction thereof to a penalty not exceeding fifty pounds. Penalty for falsifying statements, &c.

20. Every penalty imposed by this Act shall be recovered and applied in the same manner as penalties imposed by The Companies Act, 1862, are recoverable and applicable. Penalties how to be recovered and applied.

21. The Court may order the winding up of any company, in accordance with The Companies Act, 1862, on the application of one or more policy holders or shareholders, upon its being proved to the satisfaction of the Court that the company is insolvent, and in determining whether or not the company is insolvent the Court shall take into account its contingent or prospective liability under policies and annuity and other existing contracts; but the Court shall not give a hearing to the petition until security for costs for such amount as the judge shall think reasonable shall be given, and until a primâ facie case shall also be established to the satisfaction of the judge; and in the case of a proprietary company having an uncalled capital of an amount sufficient with the future premiums receivable by the company to make up the actual invested assets equal to the amount of the estimated liabilities, the Court shall suspend further proceedings on the petition for a reasonable time (in the discretion of the Court) to enable the uncalled capital, or a sufficient part thereof, to be called up; and if at the end of the original or any extended time for which the proceedings shall have been suspended such an amount shall not have been realized by means of calls as, with the already invested assets, to be equal to the liabilities, an order shall be made on the petition as if the company had been proved insolvent. Other circumstances under which company may be wound up by the Court of Chancery.

22. The Court, in the case of a company which has been proved to be insolvent, may, if it thinks fit, reduce the amount of the contracts of the company upon such terms and subject to such conditions as the Court thinks just, in place of making a winding-up order. Power to Court to reduce contracts.

23. Any notice which is by this Act required to be sent to any policy holder may be addressed and sent to the person to whom notices respecting such policy are usually sent, and any notice so addressed and sent shall be deemed and taken to be notice to the holder of such policy. Notices under this Act to policy holders.

24. The Board of Trade shall lay annually before Parliament the statements and abstracts of reports deposited with them under this Act during the preceding year. Statements, &c. to be laid before Parliament.

25. This Act shall not affect the Commissioners for the Reduction of the National Debt, nor the Postmaster General, acting under the authorities vested in them respectively by the Acts tenth George the Fourth, chapter forty-one, third and fourth William the Fourth, chapter fourteen, sixteenth and seventeenth Victoria, chapter forty-five, and twenty-seventh and twenty-eighth Victoria, chapter forty-three. Exceptions.

FIRST SCHEDULE.

Revenue Account of the ______________ for the year ending ______________.

18 . (Date.)		£ s. d.	18 . (Date.)		£ s. d.
	Amount of funds at the beginning of the year ..			Claims under policies (after deduction of sums re-assured)	
	Premiums			Surrenders	
	Consideration for annuities granted			Annuities	
	Interest and dividends			Commission	
	Other receipts (accounts to be specified) ..			Expenses of management	
				Dividends and bonuses to shareholders (if any) ..	
				Other payments (accounts to be specified) ..	
				Amount of funds at the end of the year, as per second schedule	
	£			£	

Note 1.—Companies having separate accounts for annuities to return the particulars of their annuity business in a separate statement.

Note 2.—Items in this and in the accounts in the Third and Fifth Schedules should be the net amounts after deduction of the amounts paid and received in respect of re-assurances.

SECOND SCHEDULE.

Balance sheet of the ______________ on the ______________ 18____.

Liabilities.	£ s. d.	Assets.	£ s. d.
Shareholders' capital paid up (if any) .. £		Mortgages on property within the United Kingdom ..	
Assurance fund		Do. do. out of the United Kingdom	
Annuity fund (if any)		Loans on the company's policies	
Other funds, if any, to be specified		Investments:	
		In British Government securities	
Total funds as per first schedule .. .. .. £		Indian and Colonial government securities	
Claims admitted but not paid*		Foreign government do.	
Other sums owing by the company* (accounts to be specified)		Railway and other debentures and debenture stocks	
		Do. shares (preference and ordinary)	
		House property	
		Other investments (to be specified)	
		Loans upon personal security	
		Agents' balances	
		Outstanding premiums	
		Do. interest	
		Cash:	
		On deposit .. .. £	
		In hand and on current account	
		Other assets (to be specified)	
£		£	

* *Note.*—These items are included in the corresponding items in the First Schedule.

THIRD SCHEDULE.

Revenue Accounts of the ______ for the year ending ______.

(No. 1.) Life Assurance Account.

(Date.)			(Date.)		
	Amount of life assurance fund at the beginning of the year			Claims under life policies (after deduction of sums re-assured)	
	Premiums, after deduction of re-assurance premiums ..			Surrenders	
	Consideration for annuities granted			Annuities	
	Interest and dividends			Commission	
	Other receipts (accounts to be specified)			Expenses of management	
				Other payments (accounts to be specified)	
				Amount of life assurance fund at the end of the year, as per Fourth Schedule	
		£			£

Note.—Companies having separate accounts for annuities to return the particulars of their annuity business in a separate statement.

(No. 2.) Fire Account.

	Amount of fire insurance fund at the beginning of the year			Losses by fire after deduction of re-assurances	
	Premiums received, after deduction of re-assurances ..			Expenses of management	
	Other receipts to be specified			Commission	
				Other payments to be specified	
				Amount of fire insurance fund at the end of the year, as per Fourth Schedule	
		£			£

Note.—When marine or any other branch of business is carried on, the income and expenditure thereof to be in like manner stated in a separate account.

(No. 3.) Profit and Loss Account.

	Balance of last year's account			Dividends and bonuses to shareholders	
	Interest and dividends not carried to other accounts ..			Expenses not charged to other accounts	
	Profit realised (accounts to be specified)			Loss realised (accounts to be specified)	
	Other receipts			Other payments	
				Balance as per Fourth Schedule	
		£			£

Note.—This account is not required if the items have been incorporated in the other accounts of this schedule.

FOURTH SCHEDULE.

Balance Sheet of the ____________ on the ____________ 18____.

Liabilities.		£ s. d.
Shareholders' capital		
General reserve fund (if any)		
Life assurance fund *		
Annuity fund (if any)*		
Fire fund		
Marine fund		
Profit and loss (if any)		
Other funds, if any, to be specified		
	£ s. d.	£
Claims under life policies admitted but not yet paid*		
Outstanding fire losses		
Do. marine do.		
Other sums owing by the company (accounts to be specified)		
		£

Assets.		£ s. d.
Mortgages on property within the United Kingdom		
Do. do. out of the United Kingdom		
Loans on the company's policies		
Investments:		
In British Government securities		
Indian and Colonial do.		
Foreign do.		
Railway and other debentures and debenture stocks		
Do. shares (preference and ordinary)		
House property		
Other investments (to be specified)		
Loans upon personal security		
Agents' balances		
Outstanding premiums		
Do. interest		
Cash:		
On deposit	£	
In hand and on current account		
Other assets (to be specified)		
		£

* If the life assurance fund is, in accordance with section 4. of this Act, a separate trust fund for the sole security of the life policy holders, a separate balance sheet for the life branch may be given in the form contained in Schedule 2. In other respects the company is to observe the above form. See also note to Second Schedule.

FIFTH SCHEDULE.

STATEMENT respecting the VALUATION of the LIABILITIES under LIFE POLICIES and ANNUITIES of the ______________, to be made by the ACTUARY.

(The answers should be numbered to accord with the numbers of the corresponding questions.)

1. The date up to which the valuation is made.

2. The principles upon which the valuation and distribution of profits among the policy holders are made, and whether these principles were determined by the instrument constituting the company, or by its regulations or byelaws, or otherwise.

3. The table or tables of mortality used in the valuation.

4. The rate or rates of interest assumed in the calculations.

5. The proportion of the annual premium income, if any, reserved as a provision for future expenses and profits. (If none, state how this provision is made.)

6. The consolidated revenue account since the last valuation, or, in case of a company which has made no valuation, since the commencement of the business. (This return should be made in the form annexed.)

7. The liabilities of the company under life policies and annuities at the date of the valuation, showing the number of policies, the amount assured, and the amount of premiums payable annually under each class of policies, both with and without participation in profits; and also the net liabilities and assets of the company, with the amount of surplus or deficiency. (These returns should be made in the forms annexed.)

8. The time during which a policy must be in force in order to entitle it to share in the profits.

9. The results of the valuation, showing—

(1.) The total amount of profit made by the company.

(2.) The amount of profit divided among the policy holders, and the number and amount of the policies which participated.

(3.) Specimens of bonuses allotted to policies for 100*l.* effected at the respective ages of 20, 30, 40, and 50, and having been respectively in force for five years, ten years, and upwards, at intervals of five years respectively, together with the amounts apportioned under the various modes in which the bonus might be received.

(FORM referred to under heading No. 6. in the Fifth Schedule.)

Consolidated Revenue Account of the ________________ for ________ years commencing ____________ and ending ____________.

	£ s. d.		£ s. d.
Amounts of funds on ________ 18__,—the beginning of		Claims under policies (after deduction of sums re-assured) ..	
Premiums (after deduction of re-assurance premiums) ..		Surrenders	
Consideration for annuities granted		Annuities	
Interest and dividends		Commission	
Other receipts (accounts to be specified)		Expenses of management	
		Dividends and bonuses to shareholders (if any)	
		Other payments (accounts to be specified)	
		Amount of funds on ________ 18__,—the end of the period, as per First (or Third) Schedule	
£		£	

(Form referred to under heading No. 7. in Fifth Schedule.)

Summary and Valuation of the Policies of the ______________ as at ______________ 18___.

Description of Transactions.	Particulars of the Policies for Valuation.				Valuation. Value by the ______ Table, Interest ____ per cent.			
	Number of policies.	Sums assured and bonuses.	Office yearly premiums.	Net yearly premiums, if ascertained.	Sums assured and bonuses.	Office yearly premiums.	Net yearly premiums, if computed.	Net liability.
ASSURANCES.								
I. *With participation in profits.*								
For whole term of life								
Other classes (to be specified)								
Extra premiums payable								
Total Assurances with profits								
II. *Without participation in profits.*								
For whole term of life								
Other classes (to be specified)								
Extra premiums payable								
Total Assurances without profits								
Total Assurances								
Deduct re-assurances								
Net amount of assurances								
Adjustments, if any								
ANNUITIES.								
Immediate								
Other classes (to be specified)								
Total of the results	..	.. ..						

The term "extra premium" in this Act shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country at rates of premium deduced from tables other than the European mortality tables adopted by the company, separate schedules similar in form to the above must be furnished.

(FORM referred to under heading No. 7. in Fifth Schedule.)

VALUATON BALANCE SHEET of ________ as at ________ 18___.

Dr.	£	Cr.	£
To net liability under Assurance and Annuity transactions (as per summary statement provided in Schedule 5) ..		By life assurance and annuity funds (as per balance sheet under Schedule 2 or 4)	
To surplus, if any		By deficiency, if any	

SIXTH SCHEDULE.

Statement of the Life Assurance and Annuity Business of the ________________ on the ______________ 18__.

(The answers should be numbered to accord with the numbers of the corresponding questions. Statements of re-assurances corresponding to the statements in respect of assurances, under headings 2, 3, 4, 5, and 6, are to be given.)

1. The published table or tables of premiums for assurances for the whole term of life which are in use at the date above mentioned.

2. The total amount assured on lives for the whole term of life, which are in existence at the date above mentioned, distinguishing the portions assured with and without profits, stating separately the total reversionary bonuses and specifying the sums assured for each year of life from the youngest to the oldest ages.

3. The amount of premiums receivable annually for each year of life, after deducting the abatements made by the application of bonuses, in respect of the respective assurances mentioned under heading No. 2, distinguishing ordinary from extra premiums.

4. The total amount assured under classes of assurance business, other than for the whole term of life, distinguishing the sums assured under each class, and stating separately the amount assured with and without profits, and the total amount of reversionary bonuses.

5. The amount of premiums receivable annually in respect of each such special class of assurances mentioned under heading No. 4, distinguishing ordinary from extra premiums.

6. The total amount of premiums which has been received from the commencement upon all policies under each special class mentioned under heading 4 which are in force at the date above mentioned.

7. The total amount of immediate annuities on lives, distinguishing the amounts for each year of life.

8. The amount of all annuities other than those specified under heading No. 7, distinguishing the amount of annuities payable under each class, the amount of premiums annually receivable, and the amount of consideration money received in respect of each such class, and the total amount of premiums received from the commencement upon all deferred annuities.

9. The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation.

10. A table of minimum values, if any, allowed for the surrender of policies for the whole term of life and for endowments and endowment assurances, or a statement of the method pursued in calculating such surrender values, with instances of its application to policies of different standing and taken out at various interval ages from the youngest to the oldest.

Separate statements to be furnished for business at other than European rates, together with a statement of the manner in which policies on unhealthy lives are dealt with.

Married Women's Property Act, 1870.

THIS Act, one of the most important passed during the last Session, has introduced great changes into the law of England with respect to the power of married women to hold property and contract liabilities independently of their husbands. It does not, however, fall within the scope of this *Journal* to take note of legal matters except in so far as they specially affect Insurance Companies; and we therefore reprint here only the 10th section of the Act, which relates (1) to policies of life effected by a married woman on her own life or on the life of her husband, and (2) to policies effected by a married man on his own life for the benefit of his wife and children.

33 & 34 VICT. CHAP. 93. A.D. 1870.

An Act to amend the Law relating to the Property of Married Women. [9th August, 1870.]

WHEREAS it is desirable to amend the law of property and contract with respect to married women:

Be it enacted by the Queen's most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows:—

* * * *

Married woman may effect policy of insurance.

10. A married woman may effect a policy of insurance upon her own life or the life of her husband for her separate use, and the same and all benefit thereof, if expressed on the face of it to be so effected, shall enure accordingly, and the contract in such policy shall be as valid as if made with an unmarried woman.

As to insurance of a husband for benefit of his wife.

A policy of insurance effected by any married man on his own life, and expressed upon the face of it to be for the benefit of his wife or of his wife and children, or any of them, shall enure and be deemed a trust for the benefit of his wife for her separate use, and of his children, or any of them, according to the interest so expressed, and shall not, so long as any object of the trust remains, be subject to the control of the husband or to his creditors, or form part of his estate. When the sum secured by the policy becomes payable, or at any time previously, a trustee thereof may be appointed by the Court of Chancery in England or in Ireland according as the policy of insurance was effected in England or in Ireland, or in England by the judge of the County Court of the district, or in Ireland by the chairman of the Civil Bill Court of the division of the county, in which the insurance office is situated, and the receipt of such trustee shall be a good discharge to the office. If it shall be proved that the policy

was effected and premiums paid by the husband with intent to defraud his creditors, they shall be entitled to receive out of the sum secured an amount equal to the premiums so paid.

The usefulness of the provision in the second part of the section appears to us to be needlessly restricted by the requirements that the several interests of the wife and children shall be expressed on the face of the policy. We are of opinion that it would have been better to leave the husband full power of appointing by will or deed to the several members of his family such shares as their circumstances may from time to time render desirable.

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*American Tables of Mortality.** *By* Prof. C. F. McCay.

WHEN our mutual life companies began their business in 1843, they had no American table of mortality to guide them in determining the premiums of insurance that ought to be charged at the different periods of life. There were no American statistics public or private, good, bad, or indifferent, to which they could refer, except the mortuary reports of cities, and these were so imperfect and unreliable as to be utterly useless, except to encourage the opinion that the chances of long life were about the same here as in the countries from which our people had emigrated. It was known that the numbers of the dying, as reported by our city registers, were below the real deaths; that the ages were full of errors; that the boundaries of the mortuary limits were constantly changing; that residents of the city were often buried in the country, and sometimes country people were interred in town; that the population was fluctuating; that the immigration from the rural districts and from foreign countries was large and irregular; that the census of the population, whether taken by the United States, or by the states, or by the cities themselves, was full of errors; that the ages of the living, both among males and females, were wrongly reported, sometimes intentionally, but always carelessly and thoughtlessly; and that these errors in the numbers and ages of the people and of the deaths were so numerous that no confidence could be placed in the ratio of the living and the dying at any particular age, while this ratio at all ages is an indispensable element in determining the proper premiums to be charged in any of the contracts made by our life companies. No

* Reprinted from the *Spectator* of New York and Chicago.

one was able to say whether half the children who are born in our country live to the age of twenty, or twenty-five, or thirty; whether the rate of mortality was greater at twenty or at forty; whether a man of fifty might expect to live ten, or fifteen, or twenty years; whether our mortality was greater or less than in England or in Sweden; and whether two per cent was sufficient premium for a year's risk at thirty, or at forty, or at fifty. All was uncertain and doubtful. We had no American observations which would enable us to answer these or any other questions that our life companies might wish to ask.

Under these circumstances, the companies very naturally turned their inquiries to the English tables of mortality. The British offices which had followed these had been successful and prosperous. Those who had used the high rates of the Northampton had done a profitable business; and those who had trusted to the low rate of the Carlisle had done well. The similarity between the people and the climate, and the habits and condition of our population and that of Great Britain was so great, that they concluded that our rate of mortality could not differ much from the English.

Our people were, for the most part, of the same stock, the same industrial activity, and the same degree of civilization as that of Great Britain. Our physicians followed the same rules of medical practice, and were alike observant, intelligent, and learned. Descended from them for the most part, we had adopted their rules of health, their management of the sick, their methods of preventing and curing disease. Our modes of living were similar. Neither was addicted to luxury nor exposed to fatal diseases by poverty or destitution. Both were well housed, well fed, well clothed, and well cared for in sickness and old age. Both were frugal, intelligent, and virtuous. Neither was crowded in cities, with narrow lanes reeking with the fumes of decaying animal and vegetable matter. In every respect we were more like the English than the French or Italian, and our rate of mortality would be more like that of England than any other country in Europe. The latitude of New York is, indeed, lower than that of London; but the influence of the Gulf Stream, and the prevalence of westerly winds from the warm Atlantic, bring the two places on nearly the same isothermal line. Neither is exposed to hyperborean cold or tropical heat. The depressing influence of an extremely low temperature, and the malarious effects of a vertical sun, were alike absent from both. In every thing which would promise long

life there would seem to be little, if anything, that would make the chances of living materially different in the two countries.

And, in fact, besides the English and the Swedish tables, which substantially agreed, none were to be found worthy of any confidence. The French have never had any which were much esteemed. Neither those of Deparcieux, nor Duvillard's, nor even Milne's Montpelier are of any value; and the statistics published in the Annuaire of the French government, though minute and extensive, are useless for the purposes of a life company. In no part of Germany was there any collection of vital statistics sufficiently large and accurate to determine the true expectation of human life. The same was true of Italy, Spain, and the east of Europe; so that our companies were obliged to have recourse to England for the facts and laws which should guide and regulate their contracts.

As the Actuaries' table was only published in 1843, and Dr. Farr's No. 1. in the same year, their choice was limited. They all selected the Carlisle, as it was more recent than the Northampton, and more esteemed by English actuaries, because its general results had been strongly confirmed by the experience of the Equitable and Amicable insurance companies, which were the oldest and largest in Great Britain. A better choice would have been the table constructed by Davies from the extensive and long-continued experience of the Equitable office; but the pioneers in American life insurance did not possess enough information on this subject to secure a wise decision, and followed rather the practice of the best English companies than the facts and conclusions of this able actuary. In fact, however, neither the Mutual Life of New York, nor the Mutual Benefit of New Jersey, nor the Nautilus, afterwards the New York Life, adopted the Carlisle premiums with any regular marginal addition, although all professed to follow that table. Their premiums were irregular, loaded more at one age than at another, and did not conform exactly to any life table. They were probably taken from some English company which had modified the Carlisle rates at particular ages according to its own experience, or according to some fanciful theories of its officers. The Sun and Alliance had adopted the Carlisle four per cent rates, with a marginal addition of forty per cent, but neither the New York companies nor the Girard Life at Philadelphia adopted these rates or any regular percentage of them. What they did adopt was nearly the Carlisle rates, with a margin of thirty-five per cent; but their premiums had many unjust and unequal irregularities, a few of which have been continued by some companies down to the

present time. As an example of this, the whole life rates at the ages of 34, 35, and 36 may be mentioned. Several of our oldest and best companies have 2.64, 2.75, and 2.81 for the annual life premiums at these ages, when the Carlisle four per cent, with a margin of thirty-five per cent added, will give 2.648, 2.729, and 2.816; and there is nothing in the Carlisle table of mortality or in any other which would authorise the large increment from 34 to 35, and the small increment from 35 to 36. This anomaly is without any warrant or reason, and it is surprising that so many of our companies have retained it so long.

These defects in the early premiums were, for the most part, soon corrected; and especially did the oldest and largest New York company take an active and commendable part in this improvement. Their actuary, Mr. Gill, constructed an average table from the Actuaries, the Swedish, and other good tables, and a set of "computation columns" were deduced from this which were used for premiums, and reserves, and valuations, and dividends. This was a great advance, for no better way of arriving at the true rate of mortality to be expected among the insured is possible than the method of averages; and, as good materials were used for this average, the improved table deserved much confidence. In these premiums there is to be found no irregularity, no sudden increase, no slow and rapid progress, but all is steady, regular, and harmonious.

Very few companies, however, adopted these premiums. Until 1858, the table of mortality was unknown, except to a few. The grosser errors in the premiums of most of the companies were corrected from time to time, until there was a general approximation to the best English tables.

The Mutual Life of New York did not stop with this improvement of Mr. Gill. In 1854, they published a full account of their experience for the first ten years of their operations; and, again, in 1859, their experience for fifteen years was published by Mr. Homans, the successor of Mr. Gill. This valuable contribution to American vital statistics was accompanied with an adjusted table of mortality founded on these fifteen years' experience, at the earlier ages of life, or down to the age of seventy-four, beyond which they had had only three or four living members, and a single death.

The general results of this experience conformed pretty nearly to the experience of the English life companies. Under thirty-three the American mortality was above the English; after thirty-

three, it was below. As, however, the number of deaths was only 750 in the whole fifteen years, which was fewer than was observed by Dr. Heysham at Carlisle, and much smaller than the numbers on which other life tables had been based, no great confidence can be placed on its rate of mortality at each separate age.

At the end of the next five years, Mr. Homans published a new table founded on the experience of twenty years. As this company was growing rapidly, each new period of five years doubled its experience. The deaths were now more numerous than at Carlisle; they were reported with more accuracy; the numbers of the living and the dying were both more reliable; and the table obtained from the observations was constructed with more ability and skill by Mr. Homans than by Mr. Milne. It thus surpassed the Carlisle in every particular. Compared with Dr. Farr's, it was founded on fewer observations, but these were on insured lives, and not on the general population, which made it more valuable for the uses of a life company. If we compare it with the Actuaries', it had two advantages. It was founded on American experience, and not on policies but on lives. The disadvantages were, however, serious, the observations were fewer, and the duration of the insurances shorter; and, to weigh these properly, we will consider the two tables carefully, and examine particularly the grounds on which they may claim our confidence.

Mr. Homans' American table has been much esteemed. It has been adopted by many life companies not only for the premiums, but for the valuation of policies and the distribution of surplus profits. It has been made by the state of New York the official standard by which to measure the liabilities of all the companies doing business in that state; and this example has been followed by several of the western states. It has not yet excluded the Actuaries' table from its official position in Massachusetts, but vigorous efforts have been made to effect this change; and, as American experience is attractive, and the actual experience of large and ancient life offices form the very best basis for the future expectation of mortality among the lives insured by our companies, this table presents great claims to the favor and approval of American actuaries. It is well adjusted, admirably constructed, and has added much to the reputation of the distinguished actuary of the Mutual Life of New York. With all these recommendations in its favor, it is, nevertheless, the duty of every one to consider its claims with fairness and impartiality.

The English Actuaries' was based upon policies and the

American on lives. But the results of the Economic insurance company's experience by policies and lives are almost identical; and the new Actuaries', which is founded on lives, differs so little from the old, which was founded on policies, that it may be regarded as a confirmation of the latter table, and a proof that its excellence has not been impaired by the few double and triple insurances which it contained.

The one is English and the other American; but in healthy countries, as England and the state of New York, where neither extreme heat nor extreme cold increase the mortality, and where all the social causes which influence the duration of human life are very much alike, very little difference exists in the rate or the law of mortality. The chances of dying in England, and Sweden, and Norway are nearly the same at every period of life; and so we may expect them to be in Old and New England.

The great differences, however, in these two tables are to be found in the number of the observations on which they are based, and in the duration of the policies which have been observed.

The number of deaths in the Equitable society alone was four times as great as in the New York company; and, in all the seventeen English offices, they were ten times more numerous. In the American companies the young lives were few and the old still fewer, most of the insured being in middle life. Down to 1858, the oldest member of the American company was only in his seventy-eighth year; and the number over the age of sixty-five was less than seventy. The experience was, therefore, of little value at the earliest and latest periods of life, and, at all ages, much less valuable than that of the English companies.

The greatest difference, however, in the value of these two sets of observations is in the duration of the policies. When the members of a company are recently insured, when the time which has elapsed since they were examined by the company's physician, and pronounced sound and well, has been brief, the chances of dying in a single year are much diminished. An examination has been made by the insurance commissioner of Massachusetts of the number dying in the first year after insurance, and the rate of mortality has been found less than one-half what it has been among all the members of the company. These results have been everywhere observed in all companies in Europe and America. The difference between the mortality of recent insurers and the whole body of members is greatest in the first year, decreases considerably in the second, and still more in the third, but is

appreciable in the fifth, and even down to the tenth year, after insurance. Now, the average duration of the policies in the American experience was only about three years, while in the English it was over eight. In the former, one-third of all the lives exposed to mortality were in the first year of their insurance, about one-half in the first two, and about three-fourths in the first five, leaving only one-fourth in that class whose mortality is unaffected by the medical examination on their entrance among the insured. In the latter, only one-eighth were in the first year of insurance, one-quarter in the first five years, while three-fourths were beyond the influence of the medical examination at the beginning of their insurance.

Now, the amount of the difference between the mortality of recent and old insurers at the same age being as large as twenty-five per cent, and, in the experience of some companies, as large as fifty per cent, the errors in the American table for the period of middle life must be considerable. The Mutual Life had existed for twenty years; but of the early members, three-fourths of the whole had ceased their connection with the company. The new members were very numerous, and increasing every year. Very many policies were abandoned after the first year's membership. This constant accession of fresh blood, this rapid addition of sound lives, when the whole company was yet new, must lessen very considerable the mortality at those ages where the admissions were numerous. Mr. Homans, in a recent address in New York, has mentioned that he made some allowance for this influence of new members; but if we compare the table of fifteen years' experience, where no correction was made, with that of twenty years' experience, we see that the allowance made has been very small—almost inappreciable.

The conclusion to which we, therefore, arrive is, that at the younger and older ages, the American table has no sufficient basis in American experience, and that at the middle ages, where the experience is considerable, so many of the observations are on lives recently insured that it can claim but little confidence.

If, now, this table is without sufficient warrant in the facts on which it professes to be based, let us compare it with other tables of acknowledged excellence, and note where it differs from these standards. Although there was a small experience of young lives, and none of old lives, and although the experience at middle life was composed so largely of recent insurers as to be of little value, the distinguished actuary may still have constructed a valuable and

useful table, by correcting the deficiencies at middle life and adopting proper rates for the earlier and later ages.

A glance at the curve in Fackler's book will show how admirably the table is graduated. It conforms at all ages to the law of invariable increase in the mortality at every period of life—from early youth to extreme old age. The chances of living one year invariably lessen as we grow older. At ten or twelve, when the feebleness of infancy is mastered, the body has its highest vitality. At birth, the tiny, weak frame, often diseased and imperfect, always without vital force to bear the evils to which it is exposed, falls an easy prey to its foes. Slowly it acquires the means of resistance; and, though many die in the first few years of life, by ten or twelve the rate of mortality is at a minimum, and thence increases up to the very end of life. There are no halts in this steady progress, no climacterics which it is difficult or easy to pass, no critical periods where the danger is at a maximum, no sudden changes, no slow or rapid increments, but all move on with a risk steadily increasing until the final goal is reached, and we lie down to rise no more. Mr. Homans has observed this law; and, although in his report at the close of the fifteen years' experience of his company he discovered two maxima and two minima, and found similar anomalies in Finlaison, Quetelet, and Neison, they have entirely disappeared in his twenty years' observations.

For the purpose of examining the table, and comparing it with others, we will divide it into three parts—the first extending from ten to twenty-five, when the number of observations was very limited, as few persons insure at that age; the second extending from twenty-five to seventy-five, in which nearly all the experience really occurred; the third from seventy-five to the close of life, for which period there was little or no experience among the insured, as the company had been too recently established for many of its members to reach so old an age.

In the first period, the number of lives who were exposed to mortality for one year was under four per cent of the whole number of observations in the first fifteen years; and, for the whole twenty years, the percentage was still smaller. This insignificant portion, when the whole numbers were themselves very small, can lay no claim to confidence unless confirmed by other tables.

Now, for the quinquennial period from ten to fifteen, the American table is not only above the old and new Actuaries', but also above Farr's and the Carlisle and all other good tables. At

ten and fifteen the rates in Farr's No. 3 are .0056 and .0052; in the Actuaries', 68 and 69; and in the Carlisle, 44 and 55; while in the American they are 75 and 76.

In the whole fifteen years from ten to twenty-five, the chances of dying are—in Farr's, .095; in the Actuaries', .102; in the Carlisle, .090; but in the American, .110.

The law at which the mortality increases during these three periods of five years, according to Dr. Farr and Mr. Milne, is that the rate is nearly stationary from ten to fifteen and twenty to twenty-five, with a rapid increase from fifteen to twenty. The first is slow, because near the minimum which some tables place at ten and others at eleven, or twelve, or thirteen. The approach of puberty seems to develop a new set of diseases not only among females, but males also, and the rate of mortality advances rapidly. At nineteen or twenty these large increments cease, and the onward progress from this time of life is slow and steady in all tables. The very large numbers in Dr. Farr's observations place this law of increase between ten and twenty-five beyond all question; and it is confirmed by the best experience everywhere, not only in the general population, but in the German and English life companies, and in the English and Scottish friendly societies.

Mr. Homans has not conformed his table to this law, but has begun with a high mortality at ten, which he has increased very slowly until twenty-five, when his rate falls below most other good tables. This want of conformity is, however, of little importance to insurance companies, as very few of their risks are taken under twenty where this anomaly occurs.

In a similar manner Mr. Homans has varied from the rate of mortality at the close of life. In almost every table there are some survivors after the age of 95. Out of 100,000 at ten, the Actuaries' has 37; Dr. Farr has 155; and the Carlisle 356; but Mr. Homans has none. At the age of 77 his rate is below the Actuaries'; five years later it is five per cent higher; ten years later the excess is twenty-five per cent; and fifteen years later it is fifty-seven per cent. At 77 it is three per cent below Dr. Farr's; at 82 it is seven per cent above; at 87 it is thirty-seven per cent higher; and at 92 more than a hundred per cent higher. At 76 it is below the Carlisle; at 81 it is fourteen per cent higher; at 86 the excess is thirty-seven per cent; and at 91 eighty-seven per cent. The rapid progress from the age of seventy-five upwards is even in excess of the old Northampton and of every other good table; and as the Mutual life company could not have had more

than four or five deaths at these higher ages, it is difficult to guess where Mr. Homans could have obtained any authority for these high rates at the older periods of life. There is nothing in the mortuary reports of our cities that justify them; and the private experience of almost every one will tell him that some of his acquaintances have lived beyond ninety-six, which is the extreme limit of human life according to this American table.

These high rates of mortality at the later stages of life are, however, of little importance to an insurance company. At sixty-nine and seventy the life premiums in the Actuaries' table are as high as the American; and, at the usual period of insurance, the influence of a high or low mortality at these older ages is almost inappreciable.

Turning, now, our attention to the middle period of life, where the experience of our life companies has been extensive, we observe that the American table is everywhere below all other good tables. To make this comparison more full and satisfactory, we insert below the rates in a large number of the best of tables.

Table.	30	35	40	45
I. Milne's Carlisle, ...	.0101	.0103	.0130	.0148
II. Milne's Carlisle adjusted, ...	87	110	133	145
III. Milne's for Sweden, ...	107	117	144	164
IV. Price's " "	114	121	141	179
V. Farr's English No. 1,........	102	115	129	144
VI. Farr's English No. 3, ...	101	113	130	154
VII. Davies' Equitable, ...	97	113	126	143
VIII. Actuaries', ...	84	93	104	122
IX. Average of these eight, ...	99	111	130	150
X. Mr. Gill's Average,..........	94	109	126	149
XI. Prof. McCay's Average, ...	96	108	123	143
XII. American Experience,........	84	89	98	112

Table.	50	55	60	65	70	75
I.	.0134	.0179	.0335	.0411	.0516	.0955
II.	149	183	290	423	587	878
III.	196	238	331	510	755	1008
IV.	221	267	341	501	726	111
V.	161	199	295	433	637	929
VI.	188	246	325	459	673	988
VII.	181	242	302	389	529	737
VIII.	159	217	303	441	649	956
IX.	174	221	315	446	634	945
X.	185	238	320	453	650	956
XI.	175	229	320	456	652	960
XII.	138	186	267	401	620	944

We see by examining these rates that the American experience is below the others, and that the maximum depression is about the age of forty-five, near which age is the largest accession of new members. The differences decrease until the age of sixty-five, when no more new risks are taken; but, as the influence of the

fresh and sound lives is felt more or less for a few years after admission, the equality is not restored until the age of seventy-five, when the American agrees very nearly with all three of the average tables, as well as with the Actuaries'.

Let us consider the percentage of differences between the American and these four tables. Here is the percentage for all the ten ages above mentioned, counting it upwards from the American:

Age.	*Actuaries.*	*Average of Eight.*	*Average of Gill.*	*Average of McCay.*
30	0	18	12	14
35	4	25	22	21
40	6	33	29	25
45	9	34	33	28
50	15	26	34	27
55	17	19	19	23
60	14	18	20	20
65	10	11	13	14
70	5	2	5	5
75	1	0	1	1

Now, is this depression of the American due to the influence of the new lives? We know that that cause would produce an effect of this kind. But would it produce so large an effect? The experience of the English insurance companies on the influence of selection will enable us to give an answer to this question.

The mortality of the English members who were in the first year of their insurance, and of all others, was given in Mr. Homans' report for 1858 as follows:

Ages.	*First Year.*	*Total Experience.*	*Excess.*
25–35	.584	.791	35 per cent.
35–45	.887	1.100	25 "
45–55	1.318	1.614	22 "
55–65	2.675	2.943	10 "

But in the total experience is included the mortality of the second, third, fourth, and fifth years in which the influence of the selection is largely felt.

Mr. Higham has determined the rate in the first volume of *The Assurance Magazine* for each year of insurance, and also for those members whose mortality was beyond the influence of selection. These are given below, and we have added the total rates among all the members, new and old, and also those in the American experience:

Age.	*First Year.*	*Old.*	*Old and New.*	*American.*
30	.00482	941	842	843
35	574	1,299	929	895
40	620	1,672	1,036	979
45	848	1,921	1,221	1,116
50	1,122	2,483	1,594	1,378
55	1,486	2,908	2,166	1,857

If, now, the estimate is true, or nearly true, which we made above, that one-third of the experience of the Mutual Life was of members who were in the first year of insurance, one-half in the first two years, and three-fourths in the first five, it is abundantly evident from these rates for new and old members, that the amount of diminution in the experience of the Mutual Life below the general mortality of the whole of England, and below that of the Actuaries' table, is no greater than could be expected.

We thus have a disturbing cause in the determination of the true rate of mortality from the past experience of the Mutual Life, sufficient to account for the difference between the American and the English tables; and this cause is at a maximum at the younger years of life, and decreases with the age, and is reduced to zero exactly at the period when the difference disappears. This cause is acknowledged by all writers on this subject, English and American, to be a true cause; and, as it accounts fully for the difference, it is fair to infer that it is the real cause. And we, therefore, conclude that the American table gives too low a mortality for middle life, and that the Actuaries' rate is confirmed rather than impaired by the experience of our American companies.

We have now examined the differences between the American table and the Actuaries', in each of the three periods, into which we have divided that table. At the younger and later ages, when the American experience was very small, the rate was too high; at the middle period of life, when it was based on a considerable experience, it is too low. The first two differences are of little importance, but not so with the latter. An error in the rate of mortality at or near the time when an insurance is made is of the greatest importance to a life company, especially if it is a material one. An erroneous table not only makes the amount of profits wrong, but distributes them wrongly.

In all our American companies much the larger portion of the members consists of recent insurers. Many of the companies are new, and, in the old companies, the accession of new members has been greater and greater every year. To ignore the influence of selection in estimating the expected mortality; to count as profits the gains from the low mortality of the sound and fresh lives which are added every year in increasing numbers; to make the past experience the basis of the future expectation; to suppose that the mortality of the insured who are now thirty will, at the end of fifteen years, be the same as the mortality which has been experienced among those who have been admitted to membership when they

were forty-five, or nearly forty-five, is erroneous, dangerous and unjust.

To judge of the future by the past, under the same circumstances, is wise. But our past experience, having been among new, fresh, healthy lives, is not a safe criterion for the future when disease and shattered constitutions and ruinous habits shall have impaired their vital energies and altered their chances of living.

What an insurance company really wants is the experience of mortality among select lives; not among the general population, comprising people in every degree of health and disease, but among persons who are sound and well when they enter the companies. Dr. Farr's table, or the Swedish, or the Carlisle, is not suited to their wants; nor is the American, because its experience is too limited as to age, its observations too few, its policies too recent, its insurances too brief, and its new members too large a portion of the whole number.

The Actuaries' is based on select lives; it contained a large number of persons at the oldest period of life, and also at the younger; its observations were numerous, and continued through a long series of years, comprising every variety of season and temperature, of sickness and health, of scarcity and abundance; its recent insurers were comparatively few, because the companies were old, and their increase slow and regular; its construction is admirable, and made by a committee of able and distinguished persons; and its claims are, therefore, of the highest character.

Most sincerely do we commend the efforts and labors of our American actuary who has, without exterior aid or encouragement, furnished so valuable a contribution to our vital statistics; most earnestly would we urge him to continue his investigations; most highly do we appreciate what he has already accomplished; and most confidently do we anticipate that his future observations will supply us with what we so much need, an American table, founded on insured lives, who have continued so long members of a life company as to give us the true American rate of mortality at every period of life.

We are disposed to predict that such a table will conform very closely to the Actuaries'; we are satisfied that the past experience of our American companies is a confirmation of that valuable table; we are sure that longer observations will bring our experience nearer to the English; and, though it may not accord exactly with the old or the new Actuaries', though it may develope peculiarities

either at the younger ages, or at middle life, or in old age; though it may establish for our country a higher or a lower scale of mortality than the English, we believe that, at the present time, the Actuaries' is superior to all others that are now known for all the purposes of a life Company. The reasons for that belief we have indicated, and every one will give them such weight as they deserve.

Excepting this table of Mr. Homans, we have had no other contribution to our vital statistics worthy of much attention. The New York Life and the Mutual Benefit published their experience for the first ten or twelve years, but this has not been continued. The experience of all the companies doing business in Massachusetts for several years has also been given to the public. But all these have been tainted still more than the experience of the Mutual Life with the fatal defect of being on recent risks fresh from the scrutiny of the medical examiners, whose special duty it is to reject all feeble, sickly and defective constitutions; and fresh, also, from the scrutiny of agents, friends and secretaries, which excludes all who have bad habits, hereditary tendencies to disease, or any defect which impairs their chances of long life.

It is of the highest importance that all our companies should make up, from time to time, and publish to the world a careful and exact statement of their experience, giving full particulars of the durations of the several risks, so that the influence of selections and examinations may be eliminated from the total results, and the true mortality of American insured lives obtained from the combined experience of many companies. Some of them have objections to publicity; some dislike the trouble and expense of the investigations; some are afraid of injury, or reproach, or censure; some anticipate invidious comparisons, foolish and unfounded accusation, puerile and senseless misunderstandings. But their own true interests will be best subserved by the publication of their experience, and the knowledge they will obtain of their own real condition. Truth and knowledge are always beneficial, and nowhere can they be more important than where hundreds of millions of dollars involved are payable to the aged, and the widow, and the orphan.

On the Method of Constructing Tables of Mortality. By M. M. von Baumhauer, *Director of the Statistical Department in the Netherlands.*

Extracted from the "Programme" of the 7th Session of the International Statistical Congress, held at the Hague, in Sept., 1869. *Translated by* J. Hill Williams, Esq.

The theory of the construction of Tables of Mortality offers no difficulty when applied either to the members of a Society, or to a collection of individuals, who can be traced so that all the necessary particulars as to them are known. But in the population of a country or a town, several incidental causes, such as epidemics and migrations, exercise an influence; some of them on the total numbers of the population, and others on the members living at certain ages. The mobility of a population or the constant change taking place, from one residence to another, or from one country to another, prevents us from following each individual or each group of individuals from the moment of birth to that of death.

It is the object of tables of mortality to inform us as to the average ratio of the dying at each age to the living who are exposed to the risk of death. Their construction is one of the most practical applications of the theory of averages.

The elements required for their construction are the births, the number living at each age, and the deaths at each age. Several methods have been employed; sometimes one, sometimes two, and sometimes all the three elements above mentioned, have been used. In the first instance registers of the deaths were used, stating the ages either exactly or by groups of years. This plan was first used by Halley, who deduced his table of mortality from the registers of deaths at Breslau, from 1687 to 1691. This table gives only the average mortality for each five years of age; it indicates the average rate of mortality at each age. It is not what French writers call a *Table de survie,* as it does not give the numbers living at each age, or the deaths at each age out of a given number of births. It gives the average age at death, supposing this to be the average life-time ("*vie moyenne*") of the population.

To correct the errors of these first attempts, which at best were only applicable to a stationary population, M. Moser compared the death-registers with the numbers living at each age as shown by the census, while M. Achille Guillard and the late M. Hermann of Munich, compared them with the births. These

two methods, although improvements, were both faulty. Moser's fundamental error was in not seeing that a population enumerated on a *certain fixed day* cannot properly be compared with the deaths occurring *in a year,* that those deaths are the fallen leaves, not of that population, but of that other one which comprises all persons exposed to the risk of death *during the whole year.* M. Guillard, with the object of accounting for the successive extinction of all the persons born, distributed the deaths that were omitted, or the difference between the number of deaths and of births, in a ratio compounded of the number of deaths registered for each age and the place of the age in the scale of life.

At the very outset, these calculations are open to the objection that they are based upon the arbitrary assumption that the population increases by a regular and unvarying law, not only when taken as a whole, but also when a single period of life or single age is considered separately. M. Hermann, whose example has since been followed in Austria, subtracts from the number born and surviving at each age, the number of deaths that have taken place at each age. While fully appreciating the laborious and ingenious investigations of this author I agree with the following judicious remarks of Dr. Bertillon,* "These calculations assume "in the first place that the registers of births and deaths according "to age are correctly kept, although the registers in Bavaria, being "kept by the clergy, are not under the control of the Government. "They further suppose that there is neither immigration nor "emigration. But the emigration from Bavaria is considerable; "for it has lost in 25 years (from 1835 to 1860) through the excess "of the number of emigrants over the immigrants 220,000 citizens. "Many of these must have escaped registration, and even the "ages of the immigrants and emigrants, which are indispensable "for correcting the tables of mortality and eliminating their "irregularities, are not mentioned in the registers of changes of "residence." These subtractions are, moreover, erroneous for another reason, which principally affects the first years of life. It is assumed that all the persons dying at a given age in any calendar year were born in one and the same calendar year. This is an erroneous assumption which vitiates all the calculations. Those who die in the same calendar year under the age of one year, were born partly in the calendar year before that of death, and partly in the year of death, and so on.

* *Dictionnaire Encyclopédique des Sciences Médicales, du Dr. A. Dechambre.* Art. *Bavière* (p. 618 *et seq.*). Paris, 1866.

M. Quetelet was the first to notice the blanks left by Moser's method for the first years of life, by comparing the number of the living under one year of age with the average number of births. For the first years of life he substituted the number born and the survivors of them at each age for the population. This example has been followed by M. Berg and Dr. Farr. Thus the blank has been filled up for the first years of life, while in the following years Moser's calculations have been followed without correction, and a blank consequently left in them.

By substituting the number born for the number living (as enumerated) we substitute the total number of children under one year of age who have existed during the year of 365 or 366 days for the residue of them who live to be enumerated on the day when the census is taken. In other words, we can only obtain a true table of mortality by comparing the deaths with the total number exposed to the risk of death. No just comparison can be made between the numbers of persons of each age on a certain day, and the number of persons who die at each age during the whole year, or during the series of days which make up the year.

I have given in the "*Journal des Économistes*" (July, 1868, p. 33 *et seq.*) a brief account of the method I followed in constructing new tables of mortality for the Netherlands. It consists in distinguishing the deaths occurring *before* from those occurring *after* the day of the census, and in adding the former to the living as enumerated at each age, on the ground that, although not comprised in the census, they formed part of the population exposed to the risk of death for a longer or shorter part of the year, and subsequently dividing this increased number of living by the deaths at each age. Moser confines his calculations to those living at a specified period of the year, neglecting all those who, having lived during all the other periods of the year, had died before the date of the enumeration. By substituting the number born and the survivors of them, and, generally, for each age the total number of persons exposed to the risk of death during the whole year, in place of the population enumerated at each age on a given day, we follow the method adopted in forming tables from the experience of Life Assurance Companies; for in that case, in order to ascertain the ratio of the deaths to the lives assured, the deaths are divided by the total number of the lives originally assured, whether they are living or dead at the time of the investigation, and not by the survivors only. It is by this method

that we calculate the proportional mortality of prisons, and of sanitary and other establishments, bringing into the calculation all the persons admitted therein and exposed to the risk, and not those only who are present at a given date. By this same method the annual percentage of deaths to the population should be calculated. Dr. Bertillon, in his paper upon the duration of life in France and in the Gironde,* read at the Medical Congress at Bordeaux, on the 6th of October, 1865, arrives at the same result by a different process. His formula,

$$\mathrm{S}n+t=\mathrm{S}n-\mathrm{S}n\frac{a\mathrm{D}n\ldots\ldots n+t}{\mathrm{P}n\ldots\ldots n+t+0\cdot 5adn\ldots\ldots n+t}$$

in which t is the length of the interval of age, for which the returns have registered the annual deaths $dn\ldots\ldots n+t$ (*i.e.*, the annual deaths occurring in the population comprised between the ages n and $n+t$), coincides with my formula for each age, $\frac{\mathrm{P}+da}{\mathrm{D}}$, which gives the population increased by the deaths which occurred before the date of enumeration, or the total number exposed to the risk of death divided by the deaths.

What are all the facts we should observe to ascertain the rate of mortality of a population during one or more years? A distinct answer to this question will also solve the cognate problem of the annual proportion between the deaths and the population and of the construction of tables of mortality. This construction in its simplest form consists in the investigation of the proportion that exists for each year or for each period of life between the numbers exposed to the risk of dying and the deaths, with the view of finding for each age the value of the expectation of life ("vie moyenne"), the chances of living or dying in a year, and the number of years which there is an even chance of living to or over, *i.e.*, the probable lifetime ("vie probable" or "âge médian").

The examination of the changes that take place in a population shows that the original number existing at any instant is continually changing with the lapse of time, by the exit of present members and the addition of new ones. Whether it is the population of a country, a town, a prison, a hospital, or any other establishment of living persons, matters little. In the population of a country the entries are of two kinds, the births, which are the product of the vital force of the population, and the immigrations

* Bordeaux, 1866, and "*Journal de la Société de Statistique de Paris*," March, 1866.

or increase by the settlement of strangers. It is the same with the exits: persons are withdrawn from the population under consideration, either by death or by emigration, which removes them to another population. The entries of both kinds tend to increase, and the exits to diminish, the population. In order to compare the deaths with the population, and especially to ascertain the exact number of persons exposed to the risk of death during a period of a certain number of days or of years, the correct appreciation of the influence of each element upon the population is of the highest importance. In France Dr. Bertillon,* and in Germany Dr. G. H. Knapp† of Leipzig, have quite recently given great attention to investigations of this nature.

With the view of simplifying the solution of this question, let us take a year as the period of observation, and consider the changes that take place in the population from all causes during that year. We start with the survivors from the preceding year, or the persons alive on the first of January of the year in question. Each of those persons, being exposed to the risk of death from the very beginning of the year, counts as unity among the number exposed to the risk of death. They are in fact at risk during the whole 365 or 366 days of the year. Each of the persons born in the year, and in general, each of the persons who come under observation for the first time in the course of the year, counts only as a fraction, greater or less according as he comes under observation sooner or later. While the persons born in January and the immigrants in January, *i.e.*, at the middle of January on the average, count each for eleven months and a half or for the fraction $\frac{23}{24}$, the persons born and the immigrants in December, *i.e.*, at the middle of December on the average, having been under observation on the average only for half a month, are each reckoned as $\frac{1}{24}$. To ascertain, therefore, the number exposed to the risk of death during the year, we must add to the number living on the 1st of January the number of the births and of the immigrants diminished by a fraction equal to that part of the year during which they have not been under observation. If the number of births and immigrants is the same in each month of the year, every new born child and every immigrant will be on an average exposed to the risk of death during half a year; whence it follows that in order to obtain the number exposed to the risk of death during the whole year, we must add to the number under obser-

* See, besides the memoir already referred to, his articles in the Journal of the Statistical Society of Paris, February and March, 1869.

† Ueber die Ermittlung der Sterblichkeit. Leipzig, 1868.

vation on the 1st of January one-half the number of the births and of the immigrants.

The same process may be applied for the deaths and for the emigrants. The persons who have died or been otherwise withdrawn from observation in January, *i.e.*, at the middle of January on the average, have been each exposed to the risk of death only during half a month or $\frac{1}{24}$, while those who die or withdraw in December, *i.e.*, on the average at the middle of December, have been each exposed to the same risk during eleven months and a half, or during $\frac{23}{24}$ of the year.

It is easy to show that we get the same result, or may ascertain the number exposed to the risk of death, by adding the exits, including deaths and emigrants, to the number living at the end of the year.

From the population on the first day of the year we obtain the population on the last day by subtracting the emigrants from the immigrants and the deaths from the births, and by adding the differences, in case the births and the immigrants are in excess, to the population in existence on the first day of the year, subtracting them if the case be reversed. The sum of the differences, positive or negative, of each of these two elements, will give the difference between the number of the population at the beginning and that at the end of the year. Let p be the population at the beginning, p' that at the end of the year, B the births, D the deaths, I the immigrants, and E the emigrants: we shall have

$$p' = p + (B+I) - (D+E), \text{ or}$$
$$p = p' + (D+E) - (B+I)$$
$$\therefore \quad p + (B+I) = p' + (D+E).$$

In this equation $(B+I)$ represents the addition to the population p, and $(D+E)$ the deduction from the same during the year. If the number of the deaths and emigrations has been equal in each month throughout the year, each of these elements will have been under observation on the average half a year, and we shall have, for the mean number exposed to the risk of death during the whole year, on one side $p + \frac{1}{2}(B+I)$, and on the other $p' + \frac{1}{2}(D+E)$. The influence of these two elements on the mortality is neglected if we compare the annual number of deaths during the series of days which make up the year, either with the population on a fixed date (say the 31st of December) or with half the sum of the two populations on the first and on the last day of

the year. We in fact obtain a mortality in excess of the truth by one-half percent on the average.

What we have just proved in connection with the annual changes of any population is applicable by analogy to tables of mortality, which may be regarded as the symbol of the changes of the population at each age during the period of time or during the years from which we have deduced the elements of construction. I think this demonstration establishes the formula employed by me, $\frac{P + da}{D}$, which expresses that to the population of any age on a fixed day is added half the number of deaths at that age which have not been included in such population, and that the sum is to be divided by half the number of deaths.

Although there is but little probability that we shall agree as to the true method of constructing tables of mortality, we may nevertheless recommend to the Congress that certain essential principles be embodied in the following resolutions, viz.:—

1. A thorough investigation of the identity of the elements to be compared is indispensable for a just appreciation of the elements required for the construction of tables of mortality.

2. The deaths at each age, representing in the table the deaths which have occurred at each age during the whole year or during the whole series of days which make up the year, should be compared with the total number exposed to the risk of death for each age respectively during the whole series of days which make up the year, and not with a population enumerated as of that age on a fixed day.

These propositions lead us to the work of M. Knapp. He complains justly that the identity of the elements to be compared is not observed in calculating the deaths following the births. While the births are registered according to the year and calendar month in which they have occurred, the deaths are registered according to the age at the moment of death, *i.e.* the months or years which they may have lived. M. Knapp recommends that this anomaly should be put an end to, and I think that this recommendation is worthy of consideration, and shall myself be prepared to support it.

In those countries which give the mortality during the first two years of life for every month, or for every two or three months, such as England, Belgium, France, Italy, the Netherlands, Sweden, &c., the adoption of M. Knapp's proposition is very easy. We have only to divide each column in two, one for the registration

of the dead who were born in the preceding year, the other for the dead who were born in the following year. Let us take for example the deaths in the month of January, 1869, at the age 0–1 month. These were born partly in 1868, and partly in 1869. By dividing the column into two we obtain separately the deaths referring to the births of December, 1868, and those referring to the births of 1869. As the children dying at the age 0–1 month in February were all born in 1869, the column must not be divided for the month of February as to the deaths at that age. The division of the column begins with the children dying at the age 1–2 months, of whom some were born in December, 1868, others in January, 1869. For the same reason in March the division will begin with the deaths at the age 2–3 months, in April with the deaths at the age 3–4 months, and so on, progressing successively by a month of existence for the other months of the year.

I propose therefore that the Congress should express its opinion upon the necessity of recording in the Registers of Deaths, not only the age, but the year of birth of the dying.* This is the only practical method of obtaining the deaths of each generation, at least for children of tender age.

Generally, the relative value of a table of mortality, constructed for a whole population, depends upon the value of the three elements of construction—the births, the population, and the deaths at each age. It is important therefore to provide means for making these data as complete as possible, and submitting them to continuous supervision. Disregarding the disturbing element of the registration of still-born children, we possess such means in respect of the births and deaths in the Public Registers kept by the authority of the Government, any omission in which renders both the registrar and the person who ought to make the declaration liable to punishment.

The third element, or the mean population at each age, presents the greatest difficulty. To ascertain this population we make use of two or more periodical enumerations, adding together the living at each age, and then dividing the total by the number of enumerations which have been previously added together.

Besides the omissions which must occur in all enumerations, the period at which the census is taken gives to these mean numbers of the population a doubtful and relative value. A census may

* See G. Hopf, Ueber die allgemeine Natur des Geburts u. des Sterblichkeits-Verhältnisse, pp. 32 *et seq.* Berlin, 1869.

have been taken after or during an epidemic which carries off by preference persons of a particular age, and leaves blanks at that particular age in the population enumerated. We have had experience on this point in the Netherlands at the time of the census taken towards the end of the year 1849, immediately after the occurrence of cholera in 1848 and 1849. Let us consider the influence produced on the mortality of children, and consequently on the number enumerated, by infantile diseases, such as smallpox, measles, scarlatina, croup, whooping-cough, &c., when the census is taken after or during one of these epidemics. We must therefore exercise the greatest circumspection in determining the average population by means of the data of the census, and we must carefully study and examine all those disturbing elements that may have exercised more or less influence on the results.

In the last place, we shall have to give our opinion as to the necessity of constructing tables of mortality for each sex and according to civil condition ("état civil"), of distinguishing the mortality of crowded populations from those thinly scattered, and of observing the influence of climate and of soil upon the mortality at every age.

Tables for each sex have been made in several countries; tables according to the civil condition have been constructed only in the Netherlands, from the data contained in the two last censuses of the population and the decennial Registers of Deaths (1850–1859). With tables of population and tables of deaths registered according to age and civil condition, the construction of such tables of mortality presents no difficulty.* The mortality of the married population, such at least is the experience in the Netherlands, shows great differences, when compared with that of the unmarried part of the population. The differences are the greatest in the case of women at the ages 20 to 40, and particularly 20 to 30 years, periods which are critical in the lives of married women.

Tables of mortality for large towns or for large centres of population have been constructed in several countries. England has its separate tables for healthy and unhealthy districts by Dr. Farr, the Netherlands have separate tables for the four provinces which from their damp and marshy lands exhibit the highest mortality, and also for the seven other provinces.

* See L'annuaire de Statistique des Pays-Bas, Years xiv. and xv., First Part, p. 474, and Journal des Economistes, July, 1868, pp. 38 *et seq.*

Adjusted Table of Mortality.—British Peerage, Females.

WE have received from Dr. T. M. Thiele of Copenhagen the numbers shown in column 5 of the subjoined table, which he states he has calculated by a single continuous formula for the whole of life, from the observations of Messrs. Bailey and Day on the Females of the British Peerage. For the purpose of comparison, that is to say, in order to show how nearly Dr. Thiele's results agree with the original, we have added in columns 3 and 2 the numbers surviving at each age according to the adjusted table and according to the original observations, the radix being so chosen that the numbers are equal at the age 5, when the mortality is a minimum. It will be noticed that up to about the age of 50 the adjusted results agree very closely with the unadjusted; but above that age, the agreement is not so satisfactory.

Age.	Unadjusted.	As Adjusted by Dr. Thiele.				Age.	Unadjusted.	As Adjusted by Dr. Thiele.			
	No. Living.	No. Living.	Decrement.	Log. of No. Living.	Rate of Mortality.		No. Living.	No. Living.	Decrement.	Log. of No. Living.	Rate of Mortality.
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
0	10,011	10,000	597	4·00000	·05971	31	7,569	7,561	69	3·87856	·00903
1	9,415	9,403	154	3·97326	·01635	32	7,508	7,492	69	·87462	·00926
2	9,261	9,249	65	·96610	·00702	33	7,440	7,423	70	·87058	·00949
3	9,196	9,184	47	·96304	·00517	34	7,346	7,353	72	·86644	·00974
4	9,144	9,137	44	·96079	·00480	35	7,293	7,281	74	·86219	·01010
5	9,093	9,093	43	·95870	·00475	36	7,187	7,207	74	·85778	·01040
6	9,070	9,050	44	·95663	·00485	37	7,133	7,133	77	·85324	·01072
7	9,018	9,006	45	·95452	·00491	38	7,069	7,056	78	·84856	·01099
8	8,966	8,961	45	·95238	·00503	39	7,009	6,978	78	·84376	·01120
9	8,928	8,916	46	·95019	·00519	40	6,943	6,900	78	·83887	·01136
10	8,895	8,870	47	·94793	·00535	41	6,831	6,822	78	·83391	·01145
11	8,818	8,823	50	·94560	·00560	42	6,758	6,744	78	·82891	·01156
12	8,733	8,773	51	·94316	·00586	43	6,712	6,666	78	·82386	·01167
13	8,680	8,722	54	·94061	·00615	44	6,619	6,588	78	·81876	·01181
14	8,661	8,668	56	·93793	·00652	45	6,529	6,510	78	·81360	·01206
15	8,603	8,612	59	·93509	·00686	46	6,449	6,432	80	·80833	·01236
16	8,565	8,553	62	·93210	·00723	47	6,362	6,352	80	·80293	·01267
17	8,517	8,491	64	·92895	·00755	48	6,304	6,272	83	·79739	·01313
18	8,420	8,427	66	·92566	·00780	49	6,252	6,189	83	·79165	·01354
19	8,366	8,361	67	·92226	·00798	50	6,166	6,106	86	·78573	·01406
20	8,278	8,294	67	·91878	·00809	51	6,057	6,020	88	·77958	·01465
21	8,219	8,227	67	·91525	·00814	52	5,988	5,932	92	·77317	·01542
22	8,160	8,160	66	·91170	·00816	53	5,953	5,840	95	·76642	·01624
23	8,105	8,094	67	·90814	·00819	54	5,874	5,745	99	·75931	·01723
24	8,026	8,027	66	·90457	·00823	55	5,756	5,646	104	·75176	·01839
25	7,955	7,961	66	·90098	·00828	56	5,628	5,542	108	·74370	·01961
26	7,885	7,895	66	·89737	·00836	57	5,474	5,434	114	·73510	·02092
27	7,829	7,829	66	·89372	·00851	58	5,395	5,320	118	·72592	·02229
28	7,763	7,763	67	·89001	·00860	59	5,273	5,202	124	·71613	·02371
29	7,672	7,696	67	·88626	·00876	60	5,189	5,078	128	·70571	·02517
30	7,616	7,629	68	·88244	·00889	61	5,059	4,950	132	·69464	·02678

Adjusted Table of Mortality—(continued).

Age.	Unadjusted.	As Adjusted by Dr. Thiele.				Age.	Unadjusted.	As Adjusted by Dr. Thiele.			
	No. Living.	No. Living.	Decrement.	Log. of No. Living.	Rate of Mortality.		No. Living.	No. Living.	Decrement.	Log. of No. Living.	Rate of Mortality.
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
62	4,900	4,818	138	3·68285	·02866	82	1,486	1,477	150	3·16949	·10199
63	4,737	4,680	144	·67022	·03076	83	1,297	1,327	143	·12277	·10782
64	4,634	4,536	150	·65665	·03310	84	1,176	1,184	136	·07322	·11446
65	4,444	4,386	157	·64203	·03575	85	1,114	1,048	128	·02043	·12225
66	4,230	4,229	163	·62622	·03856	86	876	920	121	2·96380	·13158
67	4,100	4,066	169	·60914	·04144	87	808	799	114	·90253	·14263
68	3,976	3,897	173	·59076	·04434	88	736	685	107	·83570	·15569
69	3,784	3,724	175	·57106	·04712	89	634	578	98	·76220	·17049
70	3,630	3,549	177	·55010	·04983	90	557	480	90	·68102	·18690
71	3,480	3,372	177	·52790	·05250	91	448	390	80	·59116	·20490
72	3,431	3,195	177	·50448	·05533	92	390	310	69	·49158	·22390
73	3,138	3,018	176	·47976	·05839	93	325	241	59	·38150	·24362
74	2,970	2,842	176	·45363	·06185	94	289	182	48	·26024	·26420
75	2,705	2,666	175	·42590	·06589	95	212	134	38	·12700	·28498
76	2,568	2,491	176	·39630	·07041	96	170	96	30	1·98132	·30598
77	2,427	2,315	174	·36459	·07530	97	113	66	21	·82269	·32626
78	2,194	2,141	173	·33059	·08057	98	57	45	16	·65118	·34583
79	2,073	1,968	169	·29411	·08590	99		29		·46687	
80	1,933	1,799	164	·25510	·09118						
81	1,716	1,635	158	·21358	·09654						

A Budget of Paradoxes. *By* Professor De Morgan.

(Continued from vol. xiv., page 121.)

No. XXVI. 1859.

The problem of squaring the circle solved; or, the circumference and area of the circle discovered. By James Smith. London, 1859, 8vo.

On the relations of a square inscribed in a circle. Read at the British Association, Sept. 1859, published in the Liverpool Courier, Oct. 8, 1859, and reprinted in broadsheet.

The question: Are there any commensurable relations between a circle and other Geometrical figures? Answered by a member of the British Association. . . London, 1860, 8vo.—[This has been translated into French by M. Armand Grange, Bordeaux, 1863, 8vo.]

The Quadrature of the Circle. Correspondence between an eminent mathematician and James Smith, Esq. (Member of the Mersey Docks and Harbour Board). London, 1861, 8vo. (pp. 200).

Letter to the British Association. . . . by James Smith, Esq. Liverpool, 1861, 8vo.

Letter to the British Association. . . . by James Smith, Esq. Liverpool, 1862, 8vo.—[These letters the author promised to continue.]

A Nut to crack for the readers of Professor De Morgan's 'Budget of Paradoxes.' By James Smith, Esq. Liverpool, 1863, 8vo.

Paper read at the Liverpool Literary and Philosophical Society, reported

in the Liverpool Daily Courier, Jan. 26, 1864. Reprinted as a pamphlet.

The Quadrature of the circle, or the true ratio between the diameter and circumference geometrically and mathematically demonstrated. By James Smith, Esq. Liverpool, 1865, 8vo.

Mr. James Smith will, I have no doubt, be the most uneclipsed circle-squarer of our day. He will not owe this distinction to his being an influential and respected member of the commercial world of Liverpool, even though the power of publishing which his means give him should induce him to issue a whole library upon one paradox. Neither will he owe it to the pains taken with him by a mathematician, who corresponded with him until the joint letters filled an octavo volume. Neither will he owe it to the notice taken of him by Sir William Hamilton, of Dublin, who refuted him in a manner intelligible to an ordinary student of Euclid, which refutation he calls a remarkable paradox easily explainable, but without explaining it. What he will owe it to I proceed to show.

Until the publication of the 'Nut to Crack' Mr. James Smith stood among circle-squarers in general. I might have treated him with ridicule, as I have done others: and he says that he does not doubt he shall come in for his share at the tail end of my budget. But I can make a better job of him than so, as Locke would have phrased it: he is such a very striking example of something I have said on the use of logic that I prefer to make an example of his writings. On one point indeed he well deserves the *scutica,* if not the *horribile flagellum.* He tells me that he will bring his solution to me in such a form as shall compel me to admit it as *un fait accompli* [*une faute accomplie?*] or leave myself open to the humiliating charge of mathematical ignorance and folly. He has also honoured me with some private letters. In the first of these he gives me a "piece of information," after which he cannot imagine that I, "as an honest mathematician," can possibly have the slightest hesitation in admitting his solution. There is a tolerable reservoir of modest assurance in a man who writes to a perfect stranger with what he takes for an argument, and gives an oblique threat of imputation of dishonesty in case the argument be not admitted without hesitation; not to speak of the minor charges of ignorance and folly. All this is blind self-confidence, without mixture of malicious meaning; and I rather like it: it makes me understand how Sam Johnson came to say of his old friend Mrs. Cobb,—"I love Moll Cobb for her impudence." I have now done with my friend's *suaviter in modo,* and proceed to his *fortiter in re:* I shall show that he *has* convicted himself of igno-

rance and folly, with an honesty and candour worthy of a better value of π.

Mr. Smith's method of proving that every circle is $3\frac{1}{8}$ diameters is to assume that it is so,—"if you dislike the term datum, then, by hypothesis, let 8 circumferences of a circle be exactly equal to 25 diameters,"—and then to show that every other supposition is thereby made absurd. The right to this assumption is enforced in the 'Nut' by the following analogy:—

> "I think you (!) will not dare (?) to dispute my right to this hypothesis, when I can prove by means of it that every other value of π will lead to the grossest absurdities; unless indeed, you are prepared to dispute the right of Euclid to adopt a false line hypothetically for the purpose of a "*reductio ad absurdum*" demonstration, in pure geometry.

Euclid assumes what he wants to *disprove,* and shows that his *assumption* leads to absurdity, and so *upsets itself.* Mr. Smith assumes what he wants to *prove,* and shows that *his* assumption makes *other propositions* lead to absurdity. This is enough for all who can reason. Mr. James Smith cannot be argued with; he has the whip-hand of all the thinkers in the world. Montucla would have said of Mr. Smith what he said of the gentleman who squared his cirle by giving 50 and 49 the same square root, *Il a perdu le droit d'être frappé de l'évidence.*

It is Mr. Smith's habit, when he finds a conclusion agreeing with its own assumption, to regard that agreement as proof of the assumption. The following is the "piece of information" which will settle me, if I be honest. Assuming π to be $3\frac{1}{8}$, he finds out by working instance after instance that the mean proportial between one-fifth of the area and one-fifth of eight is the radius. That is,

$$\text{if } \pi=\frac{25}{8}, \sqrt{\left(\frac{\pi r^2}{5}\cdot\frac{8}{5}\right)}=r.$$

This "remarkable general principle" may fail to establish Mr. Smith's quadrature, even in an honest mind, if that mind should happen to know that, a and b being any two numbers whatever, we need only assume—

$$\pi=\frac{a^2}{b}, \text{ to get at } \sqrt{\left(\frac{\pi r^2}{a}\cdot\frac{b}{a}\right)}=r.$$

We naturally ask what sort of glimmer can Mr. Smith have of the subject which he professes to treat? On this point he has given satisfactory information. I had mentioned the old problem of finding two mean proportionals, as a preliminary to the duplication of the cube. On this mention Mr. Smith writes as follows. I put

a few words in capitals; and I write *rq* for the sign of the square root, which embarrasses small type :—

"This establishes the following *infallible* rule, for finding two mean proportionals OF EQUAL VALUE, and is more than a preliminary, to the famous old problem of 'Squaring the circle.' Let any finite number, say 20, and its fourth part $=\frac{1}{4}(20)=5$, be given numbers. Then $rq(20\times 5)=rq100=10$, is their mean proportional. Let this be a given mean proportional TO FIND ANOTHER MEAN PROPORTIONAL OF EQUAL VALUE. Then $20\times\frac{\pi}{4}=20\times\frac{3{\cdot}125}{4}=20\times.78125=15{\cdot}625$ will be the first number; as $25:16::rq20:rq8{\cdot}192$: and $(rq8{\cdot}192)^2\times\frac{\pi}{4}=8{\cdot}192\times{\cdot}78125=6{\cdot}4$ will be the second number; therefore $rq(15{\cdot}625\times 6{\cdot}4)=rq100=10$, is the required mean proportional.... Now, my good Sir, however competent you may be to prove every man a fool [not *every* man, Mr. Smith! only *some;* pray learn logical quantification] who now thinks, or in times gone by has thought, the 'Squaring of the circle' *a possibility;* I doubt, and, upon the evidence afforded by your Budget, I cannot help doubting, whether you were ever before competent to find two mean proportionals *by my unique method.*"—(*Nut*, pp. 47, 48.) [That I never was, I solemnly declare!]

All readers can be made to see the following exposure. When 5 and 20 are given, x is a mean proportional when in 5, x, 20, 5 is to x as x to 20. And x must be 10. But x and y are *two* mean proportionals when in 5, x, y, 20, x is a mean proportional between 5 and y, and y is a mean proportional between x and 20. And these means are $x=5\sqrt[3]{4}$, $y=5\sqrt[3]{16}$. But Mr. Smith finds *one* mean, finds it *again* in a roundabout way, and produces 10 and 10 as the two (equal!) means, in solution of the "famous old problem." This is enough: if more were wanted, there is more where this came from. Let it not be forgotten that Mr. Smith has found a translator abroad, two, perhaps three, followers at home, and—most surprising of all—a real mathematician to try to set him right. And this mathematician did not discover the character of the subsoil of the land he was trying to cultivate, until a goodly octavo volume of letters had passed and repassed. I have noticed, in more quarters than one, an apparent want of perception of the *full* amount of Mr. Smith's ignorance: persons who have not been in contact with the non-geometrical circle-squarers have a kind of doubt as to whether anybody can carry things so far. But I am an "old bird" as Mr. Smith himself calls me; a Simorg, an "all-knowing Bird of Ages" in matters of cyclometry.

The curious phenomena of thought here exhibited illustrate, as above said, a remark I have long ago made on the effect of proper study of logic. Most persons reason well enough on matter to

which they are accustomed, and in terms with which they are familiar. But in unaccustomed matter, and with use of strange terms, few except those who are practised in the abstractions of pure logic can be tolerably sure to keep their feet. And one of the reasons is easily stated: terms which are not quite familiar partake of the vagueness of the X and Y on which the student of logic learns to see the formal force of a proposition independently of its material elements.

I make the following quotation from my fourth paper on logic in the *Cambridge Transactions*:—

"The uncultivated reason proceeds by a process almost entirely material. Though the necessary law of thought must determine the conclusion of the ploughboy as much as that of Aristotle himself, the ploughboy's conclusion will only be tolerably sure when the matter of it is such as comes within his usual cognizance. He knows that geese being all birds does not make all birds geese, but mainly because there are ducks, chickens, partridges, &c. A beginner in geometry, when asked what follows from 'Every A is B,' answers 'Every B is A.' That is, the necessary laws of thought, except in minds which have examined their tools, are not very sure to work correct conclusions except upon familiar matter....As the cultivation of the individual increases, the laws of thought which are of most usual application are applied to familiar matter with tolerable safety. But difficulty and risk of error make a new appearance with a new subject; and this, in most cases, until new subjects are familiar things, unusual matter common, untried nomenclature habitual; that is, until it is a habit to be occupied upon a novelty. It is observed that many persons reason well in some things and badly in others; and this is attributed to the consequence of employing the mind too much upon one or another subject. But those who know the truth of the preceding remarks will not have far to seek for what is often, perhaps most often, the true reason....I maintain that logic tends to make the power of reason over the unusual and unfamiliar more nearly equal to the power over the usual and familiar than it would otherwise be. The second is increased; but the first is almost created."

Mr. James Smith, by bringing ignorance, folly, and dishonesty into contact with my name, in the way of conditional insinuation, has done me a good turn: he has given me right to a freedom of personal remark which I might have declined to take in the case of a person who is useful and respected in matters which he understands. He is a glaring instance of the truth of the observations quoted above. I will answer for it that, at the Mersey Dock Board, he never dreams of proving that the balance at the banker's is larger than that in the book by assuming that the larger sum is there, and then proving that the other supposition—the smaller balance—is, upon that assumption, an absurdity. He never says to another director, How can you dare to refuse me a

right to assume the larger balance, when you yourself, the other day, said, Suppose, for argument's sake, we had 80,000*l.* at the banker's, though you knew the book only showed 30,000*l.*? This is the way in which he has supported his geometrical paradox by Euclid's example: and this is not the way he reasons at the board; I know it by the character of him as a man of business which has reached my ears from several quarters. But in geometry and rational arithmetic he is a smatterer, though expert at computation; at the board he is a trained man of business. The language of geometry is so new to him that he does not know what is meant by "two mean proportionals": but all the phrases of commerce are rooted in his mind. He is most unerasably booked in the history of the squaring of the circle, as the speculator who took a right to assume a proposition for the destruction of other propositions, on the express ground that Euclid assumes a proposition to show that it destroys itself: which is as if the curate should demand permission to throttle the squire because St. Patrick drove the vermin to suicide to save themselves from slaughter. He is conspicuous as the speculator who, more visibly than almost any other known to history, reasoned in a circle by way of reasoning on a circle. But what I have chiefly to do with is the force of instance which he has lent to my assertion that men who have not had real training in pure logic are unsafe reasoners in matter which is not familiar. It is hard to get first-rate examples of this, because there are few who find the way to the printer until practice and reflection have given security against the grossest slips. I cannot but think that his case will lead many to take what I have said into consideration, among those who are competent to think of the great mental disciplines. To this end I should desire him to continue his efforts, to amplify and develope his great principle, that of proving a proposition by assuming it and taking as confirmation every consequence that does not contradict the assumption.

Since my Budget commenced, Mr. Smith has written me notes: the portion which I have preserved—I suppose several have been mislaid—makes a hundred and seven pages of note paper, closely written. To all this I have not answered one word: but I think I cannot have read fewer than forty pages. In the last letter the writer informs me that he will not write at greater length until I have given him an answer, according to the "rules of good society." Did I not know that for every inch I wrote back he would return an ell? Surely in vain the net is spread in the eyes of anything that hath a wing. There were several good excuses

for not writing to Mr. J. Smith: I will mention five. First, I distinctly announced at the beginning of this Budget that I would not communicate with squarers of the circle. Secondly, any answer I might choose to give might with perfect propriety be reserved for this article; had the imputation of incivility been made after the first note, I should immediately have replied to this effect: but I presumed it was quite understood. Thirdly, Mr. Smith, by his publication of E. M.'s letters against the wish of the writer, had put himself out of the pale of correspondence. Fourthly, he had also gone beyond the rules of good society in sending letter after letter to a person who had shown by his silence an intention to avoid correspondence. Fifthly, these same rules of good society are contrived to be flexible or frangible in extreme cases: otherwise there would be no living under them; and good society would be bad. Father Aldrovand has laid down the necessary distinction—"I tell thee, thou foolish Fleming, the text speaketh but of promises made unto Christians, and there is in the rubric a special exemption of such as are made to Welchmen." There is also a rubric to the rules of good society; and squarers of the circle are among those whom there is special permission not to answer: they are the wild Welchmen of geometry, who are always assailing, but never taking, the Garde Douloureuse of the circle. "At this commentary," proceeds the story, "the Fleming grinned so broadly as to show his whole case of broad strong white teeth." I know not whether the Welchman would have done the like, but I hope Mr. James Smith will; and I hope he has as good a case to show as Wilkin Flammock. For I wish him long life and long health, and should be very glad to see so much energy employed in a productive way. I hope he wishes me the same: if not, I will give him what all his judicious friends will think a good reason for doing so. His pamphlets and letters are all tied up together, and will form a curious lot when death or cessation of power to forage among bookshelves shall bring my little library to the hammer. And this time may not be far off: for I was X years old in A.D. X^2; not 4 in A.D. 16, nor 5 in A.D. 25, but still in one case under that law. And now I have made my own age a problem of quadrature, and Mr. J. Smith may solve it. But I protest against his method of assuming a result, and making itself prove itself: he might in this way, as sure as eggs is eggs (a corruption of X is X), make me 1,864 years old, which is a great deal too much.

April 5, 1864.—Mr. Smith continues to write me long letters, to which he hints that I am to answer. In his last, of 31 closely written sides of note-paper, he informs me, with reference to my

obstinate silence, that though I think myself and am thought by others to be a mathematical Goliath, I have resolved to play the mathematical snail, and keep within my shell. A mathematical *snail!* This cannot be the thing so called which regulates the striking of a clock; for it would mean that I am to make Mr. Smith sound the true time of day, which I would by no means undertake upon a clock that gains 19 seconds odd in every hour by false quadrature. But he ventures to tell me that pebbles from the sling of simple truth and common sense will ultimately crack my shell, and put me *hors de combat.* The confusion of images is amusing: Goliath turning himself into a snail to avoid $\pi = 3\frac{1}{8}$ and James Smith, Esq., of the Mersey Dock Board; and put *hors de combat*—which should have been *cache*—by pebbles from a sling. If Goliath had crept into a snail-shell, David would have cracked the Philistine with his foot. There is something like modesty in the implication that the crack-shell pebble has not yet taken effect; it might have been thought that the slinger would by this time have been singing—

> And thrice [and one-eighth] I routed all my foes,
> And thrice [and one-eighth] I slew the slain.

But he promises to give the public his nut-cracker if I do not, before the Budget is concluded, "unravel" the paradox, which is the mathematico-geometrical nut he has given me to crack. Mr. Smith is a crack man: he will crack his own nut; he will crack my shell; in the mean time he cracks himself up. Heaven send he do not crack himself into lateral contiguity with himself.

HOME AND FOREIGN INTELLIGENCE.

UNIVERSITY LIFE ASSURANCE SOCIETY.

Established 1825.

QUINQUENNIAL STATEMENT 1870.

The Directors of the University Life Assurance Society have the honour to submit to the Annual General Court their report of the progress made by the Society during the past Five Years, and the result of the Quinquennial Valuation.

	Past Five Years.			Previous Five Years.	
Policies were issued for	£511,150..		as compared with..	£264,650.	
Claims by death were	179,986..		" " ..	210,175.	
The additions paid on Policies becoming Claims	76,086..		" " ..	70,955.	
Received from Premiums......	242,295	£432,442	as compared with	220,528.	£382,585
Ditto Interest and Dividends	190,147			162,057.	
The Establishment expenses including Pensions	27,918		Expenses excluding Pensions	27,856.	

The total sum invested in different securities, with the balances in the hands of the Society's Bankers, amounted, 1st May, 1870, to **£961,385**, as compared with **£858,632**, 1st May, 1865.

The average rate of Interest in the five years was **£4 10s. 10d.**, as compared with **£4 0s. 0d.** in the previous five years.

The estimated amount of Claims for the Five Years on Policies, irrespective of Additions, was **£340,000**, the actual Claims being, as stated above, **£179,986**, showing a decrease on the estimate of **£160,000.**

Average Age at which Policies have become Claims during the past Five Years has been 66 years, as compared with the average of 64 in the previous Five Years.

Average Age of present existing lives upon an amount of **£1,745,400** assured is 53¾ years, as compared with an average Age of 57 years upon the amount of **£1,505,365**, assured in 1865.

The Assets in the Balance Sheet are taken at the market value on the 3rd May last.

The Valuation of each Policy and addition is made separately.

The reserved value of the Policies is calculated upon the Office Tables (Interest 3 per cent.).

The reserved value of the additions is calculated upon the Office Tables (Interest also at 3 per cent.).

The Balance shows a surplus of **£166,619 19s. 5d.** applicable to division, as compared with a surplus of **£94,697** in 1865.

The surplus of **£166,619** may be divided into three parts; the sum of **£131,119** being the result of what may be termed the ordinary operations of the Society during the Five Years, reckoning the usual rate of investment at 4 per cent. The sum of **£22,500**, due to the increase of **10s. 10d.** in the average rate of Interest, and the sum of **£13,000** from the present market value of the Society's Investments in excess of their cost.

The Directors have referred these calculations to the opinion of Mr. Ansell. In his Report he says—

"I have carefully examined the principles and Table on which the outstanding liabilities have been valued, because that is really the subject of paramount importance in determining the condition of a Life Assurance Fund. On this point I have great pleasure in reporting that, so far as my opinion and experience extend, the method adopted is perfectly sound and correct."

* * * * *

The Directors, after careful consideration of all circumstances, recommend, with Mr. Ansell's concurrence, that a reversionary bonus of 2½ per cent. per annum for the Five Years be added to the Policies in existence on the 1st May last, and entitled to participate. This will require a sum of **£123,788 6s. 5d.** The corresponding amount due by the Society's Bye Laws to the Shareholders, viz., one-tenth of profits divided (taking into account the intermediate additions of 1¼ per cent. to Policies which have become Claims since 1865) will require the sum of **£14,151 1s. 2d.** This will give a bonus of **£2 7s. 0d.** per share, exclusive of the interest of **5s.** per share, and there will remain in reserve **£28,680 11s. 10d.**, a sum very much larger than we have on former occasions kept undivided.

Those three sums make up the surplus of **£166,619 19s. 5d.**

The amount proposed to be held in reserve is, exclusive of the sum of **£29,900**, Shareholders' Capital, paid up and invested.

In accordance with the above recommendation, the Directors submit the following resolutions for approval:—

1st. That all Policies dated previous to 1st May, 1865, which have been effected for the whole period of life, shall become entitled to additions on the amount of the policy at the rate of **£2 10s.** per cent. per annum for the Five Years, payable at the decease of the party, or to a proportionate annuity by reduction of the Annual Premium.

2nd. That all Policies dated between the 30th April, 1865, and 1st May, 1870, which have been effected for the whole period of life, shall become entitled to additions on the amount of the Policy, at the rate of **£2 10s.** per cent. per annum for each year, payable at the decease of the party as soon as five years have been completed from the date of the respective Policies, and six payments shall have been made.

3rd. That a further addition of **£1 5s.** per cent. per annum shall be made on the termination of each successive year upon all Policies effected for the whole continuance of life, which shall become claims previous to the next Division of Profits in 1875, provided such Policies shall have existed Five Years, and six payments shall have been made.

4th. That a Bonus of **£2 7s.** per share be paid to the Shareholders at the same time as the Annual Dividend.

ACCOUNT OF ASSETS AND LIABILITIES ON MAY 1, 1870.

Assets.

	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
Amount of Property and Invested Funds taken as Valued on 1st May, 1870		970,767 12 4	
ance at Bankers and in the hands of Secretary ..		2,649 7 5	
		973,416 19 9	
mium and Interest due on 1st May, in course of payment		11,057 5 4	
pended Premiums on Policies, with Interest thereon, to 1st May		4,104 16 9	
		988,579 1 10	
luct—			
Shareholders' Capital and Interest	32,003 19 11		
Claims admitted, including additions	16,847 2 6		
Office Salaries, One Month	400 0 0		
		49,251 2 5	
			939,327 19 5

Liabilities.

	No. of Policies.	Amount Assured.	Annual Premiums.	Reserved Value.	
icies for the whole term f Life in Annual Preniums	1,404 ..	1,627,392 11 5 ..	50,594 4 8 ..	453,224 16 4	
icies for the whole term f Life; Premiums paid ip in full	79 ..	96,252 0 0 ..		68,789 13 0	
the whole term of Life n No. of Payments ..	6 ..	5,000 0 0 ..	249 11 4 ..	2,291 3 9	
icies on Joint Lives ..	9 ..	11,000 0 0 .	261 2 0 ..	1,147 16 11	
to for short terms ..	4 ..	5,400 0 0 ..	116 13 10 ..	58 14 0	
to Endowment ..	1 ..	400 0 0 ..	24 0 0 ..	133 18 5	
	1,503 ..	1,745,444 11 5 ..	51,245 11 10 ..	525,646 2 5	
lue of Outstanding Additions of £330,978 16*s.* 3*d.* on 1st May ..				240,353 12 5	
to of Annuities granted in lieu of Additions to Policies				6,708 5 2	
duct Total Liabilities					772,708 0
ference applicable to Quinquennial Division					166,619 19 5

TABLE showing the AMOUNT ASSURED with BONUSES ADDED THERETO on 1st May, 1870.

Present Age.	Total Amount Assured.			Additions to 1865.			Premiums Payable Annually during Life.			Reserved Value of Policies and Outstanding Additions.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
22 to 25	18,925	0	0				414	8	6	290	2	1
26 „ 30	92,100	0	0	101	10	0	2,156	16	6	2,636	1	7
31 „ 35	88,350	0	0	1,232	11	0	2,241	8	9	4,603	3	7
36 „ 40	129,500	0	0	3,573	3	6	3,422	9	6	10,976	3	4
41 „ 45	195,417	0	0	12,083	6	4	5,473	18	1	30,184	12	0
46 „ 50	173,049	0	0	21,340	11	9	4,818	3	11	46,958	0	2
51 „ 55	226,600	0	0	34,608	1	0	6,755	17	4	78,644	13	0
56 „ 60	183,648	0	0	35,219	7	10	5,973	14	5	81,081	18	2
61 „ 65	168,552	11	5	48,238	7	9	5,076	11	0	105,808	2	0
66 „ 70	162,624	0	0	54,043	15	0	4,787	0	11	124,900	8	10
71 „ 75	188,729	0	0	78,229	3	6	6,486	4	11	169,948	19	0
76 „ 80	60,850	0	0	25,682	13	6	1,901	2	9	63,256	13	7
81 „ 85	40,300	0	0	16,626	5	1	1,335	19	5	45,370	8	2
	1,728,644	11	5				50,843	16	0	764,659	5	6
Joint Life, Short Term, & Endowment Policies	16,800	0	0				401	15	10	1,340	9	4
	£1,745,444	11	5	£330,978	16	3	£51,245	11	10	£765,999	14	10

FUNDED AND OTHER PROPERTY.

Description of Stock.	Amount of Stock.			Cost of Stock.			Estimated Income.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
3 per Cent. Consols	20,421	15	5	18,665	6	2	612	13	0
New 3 per Cent. Annuities	32,246	19	9	30,000	0	0	967	8	2
New 5 per Cent. ditto	2,852	10	0	2,852	10	0	142	12	6
Bank Stock	18,173	0	0	37,226	18	0	1,500	0	0
East India Stock	2,000	0	0	4,292	9	4	128	15	6
Anglo-Greek Bonds	2,800	0	0	2,800	0	0	140	0	0
East-Indian Railway Stock	167,690	0	0	170,585	19	6	8,234	10	0
Ditto ditto Debentures	50,000	0	0	50,405	6	6	1,764	3	8
Ditto ditto Stock, B a/c	20,000	0	0	20,108	19	3	904	18	0
	£316,184	5	2	336,937	8	9	14,395	0	10
Amount of Mortgages, Leasehold Rent-Charges, Railway Debentures, &c.. .	..	..		438,677	1	1	19,874	3	4
Loans on Society's Policies with Personal Security	..	..		99,344	14	2	4,967	2	6
Loans on Society's Policies within their surrender value	..	..		75,812	0	0	3,790	12	0
				950,771	4	0	43,026	18	8
Balances in the hands of Messrs. Drummond and the Secretary	..	..		2,649	7	5			
Total on 1st May, 1870	..	..		£953,420	11	5	£43,026	18	8

N.B.—The sum of £5,664 18*s.* 4*d.* loss on Stock sold to provide the sum of £55,000 advanced on Mortgage will be replaced under provision of mortgage deed, dated 17th February, 1866, when the loan is repaid.

* * * * *

Return of the Total Sum invested in different Securities, with the Balances in the hands the Secretary and the Society's Bankers, at the under-mentioned periods.

	£	s.	d.		£	s.	d.		£	s.	d.
May, 1851 ..	640,756	4	7	1st May, 1858 ..	796,747	1	3	1st May, 1865 ..	858,632	14	11
„ 1852 ..	667,314	14	4	„ 1859 ..	806,183	14	4	„ 1866 ..	857,436	15	8
„ 1853 ..	716,484	16	2	„ 1860 ..	833,811	4	6	„ 1867 ..	885,858	5	4
„ 1854 ..	742,972	13	5	„ 1861 ..	809,405	12	3	„ 1868 ..	909,613	17	10
„ 1855 ..	765,424	2	3	„ 1862 ..	816,022	9	9	„ 1869 ..	940,583	14	7
„ 1856 ..	763,147	1	11	„ 1863 ..	819,449	19	0	„ 1870 ..	959,161	4	10
„ 1857 ..	768,623	3	5	„ 1864 ..	825,140	5	8				

The Annual Income from Premiums and Interest amounts to **£94,700**

The total sums invested in the various Securities exclusiveof the Proprietors' Capital of **£29,900**, amounts to.. .. .. **929,261**

E EXPECTATION OF LIFE FROM THE EXPERIENCE OF THE UNIVERSITY ASSURANCE SOCIETY,

COMPARED WITH OTHER TABLES. BY CHARLES McCABE.

ES.	University Experience.	Farr's Healthy Life Tables (Males).	Carlisle Table.	Equitable Experience.	Actuaries Table 17 Offices.	AGES.	University Experience	Farr's Healthy Life Tables (Males).	Carlisle Table.	Equitable Experience.	Actuaries Table 17 Offices.
4	44·61	40·62	38·59	38·84	38·68	61	15·43	14·41	13·82	13·35	13·18
5	43·81	39·93	37·86	38·12	37·98	62	14·81	13·78	13·31	12·79	12·61
6	43·02	39·23	37·14	37·41	37·27	63	14·17	13·17	12·81	12·23	12·05
7	42·22	38·54	36·41	36·70	36·56	64	13·55	12·58	12·30	11·68	11·51
8	41·42	37·84	35·69	35·98	35·86	65	12·94	12·00	11·79	11·13	10·97
9	40·60	37·15	35·00	35·26	35·15	66	12·33	11·44	11·27	10·61	10·46
0	39·77	36·45	34·34	34·53	34·43	67	11·74	10·90	10·75	10·11	9·96
1	38·96	35·74	33·68	33·81	33·72	68	11·19	10·37	10·23	9·62	9·47
2	38·13	35·04	33·03	33·08	33·01	69	10·60	9·86	9·70	9·15	9·00
3	37·30	34·33	32·36	32·36	32·30	70	10·08	9·37	9·18	8·70	8·54
4	36·44	33·62	31·68	31·65	31·58	71	9·57	8·89	8·65	8·26	8·10
5	35·60	32·90	31·00	30·93	30·87	72	9·07	8·43	8·16	7·83	7·67
6	34·74	32·18	30·32	30·22	30·15	73	8·60	7·98	7·72	7·41	7·26
7	33·88	31·46	29·64	29·50	29·44	74	8·14	7·56	7·33	7·00	6·86
8	33·03	30·74	28·96	28·79	28·72	75	7·71	7·15	7·01	6·61	6·48
9	32·19	30·02	28·28	28·09	28·00	76	7·31	6·76	6·69	6·24	6·11
0	31·35	29·29	27·61	27·40	27·28	77	6·92	6·39	6·40	5·86	5·76
1	30·51	28·56	26·97	26·69	26·56	78	6·54	6·03	6·12	5·49	5·42
2	29·68	27·84	26·34	25·99	25·84	79	6·17	5·69	5·80	5·12	5·09
3	28·84	27·11	25·71	25·29	25·12	80	5·81	5·37	5·51	4·75	4·78
4	28·00	26·38	25·09	24·58	24·40	81	5·46	5·07	5·21	4·41	
5	27·22	25·65	24·46	23·87	23·69	82	5·12	4·78	4·93	4·09	
6	26·38	24·92	23·82	23·17	22·97	83	4·73	4·51	4·65	3·79	
7	25·57	24·20	23·17	22·47	22·27	84	4·46	4·25	4·39	3·57	
8	24·77	23·47	22·50	21·77	21·56	85	4·15	4·01	4·12	3·39	
9	23·99	22·75	21·81	21·07	20·87	86	3·84	3·78	3·90	3·21	
0	23·21	22·03	21·11	20·36	20·18	87	3·53	3·56	3·71	3·03	
1	22·44	21·32	20·39	19·66	19·50	88	3·21	3·36	3·59	2·89	
2	21·69	20·61	19·68	18·98	18·82	89	2·89	3·17	3·47	2·80	
3	20·94	19·90	18·97	18·30	18·16	90	2·54	2·99	3·28	2·56	
4	20·22	19·19	18·28	17·64	17·50	91	2·16				
5	19·50	18·49	17·58	16·99	16·86	92	1·73				
6	18·81	17·79	16·89	16·34	16·22	93	1·20				
7	18·00	17·10	16·21	15·71	15·59	94					
8	17·43	16·41	15·55	15·09	14·97						
9	16·76	15·73	14·92	14·49	14·37						
0	16·11	15·06	14·34	13·91	13·77						

* * * * * *

In 1865 the Society divided £53,215 cash, or £87,500 Reversionary Bonus, for each £1,000,000 assured.

In 1870 the Society divided £70,978 cash, or £125,000 Reversionary Bonus, for each £1,000,000 assured.

These sums are exclusive of intermediate additions of £1¼ per cent.

THE ADDITIONS TO POLICIES FOR THE LAST FORTY YEARS HAVE BEEN AT THE RATE OF NEARLY 2 PER CENT. PER ANNUM.

Since the Establishment of the Society in 1825, the amount of Additions alloted to the Assured has exceeded £941,800.

The Reversionary Bonus added to the Policy may be converted into an Annuity to be deducted from the Premium, on the following scale:—

For each £100 of Reversionary Addition—

			£	s.	d.
At the Age	30,	the Annuity is	2	10	9
,,	35	do.	2	16	11
,,	40	do.	3	4	7
,,	45	do.	3	14	1
,,	50	do.	4	7	6

Assuming that the mortality and rate of interest will in future prove as favourable as during the last forty-four years, Annual Premiums may be entirely extinguished in the under-mentioned periods.

Assurances	effected at Age	25,	Premium	extinguished in	30	Years.
,,	,,	30	,,	,,	30	,,
,,	,,	35	,,	,,	29	,,
,,	,,	40	,,	,,	28	,,

UNIVERSAL LIFE ASSURANCE SOCIETY.

Established 1834.

THIRTY-SIXTH ANNUAL REPORT OF THE BOARD OF DIRECTORS.

* * * * *

2. The estimated *Surplus Funds*, beyond the sum of **£677,430 11s. 8d.** (the calculated Reserve Fund set apart to meet all present and future liabilities), amount to **£192,350 7s. 2d.**, as shown in the annexed Balance Sheet.

3. The Board propose to allow the holders of *all Policies six years in force and entitled to participate in Annual Bonuses*, a REDUCTION of Premium for the year 11th May, 1870, to 10th May, 1871, of **50 per cent.** The present will be the *seventh year*, continuously, in which the Reduction will have amounted to FIFTY PER CENT.

4. The sums allotted to each Policy on this, as on each of the thirty preceding yearly divisions of profit, are applicable either to the reduction of the next year's Annual Premiums, or to addition to the sum originally assured of Bonuses proportionately larger than such reduction, but only payable with the Policy when a claim arises, *according to the option exercised by each Policy-holder on the first occasion of participation in profits.*

5. Under the provisions of the Deed of Settlement, One-fifth of the estimated surplus Fund of **£192,350 7s. 2d.**, subject to such deductions as the Board may consider desirable, is divisible during the current year 11th May, 1870, to 10th May, 1871. And the amount of the Surplus

Funds proposed to be so appropriated this year is £37,000. Three-fourths of this sum are applicable to the cash payments in reduction of the Assureds' participating Premiums for the current year, or to the addition of corresponding Reversionary Bonuses to Policies; and the remaining One-fourth will provide for the Annual Bonus to Proprietors. The sum of £1,936 2s. 10d. is proposed to be deducted from the Policy-holders' or Assurance Fund Suspense Account, to make up the £29,686 2s. 10d. required for the year's Reduction of 50 per cent. in the Premiums on Policies now entitled to participate in annual Bonuses.

6. The Board recommend a *Dividend*, by way of Bonus to the Proprietors, of £1 17s. *per Share, payable on the* 30*th June next, in addition to the* 10s. *per Share for Interest guaranteed by the Deed of Settlement, and due on the* 31*st December next.* Both the Dividend and Interest are free from any deduction for Income-tax.

* * * * *

8. The Average New Premium Income of each of the last five years, 1865 to 1869 has been £15,863—as against £7,917, the average of each of the preceding five years, 1860 to 1864.

9. The total Number of Policies Issued from the date of the establishment of the Society to the close of the year 1869 was:—

In *England* ..	3,412	Policies for	£3,497,688 . 3 . 3
In *India* ..	6,619	,,	5,544,659 . 8 . 8
Together ..	10,031	,,	£9,042,347 . 11 . 11

10. The Total Amount of Net Premiums (after *deduction* of all the annual reductions, averaging nearly 50 per cent.) received in *England* from 1834 to 1869, was	£937,412 . 17 . 11
In *India*, the corresponding amount was	1,480,669 . 17 . 11
Making together	£2,418,082 . 15 . 10

11. When the books were closed for the present Annual Valuation the Assurances in force were—

1,551	Policies, paying *English* Premiums, for a Sum of ..	£1,827,849 . 18 . 8
1,655	Policies, paying *Indian* Premiums, for a Sum of ..	1,274,798 . 7 . 0
Total, 3,206	Policies, for a Sum of ..	£3,102,648 . 5 . 8

* * * * *

13. The amount of Claims paid from the establishment of the Society is:—

In *England*,	711	Policies	560	Lives for	£709,271 . 11 . 10
In *India*,	1,122	,,	865	,,	1,084,129 . 0 . 0
Total	1,833	Policies	1,425	Lives for the Sum of	£1,793,400 . 11 . 10

[From the Statement of Receipts and Expenditure, it appears that the

gross premiums received in the year were £127,020, the interest on investments £39,206, and the total expenses of management, including commission, £13,518.]

36th ANNUAL BALANCE SHEET OR VALUATION
OF THE
LIABILITIES AND ASSETS OF THE UNIVERSAL LIFE ASSURANCE SOCIETY,
To 31*st December*, 1869, *in England, and to* 30*th November*, 1869, *in India.*

Dr.

		Total Liabilities. £	s.	d.
Proprietors' Fund (Capital paid up, being Ten per cent. on Half a Million Sterling fully subscribed)		50,000	0	0
Amount of calculated reserve to meet, *with future premiums*, the LIABILITIES under *English* Policies for £1,827,850, and under *Indian* Policies for £1,274,798 (after *deduction* of proportionate reserve for £167,100 *re-assured* in other Offices on *English* and £13,000 on *Indian* Policies)—				
In England	£420,108 0 0			
In India	144,445 0 0			
		564,553	0	0
Annuity Reserve		1,542	0	0
Claims under Life Policies in course of settlement		41,460	4	0
Reserve for Reductions of Premium on Policies renewable between the dates of this valuation and 11th May, 1870		13,114	15	8
Proprietors' Dividends unpaid on 31st December, 1869		1,821	4	0
Assurance Fund Suspense Account		4,064	8	0
Proprietors' Suspense Account		875	0	0
Total Liabilities		677,430	11	8
Balance or Surplus Reserve Fund		192,350	7	2
		£869,780	18	10

Cr.

	Total Assets. £	s.	d.
British, Colonial and Indian Government Securities	334,890	15	10
Mortgages of Town and District Rates	83,475	11	11
Debentures	56,854	4	6
Mortgages	257,975	10	0
Mortgages of Life Policies effected with the Society, and within their Surrender value	18,440	7	4
Reversions Purchased	13,818	0	0
Freehold Property	32,000	0	0
Loans on Personal Guarantee of Sureties, with Life Policies	13,457	11	7
Cash on Deposit at Bankers, and in hand	43,617	18	2
Agents' Balances due	278	8	1
Renewal Premiums due	2,646	9	0
Interest accrued	12,326	2	5
	£869,780	18	10

ATLAS FIRE AND LIFE ASSURANCE COMPANY.
Established 1808.

VALUATION UP TO CHRISTMAS, 1869.

The ASSETS of the Life Department at Christmas, 1869, amounted to	£1,620,366	13	5
To this must be added the Sums due or accruing on account of Interest and Dividends	33,570	1	8
Making the Total Assets	£1,653,936	15	1

The estimated Liability of the Company under its various Life Assurance and Annuity contracts, calculated on principles of computation agreeing with the Tables of Mortality on which the rates of premium charged are based, amounts to	£1,427,175 11 6
Showing a Surplus of	£226,761 3 7

Of which Surplus there has been reserved as a Cautionary Fund the sum of £68,418. The Balance, £158,343 3 7, has been divided among those Policy holders, in England and Ireland *respectively*, entitled to participate in the Profits.

This is the ninth Bonus declared by the Company; the first being up to Christmas, 1823. The total Surplus divided amongst the Assured, and including the Surplus up to Christmas last, amounts to the sum of £1,411,447 in actual money, and is represented by a still larger reversionary sum payable at death.

Bonuses have been declared on some of the Policies which remain in force to an amount greater than the sums originally assured.

Premiums have been altogether extinguished in other cases, where the parties assured have applied the Bonus in reduction of the Annual Premium.

	£	s.	d.
The total amount assured by the Company's Life Policies in force at Christmas, 1869, was	£3,320,904	17	6
Less Re-Assurances	25,199	0	0
	£3,295,705	17	6
The Reversionary Bonuses on the Policies exclusive of the Bonus for the five years ending Christmas, 1869, amounted to . .	£504,678	0	0
The Net Annual Premiums received for the year ending Christmas, 1869, less £705 for Re-Assurances, amounted to . . .	101,636	6	9
The Interest received in the year on the Invested Capital was .	70,159	13	4
Total Annual Revenue for the Life Department for 1869 . .	£171,796	0	1

The Life Funds are invested as under:—

Advances on Mortgages of Real Property				£629,368	14	8
Advances to Counties and Corporations, under the authority of Acts of Parliament				578,716	19	3
New Three per Cent. Annuities				142,104	10	6
Three per Cent. Reduced Ditto				43,597	12	9
West India Loans guaranteed by the British Government . .				135,206	17	6
Government Life Annuities purchased				18,419	3	1
Advances on Atlas Life Policies, within the Office value for surrender				60,386	0	0
Balance at Bankers	£7,075	9	11			
,, in hand	87	0	8			
On Deposit at London and Westminster Bank .	2,000	0	0			
				9,162	10	7
Balances due from Agents				3,404	5	1
Total				£1,620,366	13	5

The Investments are all of the soundest character, safety of the Assets having always been regarded by the Directors as of more importance than a high rate of interest. Nevertheless the *Invested* Funds yielded for the year 1869 a rate of interest equal to more than £4 7*s*. per Cent. per annum.

The sum of £5,119,621 has been paid during the existence of the Office for claims under Life Policies, of which amount a very considerable part was for Bonuses.

The following Statement shows the total number of Policies in force at Christmas, 1869; the amount assured and Bonus at each age, at that date; and the total amount of Annual Premiums payable in respect of those Policies.

Ordinary Whole Life Assurances.—With Participation.

Age at Christmas, 1869.	No. of Policies.	Sums Assured and Bonuses. £	s.	d.	Amount of Annual Premium payable. £	s.	d.
19 and under	42	5,715	0	0	105	3	10
20	3	800	0	0	14	19	6
21	4	5,800	0	0	115	2	0
22	10	8,817	0	0	190	3	1
23	19	6,923	0	0	175	9	7
24	22	5,750	0	0	120	10	4
25	45	22,483	0	0	485	12	4
26	42	21,355	0	0	460	7	10
27	48	17,109	0	0	377	8	8
28	55	23,738	0	0	505	16	5
29	53	22,481	0	0	511	15	0
30	61	25,965	0	0	599	10	7
31	70	39,180	0	0	941	1	9
32	71	38,424	0	0	914	10	9
33	89	36,075	0	0	844	9	9
34	89	46,373	0	0	1,183	1	5
35	95	51,945	0	0	1,286	18	10
36	86	36,860	0	0	1,010	5	7
37	91	34,458	0	0	864	8	0
38	101	47,205	0	0	1,231	18	11
39	97	55,498	0	0	1,370	6	6
40	101	40,186	19	0	1,036	5	1
41	97	57,756	0	0	1,444	11	6
42	88	50,750	0	0	1,251	13	9
43	90	48,898	0	0	1,311	18	3
44	102	68,878	0	0	1,523	19	2
45	101	61,187	0	0	1,751	1	5
46	133	75,889	0	0	2,019	2	8
47	126	68,684	0	0	1,821	0	3
48	152	92,425	0	0	2,538	6	0
49	146	83,749	0	0	2,164	0	8
50	151	74,117	0	0	2,005	9	11
51	159	73,834	0	0	1,999	18	2
52	185	92,110	0	0	2,732	17	6
53	146	76,811	10	0	2,049	0	5
54	123	61,229	19	0	1,545	17	9
Carried forward	3093	1,579,459	8	0	40,504	3	2

Age at Christmas, 1869.	No. of Policies.	Sums Assured and Bonuses. £	s.	d.	Amount of Annual Premium payable. £	s.	d.
Forward .	3093	1,579,459	8	0	40,504	3	2
55	168	72,869	0	0	2,010	9	8
56	173	97,388	0	0	2,763	18	4
57	148	84,339	0	0	2,584	5	1
58	136	84,292	19	0	2,449	10	0
59	147	96,671	19	0	2,612	4	8
60	138	93,313	0	0	2,506	9	10
61	147	77,963	0	0	2,302	16	9
62	134	83,352	0	0	2,611	16	2
63	105	71,569	0	0	1,812	9	11
64	104	65,209	2	0	1,756	0	0
65	106	102,089	10	0	3,045	13	5
66	104	76,923	0	0	1,960	19	8
67	120	79,808	0	0	2,319	19	5
68	79	49,754	14	10	1,219	4	2
69	85	64,106	0	0	1,728	7	7
70	80	89,128	0	0	2,508	18	10
71	80	86,154	0	0	2,099	16	2
72	75	69,605	0	0	2,269	5	1
73	71	73,520	10	10	2,228	2	2
74	62	58,657	15	5	964	19	5
75	59	44,985	0	0	1,025	10	8
76	51	56,572	0	0	1,522	10	9
77	57	28,855	0	0	1,127	19	10
78	47	41,774	1	6	1,364	14	8
79	36	43,302	4	7	971	9	2
80	41	41,320	0	0	989	5	0
81	25	26,710	0	0	568	5	1
82	29	25,707	0	0	639	15	7
83	12	16,396	0	0	694	19	2
84	12	14,490	12	4	379	5	8
85	9	9,519	0	0	258	16	7
86	12	15,592	0	0	218	19	9
87	1	1,995	0	0	70	13	10
88	6	1,292	0	0	57	15	3
89	5	1,511	0	0	3	4	6
90 and over	2	5,211	0	0	202	14	10
Total....	5759	3,531,404	17	6	94,355	9	10

Brought Forward.	No. of Policies.	Sums Assured and Bonuses.	Amount of Annual Premium Payable.
Ordinary whole Life Assurances,			
with Participation . . .	5759	3,531,404 17 6	94,355 9 10
without Participation . .	65	71,010 0 0	2,331 4 9
Various Special Assurances . . .	346	223,168 0 0	4,769 4 2
Assurances to secure contingent Annuities, amounting to £925 per annum . .	29	. . .	261 4 0
Total Number of Policies . .	6199		
Total Sum Assured and Bonus . .		£3,825,582 17 6	
Total Annual Premium, after deducting £2195 6 11 reductions in Premiums on account of Bonuses . .			£101,717 2 9

18 Annuities are payable, amounting to £450 per annum, the average age of the Annuitants being 62 years.

LAW LIFE INSURANCE SOCIETY.

Established 1823.

Report of the Directors on the Quinquennial Valuation to the 31st December, 1869.

The Total Assets of the Society, including both the Guarantee and Assurance Funds, on the 31st December 1869, amounted to **£5,537,281. 1*s*. 11*d*.***

The Society's Income for that year arising from Interest on Investments and Premiums was **£508,264. 6*s*. 0*d*.**

On that day the Society had been established 46½ years, and had issued 19,086 Policies, assuring upwards of **£22,500,000.**

The number of Policies remaining in force on the 31st December 1869, was 6,887, a Table containing classified particulars of which is subjoined.

These Policies assure **£8,578,390**, with Reversionary Bonuses amounting to **£1,671,574**, making the total sum at risk on that day **£10,249,964.**

The Balance Sheet appended to the Report gives the Result of the Valuation of the Assets and Liabilities of the Assurance Fund of the Society at the close of the Five Years which ended on the 31st December

* Made up as follows:—

		£ *s.* *d.*
Assurance Fund		4,655,031 11 7
Guarantee Fund, *viz.*—		
Invested on Mortgage . . .	732,061 0 5	
On Railway Debentures . . .	105,000 0 0	
Held by the Assurance Fund . .	1,745 2 4	
Total Fund . . .	838,806 2 9	
Add Interest held by Assurance Fund .	41,840 18 1	
Interest not received on the 31st December	1,602 9 6	
		882,249 10 4
		£5,537,281 1 11

1869; the Guarantee Fund, which now amounts to £838,806. 2*s*. 9*d*., on the present Division of Profits, will amount to **£962,449. 11*s*. 4*d*.**

During the Five Years ended 31st December 1869, the Society received for New and Renewal Premiums the sum of **£1,415,862,** and for Interest on the Investments forming the Guarantee and Assurance Funds, **£1,103,782.** The Total Receipts for the quinquennial period were therefore **£2,519,644.**

During the same period the Claims paid have been—		
Sums Assured . . .		£1,312,756
Bonuses thereon . . .		536,915
Claims accrued but not paid on 31st December 1869—		
Sums Assured . . .	£89,994	
Bonuses . . .	36,542	
		126,536
Total . . .		£1,976,207

The Total Claims from the commencement of the Society to the same date, have been—	
Sums Assured . . .	£6,482,005
Bonuses thereon . . .	1,950,220
Together . . .	£8,432,225

The Assets of the Assurance Fund amounted on the 31st December 1869, as shewn by the Balance Sheet, to		£4,655,031 11 7
The estimated Liabilities of the Society under all its engagements, amounted at the same date—		
On existing Policies, to	3,910,278 8 10	
On Claims reported but not then mature for payment, to . .	126,536 0 0	
		4,036,814 8 10
Shewing as Profit for the past five years, the sum of		£618,217 2 9

One-fifth of this latter amount, *viz.*, £123,643. 8*s*. 7*d*., belongs to the Proprietors, and the remaining Four-fifths, *viz.*, £494,573. 14*s*. 2*d*., belong to the Assured.

The Profits ascertained in respect of the last Five Years exceed, by nearly £100,000, the Profits which accrued during the Five Years ended on the 31st December 1864.

From the Interest accruing from the Investments of the Guarantee Fund, the Directors propose to pay to the Proprietors for the five years—1870 to 1874 inclusive—an Annual Dividend of Eighty-four Shillings per Share, free of Income Tax.

The Directors anticipate that, at the end of the current five years, there will be Profits belonging to the Proprietors more than sufficient to raise the Guarantee Fund to one Million. At that amount it will, as provided by

the Deed of Settlement, permanently remain; and all Profits assigned to the Proprietors after the completion of the Million will be divided among them.

In allotting among the Assured their share of the Profits now ascertained, the same principles of distribution as were adopted on former occasions have been adhered to.

The aggregate amount of Reversionary Bonus now proposed to be added is	£696,887
The total additions by way of Reversionary Bonus made at previous Divisions amount to . .	£4,164,147
Total . .	£4,861,034

In many cases the additions will exceed the sums originally assured.

* * * * *

CLASSIFIED PARTICULARS OF POLICIES,

In force on 31st December, 1869.

Ages on 31st December, 1869.	Number of Policies.	Sums Assured including Bonuses declared.	Annual Premiums (Tabular).
		£.	£ s. d.
Under 20 years of age	9	8,151	113 14 9
20 to 24	29	69,174	1,503 4 3
25 „ 29	130	183,595	4,020 18 2
30 „ 34	259	394,987	9,633 8 0
35 „ 39	391	561,448	13,962 18 5
40 „ 44	545	751,128	19,096 10 11
45 „ 49	630	811,677	20,801 0 2
50 „ 54	796	1,155,356	29,620 7 7
55 „ 59	934	1,345,983	35,044 2 2
60 „ 64	815	1,162,156	30,756 17 7
65 „ 69	787	1,220,816	31,418 1 2
70 „ 74	614	1,042,557	27,392 8 2
75 „ 79	353	636,618	15,864 6 3
80 „ 84	174	312,424	8,217 9 3
85 & upwards	61	89,275	2,647 11 7
	Premiums in limited number of annual payments		2,254 13 3
Non-participating Assurances and Special Classes	360	504,619	14,599 16 6

BALANCE SHEET OF THE ASSURANCE FUND,

Showing the Divisible Surplus on the 31*st December*, 1869.

LIABILITIES.

		£ s. d.
Present estimated Values of Policies..................		2,679,065 0 0
Ditto Values of Bonuses		1,231,213 8 10
		3,910,278 8 10
Claims admitted, and not due	£102,133	
Ditto reported, but not finally admitted ..	24,403	
		126,536 0 0
		4,036,814 8 10
Surplus—Profit for the Five Years		618,217 2 9
		£4,655,031 11 7

ASSETS.

					£	s.	d.
New 3 per Cent. Annuities	Stock	£190,000,	Valued at		175,100	0	0
New 5 per Cent. Annuities	„	17,300,	„		17,771	17	9
India 5 per Cent. Stock	„	25,000,	„		27,906	5	0
Railway Debenture Stock, *viz.*—							
London and South-Western..	4½	25,000,	„		25,750	0	0
Ditto ditto	4	37,500,	„		36,000	0	0
North-Eastern	4½	15,000,	„		15,600	0	0
London and Brighton	4	15,000,	„		12,750	0	0
		324,800			310,878	2	9
Loans on Railway Debentures					380,000	0	0
Ditto on Mortgages					3,587,004	6	6
Ditto on the Society's Policies					121,236	0	0
Irish Estates—Connamara				£171,170			
Mayo				41,400			
					212,570	0	0
Ditto Rents received and in the Agent's hands..					1,700	9	11
Office Premises in Fleet Street					13,100	0	0
Interest accrued to the 31st December from the dates when the last payments became due					54,903	11	2
Interest due on the 31st December (since paid)					6,317	3	2
Premiums on which the days of grace had not expired ..					14,423	0	8
Bills receivable					65	16	2
Office Cash Balance					340	16	5
Balances at Bankers					25,302	11	5
					4,727,841	18	2

From which, the following sums have to be deducted, viz.—

Outstanding Cheques for Claims, &c., due and not paid.	£29,224	6	2			
Principal belonging to the Guarantee Fund	1,745	2	4			
Interest ditto, applicable to payment of Dividends	41,840	18	1			
				72,810	6	7
				£4,655,031	11	7

GUARDIAN FIRE AND LIFE ASSURANCE COMPANY.

Established 1821.

REPORT OF THE ACTUARY ON THE QUINQUENNIAL VALUATION, CHRISTMAS, 1869.

The Valuation of the Life Branch to Christmas, 1869, has been conducted on precisely the same principles as that on which the Division of Profits was made in 1865.

The Table of Mortality used is one which was constructed by Mr. GRIFFITH DAVIES, from the Equitable Experience, but modified so as to show a rather heavier mortality than the one which appears in his printed works. The rate of interest assumed has been 3 per Cent. The value of the whole of the loading or difference between the Office premiums and the true premiums for the risk computed, for each Policy, by the above table at 3 per Cent. has been deducted from the value of the premiums, so as not to anticipate any of the future profits. Considering the fluctuations which may occur in the Claims by death, when so many of

the Policies are at such advanced ages and of such varying amounts assured, it has been thought prudent to set aside also an additional Reserve until the next valuation, when it will merge in the general profits, subject to the reconsideration of the amount at that period.

The following is the Summary of the results :—

BALANCE SHEET, CHRISTMAS, 1869.

Dr.

	£	£
To Value of £4,239,984 assured by 4094 Policies ..	2,558,271	
Deduct Value of £299,812 Re-assured	147,123	
		2,411,148
„ Value of £201,367 existing Reversionary Bonuses, and £453 Life Reductions of Premium in lieu of Bonus		148,034
		2,559,182
„ Balance of Profits, Christmas, 1869		134,807
£		2,693,989

Cr.

	£	£
By Value of £130,282 full Premiums	1,520,106	
Less Value of loading for future Profits and Expenses and further Reserve	269,392	
		1,250,714
Deduct Value of £9,362 Re-assurance Premiums	139,104	
Less Value of loading on ditto	20,958	
		118,146
		1,132,568
„ Life Assurance Fund	1,406,759	
„ Bonus Fund	154,662	
		1,561,421
£		2,693,989

There remains, therefore, on the operations of the last five years a surplus of £134,807 available for distribution, in the prescribed proportions, between the Shareholders and the Assured.

The total surplus is £33,882 in excess of what was distributed on the last occasion, although the participating Policies in force have diminished from 3417 assuring £3,417,819, to 3037 assuring £3,022,936.

The non-participating Policies have increased from 980, assuring £1,101,304, to 1057 assuring £1,217,048.

If the proposition for admitting the old series of Policy holders to share in four-fifths instead of one-half the profits, as hitherto, can be carried out on this occasion, the amount allotted to the Assured will be £107,848, and to the Proprietors £26,961 ; instead of £79,936 and £54,871, which they would respectively receive under the existing rules.

The Assets of the Company in the Life Branch, at Christmas, 1869, were £1,561,421, all invested in first-class Securities, and producing an average interest at this time of 4½ per Cent.

Class of Investments :—

LIFE ASSETS.

	£	s.	d.	£	s.	d.
Mortgages in the United Kingdom on Real Estate	735,530	6	6			
Do. do. repayable by Instalments	147,488	0	0			
Do. of Life Interests in Real Estate with Policies of Assurance	89,254	0	0			
				972,272	6	6
Mortgages of Public or Local Rates				29,620	16	8
Government Securities				141,144	5	4
Bank Stock				35,521	17	0
British Railways, Canal, and Water Works Bonds				317,550	0	0
Indian Government Guaranteed Debentures ..				5,000	0	0
Annuities and Reversions charged on Real Property				55,426	11	10
Loans on Life Policies (below the surrender value)				41,563	16	0
Outstanding Premiums on which days of Grace have uot expired				3,894	2	6
Current Interest on the above Investments ..				23,014	8	11
Agents' Balances				13,322	10	8
Cash in hand and on Deposit				25,523	9	2
Total Funds in Life Branch				£1,663,854	4	7
Claims, &c., admitted but not due ..				102,432	14	4
				£1,561,421	10	3

* * * * *

The average annnal amount of the Premiums received during the Five Years has been £130,726, and of the Interest and Dividends £63,297, together £194,023.

The average annual amount of the Claims paid, exclusive of Bonuses, has been £172,914 (after deducting the average £3,668 re-assured), of Commission on Premiums £4,501 per Annum, payments for Surrendered Policies £3,553 per Annum, and the Expenses of Management £8,602 per Annum.

Comparing the Commissions with the Premiums they will be found to be 3·44 per Cent. thereon, and the Expenses of Management 6·58 per Cent., making together just about 10 per Cent. on the premiums.

Since the establishment of the Company in 1821, the total Policies issued for Assurances on Lives amount to not far short of £14,000,000.

	No. of Policies.	Sums Assured.
Policies issued from 1821, to Christmas, 1869..	13,428	£13,836,580

	No. of Policies.	Sums Assured.		
Claims by Death	4476	£5,056,745		
Surrendered	1951	2,238,556		
Lapsed, Expired, &c.	2907	2,301,295		
			9,334	9,596,596
Life Assurances (exclusive of Bonus) in force Christmas, 1869			4,094	£4,239,984

* * * * *

Abstract of Life Assurances existing Christmas, 1869, and of the Valuations at that date.

I.—PARTICIPATING POLICIES.

Ages.	Number of Policies.	Sums Assured.	Office Premiums.	Risk Premiums.	Value of Sums Assured.	Value of Premiums.	Value of Risk Premiums.	Reversionary Bonus and Life Reductions of Premiums.	Value of Ditto.
		£	£	£	£	£	£	£	£
90 and under 94	7	7,800	344	293	7,164	772	656	2,881	2,636
80 „ 90	130	151,935	5,853	4,903	131,278	24,275	20,324	18,998	16,405
70 „ 80	501	526,303	18,194	14,850	417,986	118,822	96,933	69,181	55,170
60 „ 70	665	641,623	21,997	18,145	449,887	215,704	178,012	52,120	36,757
50 „ 60	597	576,142	18,507	15,215	346,658	243,162	199,810	28,696	17,380
40 „ 50	534	567,268	16,578	13,443	294,703	264,425	214,321	16,792	8,782
30 „ 40	326	329,771	8,088	6,378	145,168	151,090	119,119	3,119	1,415
20 „ 30	118	71,695	1,596	1,226	27.920	32,642	25,087	370	147
10 „ 20	8	1,350	24	18	425	557	420	..	..
Whole Life	2386	2,873,887	91,181	74,471	1,821,189	1,051,449	854,682	192,157	138,692
								Life £432	3,014
Other Classes	151	149,049	3,498	2,915	91,577	31,344	26,570	9,210	6,252
								Life £21	76
Total Participating....	3037	3,022,936	94,678	77,386	1,912,766	1,082,793	881,252	201,367	148,034
								Life £453	

II.—NON-PARTICIPATING POLICIES.

Ages.	Number of Policies.	Sums Assured.	Office Premiums.	Risk Premiums.	Value of Sums Assured.	Value of Premiums.	Value of Risk Premiums.	Reversionary Bonus and Life Reductions of Premiums.	Value of Ditto.
		£	£	£	£	£	£	£	£
90 and under 101......	3	600	50	41	581	58	48	......	....
80 „ 90	20	16,250	1,050	935	14,023	4,590	4,084	......	....
70 „ 80	128	109,645	4,500	4,107	85,233	31,532	28,783	......	....
60 „ 70	171	163,300	6,735	6,144	113,813	66,531	60,716	......	....
50 „ 60	208	241,448	7,754	7,274	144,943	102,440	96,111	......	....
40 „ 50	177	248,139	6,899	6,381	128,813	109,980	101,641	......	....
30 „ 40	123	133,330	2,902	2,705	58,756	54,221	50,552	......	....
20 „ 30	50	44,280	810	759	17,040	16,693	15,643	......	....
10 „ 20	9	1,400	21	19	457	470	432	......	....
Whole Life	889	958,392	30,721	28,365	563,659	386,515	358,010	......	....
Other Classes	168	258,656	4,833	4,340	81,846	50,798	45,154	......	....
Total Non-Participating	1057	1,217,048	35,604	32,705	645,505	437,313	403,164	......	....
Total................	4094	4,239,984	130,282	110,091	2,558,271	1,520,106	1,284,416	......	....
Re-assured	..	299,812	9,362	7,963	147,123	139,104	118,146	......	....
		3,940,172	120,920	102,128	2,411,148	1,381,002	1,166,270	......	....

LAW UNION FIRE AND LIFE INSURANCE COMPANY.

Established 1854.

Report of the Directors for the Third Quinquennial Period, ending 30th November, 1869.

The time having arrived for the Third Quinquennial Division of Profits, the Directors have caused a strict investigation to be made into the affairs of the Company, and they now have the pleasure to submit to the Shareholders the results of such investigation.

It will be interesting to show the results of the business for each of the Three Quinquennial Periods.

LIFE DEPARTMENT.

The new Premiums received in each of the periods referred to were as follows:—

Term.	New Premiums Received.	Average per Annum.
1854 .. 1859	£20,625 15 6	£4,125 3 1
1860 .. 1864	25,377 3 11	5,175 8 9
1865 .. 1869	**37,960 9 5**	**7,592 1 10**

The Total Income was:—

Period.	Income.	Average per Annum.
1854 .. 1859	£56,554 2 0	£11,310 16 5
1860 .. 1864	129,960 4 4	25,992 0 10
1865 .. 1869	**250,139 11 2**	**50,027 18 3**

The excess of Receipts over Expenditure was:—

Period.	Excess of Receipts over Expenditure (excluding Bonus).	Average per Annum.
1854 .. 1859	£21,222 10 10	£4,244 10 2
1860 .. 1864	52,671 0 8	10,534 4 2
1865 .. 1869	**119,713 15 1**	**23,942 15 0**

The Total Funds of this Department at the end of each period, exclusive of Share Capital and Annuity Fund, were:—

1854 .. 1859	1860 .. 1864	1865 .. 1869
£20,172 13 4	£78,391 6 11	**£189,053 10 0**

The claims paid during the last five years—less re-assurances—were £70,972.

The number of Policies issued from the formation of the Company to 30th November last (exclusive of Annuity Policies) was 3,970, insuring the sum of £2,467,281, of which 1,176 policies, insuring the sum of £589,189, have lapsed; 286, insuring the sum of £207,679, have become claims; and 194, insuring the sum of £185,438, have been surrendered, leaving in force on that day 2,314 Policies, insuring £1,484,975.

One hundred and thirty-one Life Annuities, for £8,536. 1s. 7d. per annum, and nine Survivorship Annuities, for £960. 12s. per annum, have been granted; of which thirty-four Life Annuities, for £3,794. 6s. 8d. per annum, and three Survivorship Annuities, for £535 per annum, have fallen in by deaths.

The following is a Classified List of all Life Assurances in force on 30th November last.

Class of Assurance.	Number of Policies.	Sums Assured.	Existing Bonus.	Annual Premiums.
Whole Life with Profits	1,494	£784,395	£15,937 12 10	£25,868 6 11
Do. without Profits....	546	453,193	..	15,251 16 2
Do. Limited number of Payments	5	7,250	..	380 7 6
Do. Increasing and Decreasing Premiums	15	20,550	..	502 10 11
Assurances for terms of years..	46	65,325	..	717 4 9
Do. on Joint Lives	28	15,300	70 19 11	708 9 5
Do. on last of two or more Lives......	28	16,205	76 1 5	346 7 5
Survivorship Assurances......	89	63,120	72 5 2	667 19 1
Endowment Do.	48	28,037	47 16 7	1,670 19 4
Assurances against Issue......	15	31,600	..	..
Survivorship Annuities	6	..	..	98 6 0
Extra Premiums for Foreign Residence..............	..	..	..	635 12 10
	2,320	£1,484,975	£16,204 15 11	£46,848 0 4

FIRE DEPARTMENT.

The new Premiums were as follows:—

Period.	New Premiums.	Average per Annum.
1854 .. 1859	£9,649 18 2	£1,929 19 7
1860 .. 1864	13,626 10 2	2,725 6 0
1865 .. 1869	**43,569 10 11**	**8,713 18 2**

The Total Income was:—

Period.	Total Income.	Average per Annum.
1854 .. 1859	£25,051 7 7	£5,010 5 6
1860 .. 1864	49,350 10 4	9,870 2 0
1865 .. 1869	**110,986 14 10**	**22,197 7 0**

The Excess of Receipts over Expenditure was:—

Period.	Excess of Receipts over Expenditure (excluding Bonus).	Average per Annum.
1854 .. 1859	£4,715	£943 0 0
1860 .. 1864	8,109	1,621 16 0
1865 .. 1869	**20,599**	**4,119 16 0**

The Claims paid during the last five years (less re-assurances) were £47,372.

The average rate of Interest realised on the Invested Funds of the Company was:—

1854 .. 1859.	1860 .. 1864.	1865 .. 1869.
£3. 15s. 8d. per cent.	£3. 18s. 8d. per cent.	£4. 10s. 3d. per cent.

Mr. Samuel Brown, President of the Institute of Actuaries, has examined the valuations of the assets and liabilities made by Mr. McGedy.

The following is a copy of his Report:—

" I have carefully examined the details of the valuation of the Law Union " Fire and Life Insurance Company, and made such calculations in each class of " the business as to satisfy myself of the minuteness and accuracy of the " computations for the values of the Liabilities and Assets on 30th November, " 1869.

" Every precaution has been taken to make the reserves as safe and adequate " as possible, and so as in no degree to trench on future profits. The amount " recommended for division is strictly that which is within the limits of the " profits realised up to the date of the valuation.

" The great bulk of the business is that for the whole continuance of life. The " liabilities under this head have been estimated by the Carlisle £3 per cent. Table, " to which a further reserve has been added, to provide against any fluctuation of " mortality during the next profit period. In estimating the value of the " Premiums the same rate of interest has been taken and the whole of the loading " for future profits and expenses thrown off, and not brought into the calculation. " The amount so put on one side amounts to more than 25 per cent. on the net " premiums.

" On the non-participating branch, in like manner, the full loading, amounting " to more than 15 per cent. of the net premiums, has been left out of the account.

" In all the other classes of Assurances, valued by the same Table and Rate " of Interest, equal care has been taken to exclude any portion of profits which " ought to be be reserved for the future.

" It is assumed that the List of Investments has been subjected to the

"inspection of the Auditors, and that they are of such a character as will allow at "any time of the value at which they are estimated being realised.

"With this understanding the result of the valuation is as follows:—

	£	s.	d.
"Total net Assets, 30th November, 1869	£213,719	10	7
"Net value of Liabilities under all the "Policies and Annuities	183,285	8	3
"Balance, being net divisible Surplus	£30,434	2	4

* * * * *

After writing off the Building Account the sum of £605. 11s. 6d. and making allowance for bad debts, there remains a sum of £29,038. 12s. 9d. for distribution amongst the Shareholders and Policyholders.

In accordance with the Provisions of the Deed of Settlement, the Shareholders are entitled out of that sum to £9,247. 16s., being £5 per centum upon the Life Premiums received during the last Quinquennial Period after deducting re-assurances. The Balance, namely, £19,790. 16s. 9d., is available for distribution amongst the Policyholders entitled to participate [and will give a Reversionary Bonus of £1. 5s. per cent per annum on the sum assured].

In the Fire Department the balance in favour of the Company on 30th November last, as will be seen on reference to the Balance Sheet, was £7,200, which sum, together with the amount due to the Shareholders out of the profits of the Life Department, namely, £9,247. 16s., and interest upon the paid-up capital, namely, £2,000, make a total of £18,447. 16s., out of which the Directors deem it advisable that the sum of £7,500 should be added to the paid-up capital of the Life Department, thereby raising it to £20,000; leaving £10,947. 16s. for distribution amongst the Shareholders. The Directors recommend a payment thereout of a Dividend and Bonus after the rate of *twelve per cent.* per annum for the fourteen months ending 30th November next; this will absorb £7,000, and the balance will be carried forward to the credit of the Dividend and Bonus account.

* * * * *

The income of the Company from all sources (exclusive of the purchasemoney for Annuities) was £85,211. 14s. 7d., which, added to the sum received for granting Annuities, makes a total receipt of £90,935. 18s. 9d., being £14,142. 3s. 4d. in excess of the receipts of last year.

* * * * *

BALANCE SHEET—30TH NOVEMBER, 1869.

LIFE DEPARTMENT.

DR.	£	s.	d.
To *net* Liability in respect of all sums assured (including Bonus) less Re-assurances	148,397	16	10
„ Liability under Life Annuities	34,887	11	5
„ Claims admitted but not yet payable	9,879	0	5
„ Dividends unclaimed	147	18	8
„ Sundry Outstanding Accounts	195	7	3
„ Shareholders' Paid-up Capital..................	12,500	0	0
„ Balance (being surplus)	30,434	2	4
Total..................	£236,441	16	11

Cr.

	£	s.	d.
By Mortgages	140,796	5	9
„ Amount Invested in the Purchase of Reversions	46,958	19	1
„ Ditto ditto in the Purchase of Life Interests and Policies of Assurance in other Offices	9,822	3	4
„ Government Stock (Consols and New Three per Cents.)	11,485	9	10
„ Amount Advanced upon Security of the Company's Policies (within their surrender values)	7,644	8	8
„ Proportion of Amount, invested in the Building, No. 126, Chancery Lane	7,173	1	8
„ Interest accruing due on Loans and Investments	2,192	11	5
„ Half-credit Premiums due upon Life Policies	1,585	1	5
„ Premiums due and in course of payment	2,679	19	6
„ Balance in the hands of Agents	1,663	13	2
„ Ditto with the Union Bank of London 4,406 10 2			
„ Cash in the Office 33 12 11			
	4,440	3	1
Total	£236,441	16	11

FIRE DEPARTMENT.

Dr.

	£	s.	d.
To Shareholders' paid-up Capital	37,500	0	0
„ Reserve Fund	11,368	7	11
„ Proportion of £908. 7s. 2d. written off Building Account	302	15	8
„ Dividends unclaimed	189	3	8
„ Balance of Duty due to Government	74	10	5
„ Balance	7,200	0	0
Total	£56,634	17	8

Cr.

	£	s.	d.
By Mortgages	34,060	13	7
„ Amount invested in the purchase of Reversions	11,360	1	2
„ Ditto ditto in the purchase of Life Interests	2,376	2	6
„ Government Stock (Consols and New 3 per Cents.)	2,778	10	2
„ Proportion of Amount invested in the building, No. 126, Chancery Lane	1,735	5	6
„ Interest accruing due on Loans and Investments	493	17	6
„ Balance in hands of Agents	1,133	16	10
„ Balance of Accounts with other Fire Offices due to this Company	1,622	7	8
„ Balance with the Union Bank of London	1,074	2	9
Total	£56,634	17	8

FRENCH INSURANCE STATISTICS.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—I have received from a French gentleman, who has taken great interest in the progress of Assurance in France, the following statistics of the French Fire and Life Assurance Companies for 1868. They may be useful to some readers of the *Journal*, and I beg to place them at your disposal.

I am, Sir,
Your obedient servant,
S. B.

11, *Lombard Street,*
3rd September, 1870.

FRENCH INSURANCE COMPANIES (PROPRIETARY.)

(1.) *Fire.*

No.	Company.	Established.	Business of 1868. Renewal Premiums.	Business of 1868. Premiums of New Contracts of 1868.	Business of 1868. Total Premium Income after deduction of Cancelments and Reinsurances.	Claims paid in 1868.	Sums Insured, 31st December, 1868.	Capital. Capital.	Capital. Number of Shares.	Capital. Amount of Share.	Capital. Paid up on Share.	Capital. Actual Value of the Shares.	Capital. Reserve Funds.
			£	£	£	£	£	£		£	£	£	£
1	Cie. d'Assurances Générales, 89, Rue Richelieu	1818	308,571	86,811	340,529	187,283	438,500,917	80,000	400	200	200	2,000	293,400
2	La Nationale, 23, Rue de Grammont	1819	243,893	57,709	252,967	131,386	364,456,215	400,000	2,000	200	40	400	120,000
3	Phœnix, 40, Rue de Provence	1819	274,086	65,298	281,368	168,045	361,561,804	160,000	4,000	40	40	124	80,000
4	L'Union, 15, Rue de la Banque	1828	192,271	55,807	194,726	110,896	259,371,707	400,000	2,000	200	40	280	54,600
5	Soleil, 13, Rue de Helder	1829	174,426	46,782	193,304	126,173	249,285,653	240,000	1,000	200	40	360	155,648
6	France, 14, Rue de Grammont	1837	167,782	53,555	164,050	147,361	232,964,708	400,000	2,000	200	40	220	65,028
7	L'Urbaine, 8, Rue Lepelletier	1838	155,385	43,721	159,048	94,817	183,448,115	200,000	1,000	200	40	236	69,440
8	Providence, 14, Rue de Ménars	1838	113,806	23,077	105,483	45,427	134,089,734	200,000	2,000	100	20	140	74,400
9	Paternelle, 4, Rue de Ménars	1843	129,657	30,192	125,813	60,398	168,114,935	240,000	6,000	40	16	44	12,474
10	Confiance, 21, Rue de Grammont	1844	96,667	26,885	94,918	65,452	125,162,967	240,000	1,200	200	60	200	
11	Nord, 4, Rue Lepelletier	1840	93,797	27,630	77,098	39,483	94,105,377	80,000	2,000	40	8	44	34,119
12	Aigle, 13, Rue de Helder	1843	88,436	27,828	97,037	60,205	119,279,325	80,000	400	200	80	200	32,887
13	Abeille, 52, Rue des Petites Ecuries	1857	85,976	22,750	93,460	60,098	91,986,257	480,000	12,000	40	8	40	
14	Midi, 22, Rue de Fivienne	1859	45,769	19,104	37,062	25,694	38,554,569	80,000	400	200	40	200	
15	Caisse Générale, 30, Rue de Grammont	1858	86,779	36,536	102,325	69,609	90,308,582	480,000	12,000	40	12	36	
16	Centrale, 108, Rue Richelieu	1863	38,985	21,752	41,990	31,469	49,271,929	200,000	1,000	200	60	200	
17	Monde, 106, Rue Montmartre	1864	26,899	19,357	36,642	17,041	45,197,433	200,000	10,000	20	6	20	
18	Union du Nord, à Lille	1867	16,651	11,705	24,374	12,737	29,892,896	80,000	1,000	80	20	20	
19	La Patrie, Rue des Halles	1867	9,484	7,247	15,849	8,332	16,129,893	40,000	1,000	40	10	10	
20	La Paix, Rue de Rivoli	1868	?	?	?	?	?	200,000	1,000	200	60	60	
	Total	£	2,349,320	683,746	2,438,043	1,461,906	3,091,683,016	4,480,000					991,996

French Insurance Companies—(*continued*).

(2.) *Life.*

No.	Company.	Established.	New Business of 1868. Sums Assured.	New Business of 1868. New Premiums.	Total Sum Assured on 31st December, 1868.	Annuities Payable.	Assurance and Annuity Funds.	Capital. Reserve Fund.	Capital.	Capital. Number of Shares.	Capital. Amount of Share.	Capital. Paid up on Share.	Capital. Actual Value.
			£	£	£	£	£	£	£		£	£	£
1	Cie. d'Assurances Générales, 89, Rue Richelieu..	1819	2,423,118	96,698	9,231,584	211,871	2,873,373	218,032	120,000				
2	L'Union, 15, Rue de la Banque	1828	655,693	21,881	2,828,782	51,602	794,710	334,600	400,000	2,000	200	40	320
3	La Nationale, 23, Rue de Grammont	1830	1,963,780	82,261	7,929,366	219,559	2,807,406	123,120	600,000	3,000	200	40	320
4	Caisse Paternelle, 4, Rue de Ménars..........	1841	461,592	20,974	3,125,608	28,424	263,842	6,280	240,000	12,000	20	4	24
5	Phœnix, 40, Rue de Provence	1844	734,288	29,310	2,694,288	18,862	287,693	13,547	160,000	800	200	40	240
6	L'Impériale, 182, Rue de Rivoli	1854	300,979	21,847	2,417,264	58,093	297,305	?	200,000	10,000	20	10	20
7	Caisses Gles. des familles, 4, Rue de la Paix	1858	501,085	25,412	2,810,861	40,483	321,373	368	120,000	1,200	100	20	100
8	Alliance des départments, au Havre	1864	157,793	8,127	157,793	777	10,893	256	80,000	4,000	20	8	20
9	Le Monde, 196, Rue Montmartre	1864	415.277	16,284	1,013,741	9,940	?	?	200,000	10,000	20	6	20
10	L'Urbaine, 8, Rue Lepelletier	1866	412,259	19,853	945,889	12,625	?	3,000	480,000	12,000	40	?	?
	Total	£	8,025,864	342,647	33,155,176	652,236	7,656,595	699,203					

CORRESPONDENCE.

ON THE CONSTRUCTION OF TABLES OF POLICY VALUES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—As a small contribution to Mr. Manly's subject of the construction of Tables of Policy Values allow me to send you the following formula, of which I have made use in the computation of tables for surrenders.

By being stated in terms of the annual premium it is suitable both for the purpose for which it was originally intended and (by the substitution of the net for the real or "loaded" premium) for ascertaining the values as in Mr. Manly's form, while, the *final* results being arrived at in series, a check at any point detects an error in the intermediate process of taking out the numbers to the logarithms.

Since the value of a policy for £1 at age x, the annual premium "ϖ" for which is just due, is

$$1-(1-v+\varpi)\mathrm{a}_x$$

which for brevity may be written

$$1-\mathrm{Qa}_x$$

the difference between this and the next higher value, that is

$$\mathrm{Q}(a_x-a_{x+1})$$

is the quantity to be first formed, and the addition of each term of this successively gives the final value required.

This is comparatively easy work, the logarithm (a_x-a_{x+1}) being, as in Mr. Manly's process, tabulated once for all, and Q being a constant factor for one column of the final result.

The continuous method of computation may be applied also to the formation of $\log[\mathrm{Q}(a_x-a_{x+1})]$; the differences of this series being obviously the differences of the series $\log(a_x-a_{x+1})$. Hence, if these last differences be found, $\log[\mathrm{Q}(a_x-a_{x+1})]$ may also be found by continuous addition.

I am, Sir,

Your most obedient servant,

H. AMBROSE SMITH.

2, *King William Street,*
London, 30*th April*, 1869.

NEW EXPERIENCE MORTALITY OBSERVATIONS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The tables of mortality which Mr. Woolhouse has deduced from the more important part of these observations, and which have been recently published in the *Journal* of the Institute, appear to coincide to a great extent with the tables formed by the same gentleman from the "Experience of the Seventeen Offices." In fact, this part of the investigations has been chiefly valuable as verifying and confirming the "Experience" table, and redeeming it from the reproach directed against it by many

actuaries, that it was based upon policies, not lives. We may therefore take it for granted that, with respect to ordinary tables of mortality, the New Experience Observations will not advance us far beyond the stage already reached.

There is, however, another subject of great importance to actuaries, upon which the New Experience Observations were confidently expected to throw considerable light. I refer to the "effect of selection," or in other words, to the ascertaining of how long and to what extent lives assured are healthier than those of the same age longer assured.

Now my object in writing this letter is to point out to your readers what appears to me to be an oversight in the book, affecting directly this very question of "selection." In calculating the numbers exposed to risk for each separate age at entry and year of assurance, the discontinuants in the "year 0" of assurance have been virtually neglected altogether, that is, although they must have been on the Office Registers for, on the average, a sensible portion of the "year 0" of assurance, they have not been considered as at risk at all. Thus, to make my meaning clear,

let N be the number of "entrants" at any age,
p ,, "existing" at end of "year 0,"
q ,, "discontinued" in the "year 0,"
r ,, deaths, ,,
M ,, who enter upon "year 1" of assurance;

then we shall have

$$N = p + q + r + M.$$

Now, in the Observations, the number exposed to risk for "year 0" of assurance is taken as equal to $\frac{1}{2}N - \frac{1}{2}q$.

$$\text{But} \qquad \tfrac{1}{2}N - \tfrac{1}{2}q = \tfrac{1}{2}(p + q + r + M) - \tfrac{1}{2}q$$
$$= \tfrac{1}{2}(p + r + M)$$

and is therefore independent of q.

No mention is made of this fact in the late President's able preface to the book; but, as "selection" is a subject of great interest to the profession, I have ventured to call your readers' attention to it.

Another point which I think requires to be noticed is the phrase "year 0" of assurance. The lives assured were on the average exposed to risk during six months of the calendar year of entry, so that the data at the disposal of the Institute did not enable the mortality during the first twelve months after entry to be calculated. All that could be obtained was the rate of mortality for the first six months after entry, and then the rate for each succeeding year. To obtain the rate of mortality for the first twelve months, it has been *assumed* that it will be the same as for the first six, a method, I think, by no means satisfactory.

I am, Sir,
Your obedient servant,
WILLIAM SUTTON.

18, *Lincoln's Inn Fields,*
3rd September, 1870.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On Legislation as to Life Insurance and Life Insurance Companies. By T. B. Sprague, M.A., *Vice-President of the Institute of Actuaries.*

[Read before the Institute, 28th November, 1870.]

THE past session of Parliament has witnessed the passing of an Act for the regulation of Life Assurance Companies in the United Kingdom, which, while introducing great changes in the law, still stops very far short of the system of legislation which has been for several years in operation in a few of the United States of America, and which is warmly approved of and urgently recommended for adoption by some persons in this country. The present may therefore be considered a fitting time for reviewing what has been done and considering whether any further legislation is desirable, and if any, of what nature it should be.

The object aimed at by the American legislation may be briefly described as an attempt to secure the solvency of all Life Insurance*

* It will be noticed that I do not accept the distinction laid down by some authors as to the use of the words *assurance* and *insurance*, by which the former is restricted to life and the latter to fire risks. The more correct distinction I believe to be that a man *insures* the life of himself or of some other person, or his house, or his ships, or the fidelity of his servants; and that the Office *assures* to him in each of these cases a sum of money payable in certain contingencies. Hence the Office is called the *assurer*, or *assurers*, and the man the *assured;* while we may speak either of *the life assured*, or *the life insured;* also of *the sum assured*, or *the sum insured;* according as we take the

Companies doing business in the respective States, so that all the contracts of assurance entered into by them shall be duly fulfilled. The advocates of that legislation consider that the business of life assurance is a matter of such vast national importance that it must be put under very special and exceptional regulations. It is not to be tolerated, they urge, that a Life Insurance Company shall ever fail to meet its engagements, if that can possibly be prevented by legislative enactments. With this view, regulations have been made by the legislatures of some of the States, more especially Massachusetts and New York, which are believed to be effectual for the purpose. The general nature of these regulations is so well known that it is not necessary to spend time in describing them, and I will therefore at once proceed to give my reasons for believing

Firstly, That no legislative enactments whatever can succeed in absolutely securing the solvency of Life Insurance Companies;

Secondly, That even if this could be done, it would be highly undesirable that it should be attempted.

First, then,—It is not possible for legislation to secure the solvency of Life Insurance Companies. It is, indeed, easy to provide, as has been long done in America, and as has been now done for the first time in England, that all Life Offices shall publish full and detailed accounts in a prescribed form, so that complete publicity is given to their income and expenditure, assets and accrued liabilities. It is also easy to provide that all Life Offices shall return sufficient particulars of all policies granted by them, to enable a skilled actuary to form his opinion as to the liability of each Office under its contracts. This is done in America; but the Act lately passed in England does not go quite so far. But when all this has been done, the object aimed at is far from being attained. In the following remarks we will more particularly consider, for the sake of simplicity, the case of Mutual Insurance Companies; but our conclusions will apply with the slight needful changes to the case of Proprietary Companies.

The insolvency of a Life Office may arise from any one or more of several causes, viz:—

1. From an excessive expenditure in obtaining or conducting the business.

point of view of the Office or of the individual. So also we may speak either of "Life Insurance" or of "Life Assurance"; as, for instance, we may say that a man believes in the duty and advantage of life insurance, or that a certain Company finds the business of life assurance very profitable.

2. From the declaration of excessive bonuses, which the profits of the Office do not justify.

3. From loss on the investments.

4. From an excessive rate of mortality.

There seems little doubt that the system pursued in America will be effectual in preventing insolvency from the first of these causes. If the expenditure of an Office has been excessive for a number of years, so that the money which ought to be invested and accumulated to meet the liabilities, has been paid away to agents, or squandered in unproductive buildings, &c., it will be observable year by year in the Reports made by the Insurance Commissioner, that the Company is drawing nearer to insolvency as defined by the law; and at last a time will come when the Commissioner will declare the Company insolvent according to the standard prescribed by law, and order it to be wound up. If then the business has been in other respects well managed, that is to say, if the lives have been accepted with due care, and the investments realize their nominal amount, the Company on having its excessive expenditure forcibly put a stop to, can be either wound up so as to pay its policies, or the policies can be transferred to another Office without reduction, the latter being probably in all instances the preferable course.

Similar remarks apply to the insolvency of an Office caused by the declaration of excessive bonuses. If a bonus has been declared so large that in the opinion of the Commissioner, or more precisely, according to the legal standard of solvency, the resources of the Company do not admit of its being paid, the Commissioner might in this case also order the Company to be wound up; and still supposing the Company to have been in other respects well managed, the only loss that would under these circumstances be incurred would be the reduction of the bonus that had been improperly declared.

But if the insolvency has arisen from either of the other two causes above mentioned, the case is not so simple. If a portion of the assets that are absolutely essential to enable a Company to meet its liabilities, has been lost by injudicious investments, there will necessarily be a deficiency on the winding up of the Company; and the sums assured under the policies cannot be paid in full, and must be reduced, if all the loss is not to fall on those of the assured who may happen to live the longest. It might easily happen, or is at all events conceivable, that a Company might make a very fair show for many years in the Commissioner's

Reports, but that on being ordered to be wound up, a great part, or even the whole, of its securities might turn out unsatisfactory, and a great loss occur on their realization. In this case the Commissioner would be powerless to prevent loss to the assured, and his function would be restricted to stopping the business when insolvency could no longer be concealed, and shortening the career of the dishonest or incompetent managers. If the whole of the assets have not been dissipated, the wreck will be saved for the assured; but there can be no certainty that a total loss will be prevented. In order to meet this danger, it is clear that the powers of the Commissioner must be greatly increased. He must be authorized to sit in judgment on the securities held by the Company, and to decide which of them are admissible as assets against the liabilities, and which must be rejected. This has not been attempted in Massachusetts; but it is done in New York; and the Superintendent of the Insurance Department in that State accordingly issues from time to time a circular of the most inquisitorial character, requiring particulars as to the persons to whom loans have been granted, and the securities for them. If a security is considered unsatisfactory by the Superintendent; if, for instance, a loan has been granted on personal security, or a sum invested in commuting the commission payable to an agent; he strikes such security out of the list of assets, and requires the Company to have the full amount of the legal reserve for its liabilities invested in other securities approved by him. Even this, however, will be found on examination to be insufficient to prevent insolvency and loss to the assured; for managers of a speculative and sanguine turn of mind might easily succeed in working irremediable mischief, in the interval between two of the periodical inquiries of the Commissioner. To sum up, the powers given to the New York Superintendent, wide as they are, are still inadequate to prevent loss to the assured. At the same time it must be admitted that loss from improper investments is in general gradual, and therefore the chance of such loss is much diminished by the New York legislation. In order however entirely to remove the risk of loss from investments, it will be necessary to go still further, and provide that Life Insurance Companies shall invest their funds only on certain specified securities, as for instance, Government Stock, mortgages of freehold land, railway debentures, &c. When all this has been done, it will still be possible, if a Company is improperly managed, that it may incur losses which will render it insolvent. It may

have invested a large proportion of its funds in Government Stock at a high price, and be forced by a series of unfavourable years to sell a great portion of the stock at a sacrifice. Even in mortgages on apparently unexceptionable landed security, it sometimes turns out, after every care has been taken, that the title of the borrower is bad; and if a Company had invested a considerable portion of its money in such a security, it might very easily become insolvent. To guard against this danger, therefore, it would be necessary that the law should lay down regulations to provide what proportion of its funds an Office should invest in a single security. In fact, the further we trace the subject, the more minute we find must be the regulations; while however minute and exact they may be, we are forced to the conclusion that they cannot under all conceivable circumstances secure their object.

We now come to the last of the abovementioned causes of insolvency—an excessive mortality. On considering the matter, it very soon becomes evident that no amount of Government supervision can guard against the danger of insolvency from this cause. If the managers of a Company, whether from fraud or ignorance, have accepted proposals on unsound lives to a dangerous extent, insolvency is sure to occur; and when the Company is ordered by the Commissioner to be wound up, a reduction of the sums assured becomes inevitable. The same is the case if the Company has accepted at inadequate rates proposals on lives exposed to extra risks from residence or occupation. The mischief will probably not show itself for some years; but when the scrutiny of the Commissioner detects insolvency, it will be found impossible to wind up the Company, or transfer its business, without a heavy loss to many of the assured.

The above may suffice to establish the first of my propositions, that no amount of Government supervision can secure in all conceivable cases the solvency of all the Life Insurance Companies in a country. If, then, the Government desire to secure to persons wishing to insure their lives the certainty that their policies will be paid on becoming claims, the only means by which this can be done is that the Government shall itself undertake the business of life assurance, receiving and investing the premiums, and pledging the national credit for due payment of the policies. There are two ways in which this might be done. The Government might pursue the course which has long been pursued with reference to the sale of annuities, and which has been already adopted with respect to small insurances; that is to say, open a new Government Insurance

Office to compete with the existing Offices. If this were done, persons desiring Government security would be able to obtain it, while the business of the existing Companies would be left untouched, except that they would have a new competitor for the public favour. If the Government should consider this course desirable, I do not think any valid objection could be made to it; and probably there would be no opposition on the part of existing Companies, who would feel that in every respect, whether as regards the magnitude of the premiums charged, the facilities offered for the conduct of business, or the amount of profits returned to the assured, they would be able to offer more real and solid advantages than the Government. In fact, those who wished for Government security would have to pay for it, just as persons who desire Government security for their investments and purchase consols for that purpose, receive a smaller return on their investments than the holders of any other kind of property. Of course it would be essential that the business should pay its own expenses, so that the experiment should cause no direct loss to the nation. As regards the small assurances at present granted by the Government, I believe I shall be correct in saying that the above condition is not fulfilled; and as the experiment has now had a fair trial, it should seem that the time has arrived when the Government ought to acknowledge they have made a mistake, and retire from the business. Even as regards the grant of annuities, it appears to me very doubtful whether the Government does not carry on the business at a loss to the country. The rates charged are, it is true, deduced from the mortality experienced among Government annuitants in past years; but without urging that sanitary improvements have increased the average duration of life at the present time, I would simply point out that the rates in use were deduced from the total experience, without making allowance for the influence of the self-selection of the annuitants, and the consequent light mortality in the years immediately succeeding the grant of the annuity. For this reason, it would be very desirable that there should now be made a new investigation into the mortality of the Government annuitants, with the view of settling this important question, and ascertaining whether the country is a loser by the grant of annuities on the present terms.

The other course which might be pursued by the Government would be to undertake the business in the same way as they have done that of telegraphic communication, by buying up all the existing Companies compulsorily and obtaining a monopoly of the

business. There would probably be no objection made to this course on personal grounds, of course assuming that the managers would be pensioned off on full pay. But on every other ground such a course is much to be deprecated. There are many objections to the Government undertaking so complicated a business as the insurance of lives, which will be familiar to all students of political economy, and which are well summed up by Mr. J. Stuart Mill in the following passages.

"In all the more advanced communities, the great majority of things are worse done by the intervention of Government, than the individuals most interested in the matter would do them, or cause them to be done, if left to themselves. . . . . . . Government by excluding or even by superseding individual agency either substitutes a less qualified instrumentality for one better qualified or at any rate substitutes its own mode of accomplishing the work for all the variety of modes which would be tried by a number of equally qualified persons aiming at the same end; a competition by many degrees more propitious to the progress of improvement than any uniformity of system."

"The true reasons in favour of leaving to voluntary associations all such things as they are competent to perform, would exist in equal strength if it were certain that the work itself would be as well or better done by public officers. . . . . . . the mischief of overloading the chief functionaries of government with demands on their attention, and diverting them from duties which they alone can discharge, to objects which can be sufficiently well obtained without them; the danger of unnecessarily swelling the direct power and indirect influence of government, and multiplying occasions of collision between its agents and private citizens; and the inexpediency of concentrating in a dominant bureaucracy, all the skill and experience in the management of large interests, and all the power of organized action, existing in the community; a practice which keeps the citizens in a relation to government like that of children to their guardians, and is a main cause of the inferior capacity for political life which has hitherto characterized the overgoverned countries of the Continent, whether with or without the forms of representative government." *Principles of Political Economy*, Book v §§ 5 and 11.

I will only add to these a few objections which arise from the special nature of the business. First, let us consider the case of persons whose proposals for insurance might be declined by the supposed Government Life Office. At the present time, if a life is declined at one Office, there are one hundred others that may be applied to; and it is no uncommon thing for a life declined by one Office to be accepted by another at a special rate, sometimes, indeed, at the ordinary rate. But if all the existing Companies were bought up, and the formation of new ones forbidden, lives rejected by the Government would virtually have no chance of assuring at all. It would be clearly impossible to endeavour to

provide against this by allowing Companies to insure lives at special rates; for a Company might publish a very low scale of premiums as the ordinary one, that would be charged if it were permitted to grant insurances at the ordinary rate; and while professing to charge all persons an extra premium, might really insure them at rates lower than the Government. Again, it cannot be doubted that the rates charged by Government would exceed the average of those now charged; and thus the cost of insurance would be increased. Thus, then, those persons who would have been quite satisfied with the security offered by the Companies, would have to pay an increased premium, in order that others might obtain the security of Government for their policies. Thirdly, the existing Companies discharge a very useful public function in purchasing and making advances upon life interests and reversions; also in lending money on personal security; these transactions being generally mutually advantageous to the Companies and to their clients. The Government would certainly not undertake these classes of business; and if the Insurance Companies were bought up, the whole of it would be left to the Reversionary Companies and similar Societies. The demand for securities of this kind being thus very greatly reduced, while the supply remained the same, and any new Offices that might be started being precluded from making a profit as at present by the grant of life policies, it would naturally follow that much lower values would be given for the purchase of such property, and a much higher rate of interest charged for loans, and thus the venders* or borrowers would be greatly prejudiced by the supposed change. To sum up the whole, the baneful effects of allowing a monopoly of any business to a single institution, would probably be even more apparent in this case than in most others. In the case of the telegraphs already mentioned, the constant complaints which appear in the newspapers show that in that department the monopoly has been productive of much mischief; and even as regards the carriage and delivery of letters, it appears that the time has come when the Government monopoly ought to be no longer tolerated.

* As regards the spelling adopted here and subsequently, I submit that the analogy of other English words requires that a person who vends, should be called a *vender;* a person who invests, an *invester;* and a person who deposits, a *depositer;* just as we speak of a *seller*, a *lender*, or a *borrower*. The adoption of this spelling in these and other cases where it is applicable (as in the words *visiter, governer, directer*) would remove one of the many anomalies prevailing in the English language; and in comparison with this object, the argument from the derivation of the words appears quite futile. In fact, it should be the aim of every lover of his native language to efface as quickly as possible, rather than carefully perpetuate, every mark of the different sources from which the words he uses have been derived.

I now pass on to my second proposition, which is, that even if it were possible for Government, by making extremely minute regulations for the conduct of the business, and exercising a keen and constant supervision, to secure the solvency of the Life Offices, still it would be very undesirable on grounds of public policy that this should be done.

What would have been the history of life insurance up to the present time, if the modern notions of paternal supervision had prevailed when the Equitable Office was projected? It will be remembered that when the projecters applied to the Government for a charter, the law officers of the Crown represented that the project was so hazardous and unlikely to succeed that they could not advise the Crown to grant one. Nor was this decision so absurd as we may now be disposed to think it. The promoters of the Equitable were proposing to enter upon a new and untried speculation. The premiums they proposed to charge were greatly lower than those which had been hitherto charged by the few Offices granting assurances on lives; and although Dr. Price had well founded confidence in his calculations, it could not be expected that this would be shared by the law officers of the Crown. We cannot help thinking that the decision arrived at was an eminently proper one. The new venture was not forbidden. The sanguine promoters might start their novel Society at their own risk; but they should not receive the countenance and encouragement which the grant of a royal charter would imply. This, it appears to me, is a precedent that should be borne in mind and well considered at the present time.

I venture to lay down the general principle that it is to the advantage of the community at large that all commercial ventures, started in good faith, should be allowed a fair trial, however rash, and even hopeless, cautious men may think them. Then if the venture should turn out disastrously, and bring ruin on the promoters, still the general benefit to the community at large, far outweighs these losses. It is, in fact, impossible to say beforehand what speculations may succeed, and which will certainly fail; and if the Government consider that it is their duty to discourage speculations that appear to be rash and unlikely to succeed, there is the greatest probability, or even certainty, that many useful improvements will be prevented. In fact, an examination of the great improvements of modern times will prove that they have generally originated with sanguine men, who were considered by their contemporaries to be extremely rash, and who have, indeed, very frequently ruined themselves while benefiting the community.

Of course, in laying down these principles, we assume that it is no part of the duty of a Government to teach prudence to its subjects; nor to advise them as to the most advantageous or the safest way of employing either their money or their talents; nor to protect them from losses that may be caused by their errors of judgment. These are all propositions in political economy which it is here only necessary to state, and for the proof of which I may again refer to the standard works on political economy. As far as I can see, there is no reason why these principles should not be applied to the business of life assurance as well as to commercial enterprises of other kinds. I am aware that there are some persons who would regard the business of life assurance as standing on a different footing from all others, who would argue that life assurance is a trust and not a business; but to me these views appear quite untenable.

We have seen how strict and minute the regulations would have to be made, in the event of the Government of a country attempting, with any real prospect of ultimate success, to secure the solvency of the Life Offices in it. Those regulations would extend to the magnitude of the premiums to be charged, the manner of conducting the business, the securities in which the funds shall be invested, and the amount of the reserve to be made for the liabilities. In fact, we may say the Offices would be bound hand and foot. It is quite unnecessary to demonstrate that the existence of such regulations enforced by the Government would have the most powerful operation in preventing the introduction of possible improvements, and would in fact amount to prohibition of any radical alteration in the way of conducting the business. We thus arrive at the conclusion embodied in my second proposition, viz. that even if it were in the power of a Government to secure the solvency of the Life Offices under its control, the disadvantages of that course would be so great as to more than outweigh the benefits.

The life insurance legislation in the United States has certainly had the effect of checking improvements there; for there can be no doubt that, with the high rates of interest prevailing there and almost certain to prevail for many years to come, the cost price of insurance must be far less than in England, and the rates of premium ought certainly to be greatly reduced. It is probable that the only reason this has not already been done is to be found in the restrictions under which Life Insurance Companies have there been placed. Even in England we cannot at the present

time say with any certainty that life insurance has received its latest developments, whether as regards the premiums charged, or the general manner of conducting the business.

First, as regards the premiums charged. When we look at the very large bonuses divided by some Offices, it is scarcely possible to resist the conviction that the premiums charged by all Offices are greatly in excess of those necessary for the insurance of well selected lives, and that the means will some day be found of greatly reducing them consistently with safety. Such a reduction of premium might be justified either in consequence of the rate of mortality among the persons whose lives are assured being considerably less than that shown by the tables employed in the calculation of life assurance premiums, or by the fact of the Company being able permanently to realize a considerably higher rate of interest than the three or four percent now used in the calculation of the premiums.

If now a Company were projected to insure only unexceptionable lives, in fact, lives whose expectation, as far as can be judged, is greatly above the average, charging premiums considerably lower than those at present charged, would it not be clearly for the public interest that such an experiment should have a fair trial? Again, if the projecters of a new Company honestly believe that they have succeeded in elaborating a system by which funds collected in this country can be transmitted to the colonies or foreign countries, where the current rate of interest is very much higher than here, and there safely invested on good security at six, seven, or eight percent interest, is it not clearly for the advantage of the public that this experiment also should have a fair trial? Again, there is very little doubt that by judicious investments in foreign stocks, at least six or seven percent interest may be continuously realized with safety, after allowing for a proper sinking fund to provide against the loss of capital that may be considered as certain to arise sooner or later in these investments. If then a new Insurance Company were founded, with the avowed intention of investing its funds in this manner, can it be considered as falling within the legitimate province of Government to forbid the making of the experiment? In the case of such a Company, the occasional loss of capital that would cause serious inconvenience to an individual holding only one kind of stock, would be looked upon as one of the losses to be expected in the usual course of business; and the only question for consideration would be whether the excess in the rate of interest

is a sufficient equivalent for the risk of loss. The case is in fact exactly parallel to that of an ordinary insurance. An Insurance Company is willing to take upon itself, for a consideration, the risk which the individual considers it unwise to run, of loss in consequence of premature death, fire, or perils of the sea; and the only question in all these cases is whether the premium received by the Company is adequate to the risk.

Next, as regards the general scope of the business. At the present time, Life Insurance Companies from one point of view may be considered as receiving the investments of the public, and, on the average, returning them with interest on the occurrence of certain events. Under this aspect, life insurance is an investment; and I cannot help thinking that this aspect of life insurance has been too much lost sight of. For a man who has an income dependent upon his exertions and his life, there can certainly be no better investment for the benefit of his family than that afforded by life insurance. A man in this position will, however, of course wish to save and invest a portion of his income as a provision for himself in old age. At present, the Life Offices offer but small advantages in this respect. A man could buy a deferred annuity, but in doing so he will on the average obtain less than three percent interest on his money, which is a prospect far from tempting. He may also effect an endowment assurance instead of an ordinary assurance, but the premium then becomes so heavy that he must be very much indeed persuaded of the advantages of life insurance to avail himself of this method. Besides, the invester will greatly prefer that the investments he has made as a provision for his old age shall be available on the happening of any of the many contingencies in which he may be glad of ready money. Now it appears to me that Life Offices may very properly and usefully, and with advantage both to themselves and their supporters, extend their operations in this direction. It is, of course, out of the question for them to receive unlimited sums of money on deposit repayable at fixed notice. If they were to do this, they would infallibly, some day or other, find themselves exposed to the inconvenience that lately threatened several of the Railway Companies in this country, which, having borrowed large sums of money on debentures, found the greatest difficulty in getting the debentures renewed when they became payable. Similarly, the Life Offices would certainly find themselves called upon to repay large amounts to their depositers, which they would be either wholly unable to do, or only with the greatest inconvenience and loss. But the same

objection would not apply to their receiving money on deposit, to be repaid at the death of the invester, allowing interest at an agreed rate, which might either be fixed, or vary from time to time with the profits of the Company; and might either be payable half-yearly to the invester, or payable by way of compound interest together with the original deposit on his death. A prudent Office transacting this class of business would not pledge itself to return the deposits on demand during the life of the assured, even subject to a discount; but it would always be open to entertain applications of that kind; and if the state of its finances at any time rendered it undesirable to repay immediately all the sums which depositers might wish to withdraw, this state of things would only be temporary, and temporary advances could without difficulty be obtained from bankers on security of such deposit-policies. As a step in this direction, Life Offices might first of all receive deposits only from persons whose lives are insured with them, and only to the extent of the sum assured. This suggestion is not so revolutionary as it may at first sight appear, and is indeed only an extension of the principle of allowing reversionary bonuses to be surrendered for their cash value at any time, and of the making of loans on the security of policies within their value.

Assuming this kind of business to be undertaken by Life Offices, and some legal difficulties to be got over, I believe that Life Insurance Companies would be found to be extremely convenient agencies for the investment of trust monies. These are generally to be invested at interest during the life of a certain person, and on the death of that person to be realized and distributed. If these monies are not invested at the minimum interest in the public funds, inquiry has to be made for an eligible mortgage of the right amount, and if a mortgage is found otherwise suitable, it is constantly found either that the mortgager does not want the whole of the money which the trustees are prepared to lend, or that he wants rather more. If, however, the trust funds were placed in the hands of an Insurance Company on agreed terms as to interest, this difficulty would be entirely removed, the Company being able without the least inconvenience to themselves to advance the exact amount required by each mortgager. In addition, the interest payable by the Company would always be paid to the day, while borrowers are sometimes a good deal behindhand. If it should be held that such deposit policies come within the scope of the recently passed Married Women's Property Act, a very ready means would be afforded by them of making a

valid post-nuptial settlement of any amount of money on the depositer's wife and children.

There can be little doubt that the principle of association is in its infancy, and that it will be found applicable to many purposes to which it has not yet been applied. My subject leads me naturally to specify one class of business of practically unlimited extent, which I cannot doubt will be entered upon and actively prosecuted before many years are past. I refer to the formation of Societies on the mutual principle, whose business it shall be to receive the savings of the public and invest them in certain specified classes of securities, allowing the invester a rate of interest varying from time to time with the profits of the Society. The securities upon which such Societies will invest the funds of their members, may be, in one instance, house property and ground rents; in another, foreign railways; and in a third, the stocks of foreign Governments. There are many advantages to be gained by investments in such securities being made by a Society instead of by an individual; and when once such a Society shall have been established, and have gained public confidence by judicious management and complete publicity in its operations, its great convenience will probably be quickly recognized, and there will no doubt be a rush for the formation of similar Societies. There are probably many persons now investing their money in house property, who would be glad to receive a somewhat lower return on their money, in consideration of being relieved of all the trouble, anxiety, and uncertainty which attach to such investments. At the present time, the large Railway Companies in this country and in India have indirectly been agencies for the investment of the savings of the public. So long as large sums continue to be required for the completion of the Indian railways, the need for the Societies I have mentioned will not be urgently felt. Other Societies which have more directly, but to a limited extent only, fulfilled the same function, are the Building Societies and the Reversionary Interest Societies. It is a very old suggestion that Life Insurance Companies may properly transact the whole of the business at the present time transacted by the latter class of Companies, and as I have pointed out above, I believe they might with advantage and propriety go still further. But the extension of their operations in any of the directions suggested above would almost certainly be prevented by any legislation after the American pattern.

Any interference of Government with the business of life in-

surance is an interference with the freedom of contract among its subjects, and ought therefore to be justified by very strong arguments. It cannot be said beforehand of all interference of this kind, that it is injurious; but each particular instance must be considered on its own merits, with the view of determining whether Government interference will produce on the whole more harm or good. It has been enacted by the recent Act, that no new Life Insurance Company shall be permitted to commence business until it has deposited £20,000 with the Government, which sum is to be repaid to the Company when its accumulations from premiums amount to £40,000. This provision is probably open to as little objection as any of the kind could be; but the policy of it seems to me extremely doubtful. How many Life Insurance Companies now flourishing have been started with a far much smaller amount of capital paid up, or even only subscribed! As regards Proprietary Life Insurance Companies, the provision merely seems to amount to this, that they shall not be allowed to carry on business unless they are of a certain magnitude; but as regards Mutual Insurance Companies, the provision is virtually prohibitory. The Act, besides thus restricting the formation of new Companies, has in several other important particulars placed restrictions on the freedom of contract. Any provisions of a Company's deed of settlement authorizing the amalgamation of the Company under certain conditions, are now entirely overruled by the provisions of the Act as to amalgamations. The propriety of this is, I think, unquestionable.

In some other respects the Act might with great advantage have placed additional restrictions on the freedom of contract, by forbidding the insertion in policies of some very common provisions as to the liability of the shareholders. There can, of course, be no reasonable objection to the common, but not universal, stipulation, that the liability of the shareholders shall be limited to the amount of the capital subscribed by them; but taking a strict view of matters, can anything be more unjust, and in fact dishonest, than the common stipulation that the liability of the shareholder shall entirely, and in all events, cease when the shares are transferred to a new holder? The consequence of this is, that a number of wealthy shareholders may start a Company, and obtain insurances on the faith of the large amount of capital subscribed by them; and after a few years, if they should have reason to believe that the Company is not prospering, they can under the above-mentioned stipulation transfer their shares to men of straw, thus effectually relieving themselves

of all further liability, and depriving the assured of the security on the faith of which they were induced to join the Company. It appears to me that it would greatly conduce to commercial morality, if it were enacted that notwithstanding any provisions of policies and deeds of settlement, the liability of shareholders in a Life Insurance Company shall continue until all contracts entered into or current while they were shareholders, have terminated. It is of course assumed that the new holder to whom the shares have been transferred would be primarily responsible, and that the original holder should be only liable in the event of the new holder being unable to pay up the subscribed amount of the shares.

Legislation may also, without being compulsory, exercise a most useful influence on the law of contracts, by providing for contingencies that are not provided for in the original contract. Thus with regard to life insurance, the law might very usefully prescribe a general form of policy, the conditions of which should prevail in all cases where there was no special contract to the contrary. It might then suffice to give the assured, instead of the long and complicated policies at present in use, a simple certificate that he has insured his life in such and such an Office, the conditions of the insurance being supplied by the parliamentary form. This is the principle of the Statute of Distribution, which makes a will for every man who has not made a will for himself; and it might be very usefully extended to a variety of other contracts, such as marriage settlements, leases, and the sale and purchase of property. In all these cases it might suffice for the purpose to make a short memorandum of the contract, leaving the usual lengthy covenants and provisions of various kinds to be supplied by the parliamentary forms.

I must now draw attention to the limitation I introduced above in arguing that new ventures should not be discouraged by the Government; namely, that they should be started in good faith. This condition is all-important, and no objection can properly be taken to any provision which is necessary for testing the good faith of the promoters of a new concern. No doubt many persons will consider that the deposit of £20,000 now required from new Life Insurance Companies is a proof of the good faith of the promoters; but I cannot myself agree in that opinion. It proceeds on the assumption that persons intending to defraud the public by means of a bubble Life Insurance Company will never be prepared to stake £20,000 themselves in the hope of making a much larger profit; nor be able to induce others to embark that amount; but

this is obviously not a safe assumption. The effect of the deposit is therefore simply that the swindling, if there is any, shall be on a large scale. The policy of the Government as regards new enterprises should be neither to foster nor discourage them; but it clearly falls within the duty of the Government to take every precaution to guard against their appealing to the public for support under false pretences. When the promoters of a new Company invite the public to become shareholders, they should be required to state in the most precise and distinct manner the exact nature of the business in which it is proposed to embark. They should not be allowed to keep out of sight important provisions as to the payment of promotion money, or, as was done in a recent well-known instance, to deceive intending shareholders as to the value of the assets transferred or the nature of the risks of the business to be undertaken. It may be gathered from that case, in which there was no technically fraudulent intent, that the provisions of the law as to false pretences, and misleading prospectuses, require to be made more comprehensive and more stringent.

Similarly, as regards the policyholders in Insurance Companies, I believe that the Government may properly, and indeed ought to insist on policyholders being furnished with such information as will enable them to ascertain the exact nature and extent of the security offered to them. It is unquestionably a fraud of the grossest character, for a Life Insurance Company to keep its doors open, holding itself out to the public as solvent, and inviting new insurances, for years after it has become hopelessly insolvent, as was done in a recent notorious case; and the only way to prevent this fraud being committed, is to require all Life Offices, without exception, to publish precise and detailed statements of their affairs. It is also in the nature of a fraud towards existing policyholders for the Company to continue its business under the circumstances supposed; for when it is clear that the assets will not suffice to pay all liabilities in full, it is certainly giving an unfair advantage to the old and invalid lives to continue to receive the premiums as formerly from the young and healthy policyholders, who continue to pay in the belief that the contracts made with them will be duly fulfilled. The claim of existing policyholders in a Company to full information as to its position is strengthened by the fact that they are not merely creditors, but to a certain extent partners in the Company. This is, of course, obvious as regards participating policyholders; and as regards the non-participating policyholders, it will be seen on consideration,

that the fulfilment of the contracts made with them must depend to a very great extent upon the success of the Company's trading. The abstract right of existing or intending policyholders in a Company to be informed as to its real position being admitted, it is extremely desirable that the extent of this right should be defined by law. It would be intolerable for every such person to be at liberty to enter the office of the Company and examine its books; and the obvious deduction is, that the Government should specify what particulars shall be published by each Company, and how their accuracy shall be verified. On the other hand, if a person interested in a Company, whether as a policyholder or a shareholder, wishes to inform himself as to the real position of the Company, this being a very legitimate wish, it is desirable on grounds of public policy that in the event of the managers being unwilling to give him the information he desires, he should not be left to fight the Company single-handed, but that the law should first define his rights, and then assist him to enforce them.

To sum up, the Government of a country may properly take steps to discourage and punish fraud and false pretences; and indeed there can be no more legitimate function of a Government. This is the true principle which must guide us in our present enquiries, and it admits of application far beyond the limits of our present subject, and may indeed be extended into every department of trade. To take a familiar illustration, the Government of this country does not attempt to regulate the price of bread, but it has provided that bread shall be sold by the pound. There is nothing to prevent any baker from charging as much or as little as he likes for the 4lb. loaf. He may on the one hand charge a higher price than his rivals in trade—say 8*d.* for the 4lb. loaf when other bakers are charging 7½*d.*; or he may undersell them, and charge only 6*d.*; but in the latter case, he must not reduce the weight. If he does this, he commits an offence against the law, and is liable to punishment. Unfortunately the punishment awarded to persons guilty of such fraud and false pretence in this country is far too light. Strict justice requires that there should be no distinction made between the thief who picks his neighbour's pocket and the dishonest tradesman who uses false weights and measures.

As regards the business of life assurance, the passing of the new Act is a great step in the direction of the policy I am now advocating. It is no longer allowed to Life Offices to point to their list of directers, who are all men of known integrity, in fact

all honourable men, and assure the public that their names are a sufficient guarantee that the affairs of the Company are honestly and judiciously managed. The veil is withdrawn for the first time from the operations of many ancient Companies, and the returns that are to be made as to the income and expenditure, assets and liabilities, will serve to enable the public whose support is courted to judge much more accurately than they could possibly have done hitherto, of the security offered by those Companies. Altho' the returns are not sufficient to enable the value of the liabilities of each Company to be rigorously estimated, yet they are quite sufficient to enable actuaries, and through them, the public, to judge of the stability and future prospects of the Life Offices of the country, on far surer grounds than were formerly available. Hitherto, when an actuary has been asked his opinion of certain Life Offices, all he could say was that he had no reason to doubt their solvency, but he really knew nothing of them but by popular repute, which is in most cases right, but is sometimes quite wrong. As regards other Offices, again, he could only say that he entertained grave doubts as to their solvency, but could give no conclusive proof that they were insolvent. In future, an actuary, when thus consulted, will be able to speak with more confidence and with greater plainness; and this cannot fail to be advantageous both to the public and to the well managed Offices. The sound Life Offices, which form the overwhelming majority, have nothing to fear, but on the contrary everything to gain from complete publicity, and it is to be expected that their prompt and strict compliance with the requirements of the Act will have the effect of increasing the confidence of the thinking part of the public in them and in the system of life insurance generally. In no business probably is publicity so desirable and so likely to produce good results as in life assurance, and the new Act will ensure a healthy publicity in future. From this point of view I think the passing of the new Act is a matter for unqualified congratulation.

In other respects, besides the publication of accounts, the Act will operate in the same direction. It provides that all Life Offices shall print and sell to policyholders their deeds of settlement. Thus policyholders will be able for the first time to ascertain with certainty the powers of the directers of the Offices in which they are insured, the nature of the investments in which their funds may be employed, with many other particulars of a like kind. They will also now learn the regulations of the Office as to the surrender value of policies, which is certainly a point on which the assured

have hitherto been kept far too much in the dark. There can be little doubt that a Life Insurance Company after making its return under the Sixth Schedule to the Act, and stating therein how it calculates the minimum surrender values of its policies, will be bound to allow values not less than those so calculated to all existing policyholders, and to all new policyholders who may join the Company before a new rule shall have been promulgated in a subsequent return. Thus, in fact, unless a Company chooses to guard itself very carefully in making the return, and to reserve to itself the right of altering its rule, the surrender values of policies will be in future a matter of strict contract. But I think still more may properly be done in the same direction. As suggested by Finanzrath Hopf in his paper read before the Institute last session, it might properly be made a part of the contract of life insurance, that the reserve made by the Company for its liability under each of its policies should not fall below a certain amount, calculated according to a rule made known beforehand. So also it might with great propriety be made a part of the contract, that the assurance fund of the Company should be invested in certain specified classes of securities and in no others.

We have seen that it is a fraud for a Life Insurance Company to keep its doors open and invite new insurances after it has become undoubtedly insolvent. So also we may remark, in passing, it is a fraud for a bank to continue its business in similar circumstances; and in both cases it should be made, if indeed it is not already, a criminal offence, for the business to be continued as usual when such a state of things has arisen. We have also seen that it falls within the province of the Government to discourage and punish fraud; and it therefore follows that it is one of the proper functions of the Government, either forcibly to put a stop to the business of the Company in the case supposed, or to afford the utmost facility to its creditors to prove insolvency against it, and have it wound up. It may be thought that in admitting this we admit the propriety of the American life insurance legislation; but this is not the case. The difficulty is, that whereas it is a comparatively easy matter to say when a bank is insolvent, it is by no means so easy to say when a Life Insurance Company is insolvent. Indeed, within very wide limits the solvency of a Life Insurance Company must be simply a matter of opinion. Supposing a Life Office to be liable under certain policies of insurance, issued at certain dates, and subject to certain premiums, the value of its liability thereunder admits of

being estimated in a variety of different ways; and the estimates made by different actuaries will in practice be found to differ very considerably, those estimates depending of course on the datums used in the valuation, viz., the rate of interest and the table of mortality; and it is impossible to say authoritatively which of such estimates is to be considered the correct one. All that can be said is, that assuming a certain rate of mortality will prevail in the future, and a certain rate of interest will be realized on the investments, and making a certain provision for future expenses of conducting the business, the liability of the Company is so much. What table of mortality, and what rate of interest, should be used in the calculations, or what allowance should be made for future expenses of the business, are all matters of opinion; and it is thus impossible to say that one particular estimate of the value of the liabilities is right and all others wrong. It is of course easy to find limits above and below which difference of opinion ceases, so that if a Company with known liabilities possesses a certain amount of assets, all actuaries will agree in saying that it is solvent, while on the other hand if it possesses less than a certain other amount of assets, all actuaries will agree in saying that it is insolvent. But, as stated above, these limits would be very wide; and if the Company possesses an intermediate amount of assets, probably some actuaries will say that in their opinion it is solvent, while others will say that in their opinion it is insolvent. The conclusion we arrive at therefore is, that the solvency of Life Insurance Companies being so much a matter of opinion, the Government cannot with propriety attempt to decide the question by laying down a legal standard of solvency. For the present, at all events, the Government should limit its action to requiring Life Offices to publish certain specified particulars, leaving all persons interested in each Office to form their own opinion as to its stability. This has been done by the passing of the new Act, and it now remains to be seen how this will work. I purposely abstain from all criticism of its details. Objections have been raised in some quarters to certain of the returns required to be made, on the grounds that they are both useless and inquisitorial; but it appears to me that it is unbecoming for the representatives of the Life Offices to indulge in such remarks. The public having by their representatives discussed and decided what returns would in their opinion secure the objects desired, it is both the duty and the interest of the Offices to comply cheerfully and unreservedly with the requirements of the Act, reserving any suggestions for its improvement until a suitable

opportunity shall arise. As regards the further step of fixing a legal standard of solvency, it is not to be denied that very strong arguments may be adduced in favor of it, or, at all events, of fixing a standard of insolvency, on reaching which a Company's operations shall be forcibly put a stop to. I hope shortly to return to this part of the subject, and to explain what would in my opinion be the most satisfactory practical manner of fixing such a standard, if it were on the whole thought desirable.

It will scarcely be necessary for me to add in conclusion, that I consider that the happy mean has been attained by the new Act. While it has greatly facilitated the exposure and suppression of unsound Companies, it throws (if we except the £20,000 deposit) the smallest possible impediments in the way of legitimate enterprise. The Government has, on the one hand, taken effectual measures to ensure that the public shall no longer be misled by false pretensions of solvency and prosperity, but shall be able to put all such pretensions to the test. On the other hand, it has avoided what I consider the cardinal error of the American system, namely, the attempt to secure the solvency of Life Offices. If persons, in the future, suffer loss in consequence of insuring in unsound Life Offices, they will have nobody to blame but themselves; and the Government will be quite free from the moral responsibility to make good that loss, which would certainly attach to it, if it held out to the public that it had taken steps to secure the solvency of the Life Offices of the country. I believe that the greatest good will result from the operation of the Act; and altho' experience may probably suggest improvements, they will, I believe, be in the details of the Act, and not in its principles.

On the Method of Calculating the Differential Coefficients of a Function from its Differences; and on their application to the Interpolation of functions of one, two, or three variables. By WILLIAM MATTHEW MAKEHAM, *Fellow of the Institute of Actuaries.*

THE development of the nth differential coefficient of a function in terms of the finite differences is involved in the expansion of the multinomial $\left(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\ldots\right)^n$. A general expression for the coefficient of each power, to the seventh inclusive, is given in De Morgan's *Diff. and Int. Calculus*, p. 264,—but the extremely

intricate character of such general expression renders it desirable to obtain a more convenient practical method of computing these coefficients to any extent that may be required.

Putting $n=2$ and multiplying out we get

$$
\begin{aligned}
&\Delta^2-\frac{1}{2}\Delta^3+\frac{1}{3}\Delta^4-\frac{1}{4}\Delta^5+\ \ldots\ldots\pm\frac{1}{m-1}\Delta^m\\
&\quad-\frac{1}{2}\Delta^3+\frac{1}{2.2}\Delta^4-\frac{1}{2.3}\Delta^5+\ldots\pm\frac{1}{2(m-2)}\Delta^m\\
&\qquad\qquad+\frac{1}{3}\Delta^4-\frac{1}{3.2}\Delta^5+\ldots\pm\frac{1}{3(m-3)}\Delta^m\\
&\qquad\qquad\qquad\qquad-\frac{1}{4}\Delta^5+\ldots\pm\frac{1}{4(m-4)}\Delta^m\\
&\qquad\qquad\qquad\qquad\qquad+\ldots\pm\ldots\\
&\qquad\qquad\qquad\qquad\qquad\quad\pm\frac{1}{m-1}\Delta^m
\end{aligned}
$$

Let $S_{m-1}=1+\frac{1}{2}+\ldots+\frac{1}{m-1}$,

and repeating the series in the reverse order

$$S_{m-1}=\frac{1}{m-1}+\frac{1}{m-2}+\frac{1}{m-3}+\ldots+1$$

we have by the addition of term by term

$$2S_{m-1}=\frac{m}{m-1}+\frac{m}{2(m-2)}+\frac{m}{3(m-3)}+\ldots+\frac{m}{m-1}$$

Therefore

$$\frac{2}{m}S_{m-1}=\frac{1}{m-1}+\frac{1}{2(m-2)}+\frac{1}{3(m-3)}+\ldots+\frac{1}{m-1}$$

But $\frac{2}{m}S_{m-1}=\frac{2}{m}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{m-1}\right)$

The coefficient of Δ^m in the expansion of $\left(\Delta-\frac{1}{2}\Delta^2+\frac{1}{3}\Delta^3-\frac{1}{4}\Delta^4+\ldots\right)^2$ is therefore $\frac{2}{m}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{m-1}\right)$, and we have

$$
\begin{aligned}
\frac{1}{2}\left(\frac{d}{dx}\right)^2=\frac{\Delta^2}{2}-\left(1+\frac{1}{2}\right)\frac{\Delta^3}{3}+\left(1+\frac{1}{2}+\frac{1}{3}\right)\frac{\Delta^4}{4}-\ldots\\
\pm\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{m-1}\right)\frac{\Delta^m}{m}\mp\ldots
\end{aligned}
$$

If the development of the nth differential coefficient be represented by the series

$$\Delta^n - A'_n\Delta^{n+1} + A''_n\Delta^{n+1} - \ldots \pm \overset{(r)}{A}_n\Delta^{n+r}$$

and the $n+1$th by

$$\Delta^{n+1} - A'_{n+1}\Delta^{n+2} + A''_{n+1}\Delta^{n+3} - \ldots \pm \overset{(r)}{A}_{n+1}\Delta^{n+r+1}$$

we have seen that in the particular case of $n=1$ the latter series becomes

$$(n+1)\left\{\frac{\Delta^{n+1}}{n+1} - (1+A'_n)\frac{\Delta^{n+2}}{n+2} + (1+A'_n+A''_n)\cdot\frac{\Delta^{n+3}}{n+3} - \ldots\right\}$$

the law connecting the coefficients in the expansion of $\left(\frac{d}{dx}\right)^n$ with those of the expansion of $\left(\frac{d}{dx}\right)^{n+1}$ being expressed by the equation

$$\overset{(r)}{A}_{n+1} = \frac{n+1}{n+r+1}(1+A'_n+A''_n+\ldots+\overset{(r)}{A}_n)$$

I proceed now to show that this law holds good for *all* values of n.

Multiplying together the two equations

$$\left(\frac{d}{dx}\right)^n = \Delta^n - A'_n\Delta^{n+1} + .A''_n\Delta^{n+2} - \ldots \pm \overset{(r)}{A}_n\Delta^{n+r}$$

$$\frac{d}{dx} = \Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \ldots \pm \frac{1}{r+1}\Delta^{r+1}$$

$$\left(\frac{d}{dx}\right)^{n+1} = \left\{\begin{array}{l} \Delta^{n+1} - A'_n\Delta^{n+2} + A''_n\Delta^{n+3} - \ldots \pm \overset{(r)}{A}_n\Delta^{n+r+1} \\ \qquad - \frac{1}{2}\Delta^{n+2} + \frac{1}{2}A'_n\Delta^{n+3} - \ldots \pm \frac{1}{2}\overset{(r-1)}{A}_n\Delta^{n+r+1} \\ \qquad\qquad + \frac{1}{3}\Delta^{n+3} - \ldots \pm \frac{1}{3}\overset{(r-2)}{A}_n\Delta^{n+r+1} \\ \qquad\qquad\qquad - \ldots \pm \ldots. \\ \qquad\qquad\qquad\qquad \pm \frac{1}{r+1}\Delta^{n+r+1} \end{array}\right.$$

from which it appears that $\overset{(r)}{A}_{n+1}$, the coefficient of $\pm\Delta^{n+r+1}$ in $\left(\frac{d}{dx}\right)^{n+1}$, is $\frac{1}{r+1} + \frac{1}{r}A'_n + \frac{1}{r-1}A''_n + \ldots + \frac{1}{2}\overset{(r-1)}{A}_n + \overset{(r)}{A}_n$. Consequently, if the supposed law be true, but not otherwise, we shall have

$$\frac{1}{r+1}+\frac{1}{r}A'_n+\frac{1}{r-1}A''_n+\ldots+\frac{1}{2}\overset{(r-1)}{A}_n+\overset{(r)}{A}_n$$
$$=\frac{n+1}{n+r+1}(1+A'_n+A''_n+\ldots+\overset{(r)}{A}_n)$$

or
$$r\overset{(r)}{A}_n=\frac{n-(r-1)}{2}\overset{(r-1)}{A}_n+\frac{2n-(r-2)}{3}\overset{(r-2)}{A}_n+\ldots$$
$$+\frac{(r-1)n-1}{r}A'_n+\frac{rn}{r+1}$$

and this equation therefore will form a test of the truth of the law in question.

Taking $r=1, 2, 3$, &c., successively in the last equation we have

$$A'_n=\frac{n}{2}$$
$$2A''_n=\frac{2n}{3}+\frac{n-1}{2}\cdot\frac{n}{2}=\frac{n}{3}\cdot\frac{3n+5}{4}$$
$$\therefore\quad A''_n=\frac{n}{3}\cdot\frac{3n+5}{8}$$
$$3A'''_n=\frac{3n}{4}+\frac{2n-1}{3}\cdot\frac{n}{2}+\frac{n-2}{2}\cdot\frac{n}{3}\cdot\frac{3n+5}{8}=n\frac{(n+2)(n+3)}{16}$$
$$\therefore\quad A'''_n=\frac{n}{4}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}$$
$$\ldots=\ldots\ldots\ldots$$

these several equations being nothing more than the general equation in another shape.

Now substituting in

$$\overset{(r)}{A}_{n+1}=\frac{n+1}{n+r+1}(1+A'_n+A''_n+\ldots+\overset{(r)}{A}_n)$$

the values just found, we have

$$\overset{(r)}{A}_{n+1}=\frac{n+1}{n+r+1}\left(1+\frac{n}{2}+\frac{n}{3}\cdot\frac{3n+5}{8}+\frac{n}{4}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}+\ldots\right)$$

and taking successive values of r

$$A'_{n+1}=\frac{n+1}{n+2}\left(1+\frac{n}{2}\right)=\frac{n+1}{2}$$
$$A''_{n+1}=\frac{n+1}{n+3}\left(1+\frac{n}{2}+\frac{n}{3}\cdot\frac{3n+5}{8}\right)=\frac{n+1}{n+3}\cdot\frac{3n^2+17n+24}{24}$$
$$=\frac{n+1}{3}\cdot\frac{3(n+1)+5}{8}$$

$$A'''_{n+1} = \frac{n+1}{n+4}\left(1 + \frac{n}{2} + \frac{n}{3}\cdot\frac{3n+5}{8} + \frac{n}{4}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}\right)$$

$$= \frac{n+1}{n+4}\cdot\frac{n^3+11n^2+40n+48}{48}$$

$$= \frac{n+1}{4}\cdot\frac{n^2+7n+12}{12} = \frac{n+1}{4}\cdot\frac{n+3}{3}\cdot\frac{n+4}{4}$$

Comparing side by side the two series of equations last obtained

$$A'_n = \frac{n}{2} \qquad A'_{n+1} = \frac{n+1}{2}$$

$$A''_n = \frac{n}{3}\cdot\frac{3n+5}{4} \qquad A''_{n+1} = \frac{n+1}{3}\cdot\frac{3(n+1)+5}{8}$$

$$A'''_n = \frac{n}{4}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4} \qquad A'''_{n+1} = \frac{n+1}{4}\cdot\frac{n+3}{3}\cdot\frac{n+4}{4}$$

we see that the second is the same function of $n+1$ that the first is of n; and putting the second series of equations in the *general* form we have

$$r\overset{(r)}{A}_{n+1} = \frac{(n+1)-(r-1)}{2}\overset{(r-1)}{A}_{n+1} + \frac{2(n+1)-(r-2)}{3}\overset{(r-2)}{A}_{n+1} + \dots + \frac{r(n+1)}{r+1}$$

It therefore appears that if the equation

$$r\overset{(r)}{A}_n = \frac{n-(r-1)}{2}\overset{(r-1)}{A}_n + \frac{2n-(r-2)}{3}\overset{(r-2)}{A}_n + \dots \frac{(r-1)n-1}{r}A'_n + \frac{rn}{r+1}$$

or its equivalent

$$\overset{(r)}{A}_{n+1} = \frac{n+1}{n+r+1}(1 + A'_n + A''_n + \dots + \overset{(r)}{A}_n)$$

be true for any one value of n it must also be true for the succeeding value. But we have seen that it is true for $n=1$, and therefore it is true for all values.

We have thus two expressions for the coefficients required in the development of $\left(\frac{d}{dx}\right)^n$,—by the first of which we may compute successively the coefficients for any assigned value of n, and by the second a complete series to any given number of terms for successive values of n from unity upwards. The following is an example of the latter process:—

1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
(1)	$(\frac{3}{2})$	$(\frac{11}{6})$	$(\frac{25}{12})$	$(\frac{137}{60})$	$(\frac{49}{20})$	$(\frac{363}{140})$	$(\frac{761}{280})$	$(\frac{7129}{2520})$	
1	1	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{137}{180}$	$\frac{7}{10}$	$\frac{363}{560}$	$\frac{761}{1260}$	$\frac{7129}{12600}$	
(1)	(2)	$(\frac{35}{12})$	$(\frac{15}{4})$	$(\frac{208}{45})$	$(\frac{469}{90})$	$(\frac{29531}{5040})$	$(\frac{6515}{1008})$		
1	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{15}{8}$	$\frac{29}{15}$	$\frac{469}{240}$	$\frac{29531}{15120}$	$\frac{1303}{672}$		
(1)	$(\frac{5}{2})$	$(\frac{17}{4})$	$(\frac{49}{8})$	$(\frac{967}{120})$	$(\frac{801}{10})$	$(\frac{4523}{378})$			
1	2	$\frac{17}{6}$	$\frac{7}{2}$	$\frac{967}{240}$	$\frac{89}{20}$	$\frac{4523}{945}$			
(1)	(3)	$(\frac{35}{6})$	$(\frac{28}{3})$	$(\frac{1069}{80})$	$(\frac{1425}{80})$				
1	$\frac{5}{2}$	$\frac{25}{6}$	$\frac{35}{6}$	$\frac{1069}{144}$	$\frac{1425}{160}$				
(1)	$(\frac{7}{2})$	$(\frac{23}{3})$	$(\frac{27}{2})$	$(\frac{3013}{144})$					
1	3	$\frac{23}{4}$	9	$\frac{3013}{240}$					
(1)	(4)	$(\frac{39}{4})$	$(\frac{75}{4})$						
1	$\frac{7}{2}$	$\frac{91}{12}$	$\frac{105}{8}$						
(1)	$(\frac{9}{2})$	$(\frac{145}{12})$							
1	4	$\frac{29}{3}$							
(1)	(5)								
1	$\frac{9}{2}$								
(1)									
1									

The calculation is here carried as far as the tenth order of differences. The line within brackets (   ) contains the sum of the line immediately above it,—and being multiplied by $\frac{n+1}{n+r+1}$ gives the next series of coefficients, which forms the line immediately following.

In a paper on the Law of Mortality read by me before the Institute, and published in vol. 13 of this *Journal,* I showed that the first order of differential coefficients could be developed in the form

$$\frac{d}{dx}=\Delta_{-\frac{1}{2}}-\frac{1}{24}\Delta^3_{-\frac{3}{2}}+\frac{3}{640}\Delta^5_{-\frac{5}{2}}-\ldots$$

and having been favoured by Mr. Sprague with a demonstration of that theorem, founded upon the calculus of operations, I avail myself of the method adopted by that gentleman, with some slight abbreviation, and with an extension of it to the case of higher differentials than the first.

The abbreviation consists in the assumption that the nth differential coefficient can be developed in the form

$$\left(\frac{d}{dx}\right)^n=\Delta^n_{-\frac{n}{2}}-\text{A}.\Delta^{n+2}_{-\frac{n+2}{2}}+\text{B}.\Delta^{n+4}_{-\frac{n+4}{2}}-\ldots$$

or, as it may be expressed by the calculus of operations,

$$\left(\frac{d}{dx}\right)^n=\{\Delta(1+\Delta)^{-\frac{1}{2}}\}^n-\text{A}\{\Delta(1+\Delta)^{-\frac{1}{2}}\}^{n+2}$$
$$+\text{B}\{\Delta(1+\Delta)^{-\frac{1}{2}}\}^{n+4}-\ldots$$

leaving the proof of the truth of the assumption of the vanishing of the alternate terms to be developed in the process of determining the values of the coefficients A, B, &c.

Expanding each term of the right-hand member of the last equation, we have

$$\begin{aligned}\left(\frac{d}{dx}\right)^n=&\left\{\Delta^n-\frac{n}{2}\cdot\Delta^{n+1}+\frac{n}{2}\cdot\frac{n+2}{4}\cdot\Delta^{n+2}-\frac{n}{2}\cdot\frac{n+2}{4}\cdot\frac{n+4}{6}\cdot\Delta^{n+3}\right.\\&\left.\qquad+\frac{n}{2}\cdot\frac{n+2}{4}\cdot\frac{n+4}{6}\cdot\frac{n+6}{8}\cdot\Delta^{n+4}-\ldots\right\}\\-\text{A}&\left\{\Delta^{n+2}-\frac{n+2}{2}\cdot\Delta^{n+3}\right.\\&\left.\qquad+\frac{n+2}{2}\cdot\frac{n+4}{4}\cdot\Delta^{n+4}-\ldots\right\}\\+\text{B}&\{\Delta^{n+4}-\ldots\}\end{aligned}$$

or collecting the terms,

$$\begin{aligned}\left(\frac{d}{dx}\right)^n=&\Delta^n-\frac{n}{2}\Delta^{n+1}+\left(\frac{n}{2}\cdot\frac{n+2}{4}-\text{A}\right)\Delta^{n+2}\\&-\left(\frac{n}{2}\cdot\frac{n+2}{4}\cdot\frac{n+4}{6}-\frac{n+2}{2}\text{A}\right)\Delta^{n+3}\\&+\left(\frac{n}{2}\cdot\frac{n+2}{4}\cdot\frac{n+4}{6}\cdot\frac{n+6}{8}-\frac{n+2}{2}\cdot\frac{n+4}{4}\text{A}+\text{B}\right)\Delta^{n+4}-\ldots\end{aligned}$$

Now we have seen that

$$\left(\frac{d}{dx}\right)^n=\Delta^n-\frac{n}{2}\Delta^{n+1}+\frac{n}{3}\cdot\frac{3n+5}{8}\Delta^{n+2}-\frac{n}{4}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}\Delta^{n+3}+\ldots$$

and comparing the two series together we see that the coefficients of the first and second terms in each are identical. This proves the assumption that the coefficient of $\{\Delta(1+\Delta)^{-\frac{1}{2}}\}^n$ is unity and that of $\{\Delta(1+\Delta)^{-\frac{1}{2}}\}^{n+1}$ zero. Again, equating the coefficients of Δ^{n+2}, we find that $A=\frac{n}{24}$, and substituting this value in the coefficient of Δ^{n+3} in the first series the coefficient in question becomes $\frac{n}{2}\cdot\frac{n+2}{4}\cdot\frac{n+4}{6}-\frac{n}{24}\cdot\frac{n+2}{2}=\frac{n}{4}\cdot\frac{n+2}{3}\cdot\frac{n+3}{4}$, which is identical with the corresponding coefficient of the second series, thus proving the assumption that the coefficient of $\{\Delta(1+\Delta)^{-\frac{1}{2}}\}^{n+3}$ is zero. Proceeding in the same way with the following terms, it will be found that the nth differential coefficient developed in terms of the central differences becomes

$$\Delta^n_{-\frac{n}{2}}-\frac{n}{24}\Delta^{n+2}_{-\frac{n+2}{2}}+\frac{n(5n+22)}{5760}\Delta^{n+4}_{-\frac{n+4}{2}}$$
$$-\frac{n(35n^2+462n+1528)}{2903040}\Delta^{n+6}_{-\frac{n+6}{2}}+\ldots$$

by which it will be seen that not only does the series consist of but half the number of terms (by the vanishing of the alternate terms) but the coefficients of the remaining terms are much simplified.

The following is an example of the process of determining the several orders of differential coefficients in a case where the fifth and higher differences are insignificant:—

24	30·421862	3·185337	·126166	4996	192
		3·185129	·126150	208	16
25	32·030300	3·249044	·128687	5094	204
		3·248832	·128670	212	17
26	33·670906	3·314024	·131260	5200	211
		3·313807	·131242	217	18
27	35·344324	3·380304	·133887	5305	208
		3·380083	·133870	221	17
28	37·051210	3·447911	·136565	5408	212
		3·447686	·136547	225	18
29	38·792235	3·516869	·139295	5517	220
		3·516639	·139277	230	18
30	40·568079	3·587206	·142082	5628	222
		3·586972	·142063	234	19

The first column contains the value of the variable, the second that of the given function. The next is the first order of differences of the latter, but taking *alternate* instead of *successive* terms. Thus, against the argument 25 we find 3·249044, which is the difference between the values of the function corresponding to the arguments 26 and 24; and the succeeding orders of differences are

derived in the same manner. The terms interposed (3·185129, &c.) form a part of the subsequent process.

In the formula for the development of $\left(\frac{d}{dx}\right)^n$ in terms of the finite differences the increment of the variable is supposed to be unity. If, however, it be supposed to be any other given quantity, h, we shall have $\left(\frac{d}{dx}h\right)^n$ in lieu of $\left(\frac{d}{dx}\right)^n$, and consequently the result has to be divided by h^n. In the preceding case $h=2$, and the formulæ become

$$\frac{d}{dx} = \frac{1}{2}\left(\Delta_{-1} - \frac{1}{24}\Delta^3{}_{-3}\right)$$

$$\left(\frac{d}{dx}\right)^2 = \frac{1}{4}\left(\Delta^2{}_{-2} - \frac{1}{12}\Delta^4{}_{-4}\right)$$

$$\left(\frac{d}{dx}\right)^3 = \frac{1}{8}\cdot\Delta^3{}_{-3}$$

$$\left(\frac{d}{dx}\right)^4 = \frac{1}{16}\Delta^4{}_{-4}$$

The terms of the fourth order of differences are each divided by 12, and those of the third order by 24, the quotients being inserted under each term in old type. These quotients are then deducted from the corresponding term in the first and second order, and the remainder appears (in ordinary type) under the term from which the subtraction is made. These processes give the means of constructing the Table required for use, a specimen of which follows (y_x being the value of the function, y'_x the first order of differential coefficients, &c.):—

x	y_x	y'_x	$\frac{1}{2}y''_x$	$\frac{1}{6}y'''_x$	$\frac{1}{24}y^{\text{IV}}_x$
24	30·421862	1·592564	·15769	104	1
25	32·030300	1·624416	·16084	106	0
26	33·670906	1·656904	·16405	108	1
27	35·344324	1·690041	·16734	111	0
28	37·051210	1·723843	·17068	113	1
29	38·792235	1·758319	·17410	115	0
30	40·568079	1·793486	·17758	117	1

The correctness of the several calculations is proved by the sum of each lateral series giving the first term of the next.

We have by a well-known theorem

$$y_{x+n} = y_x + n.y'_x + n^2\frac{y''_x}{2} + n^3.\frac{y'''_x}{2.3} + \ldots$$

an equation which will afford a convenient formula of interpolation for fractional values of n, when the Table is constructed according to the method above exemplified.

For instance let it be required to find the value of y_x for $x=24+\dfrac{1}{5}$.

Owing to the rapid diminution of the powers of n, three terms of the series will suffice. The calculation will therefore stand thus:

$$\begin{array}{r} 15769 \times \frac{1}{5} \\ \hline +3154 \\ 1{\cdot}592564 \\ \hline 1{\cdot}595718 \times \frac{1}{5} \\ \hline +{\cdot}319144 \\ 30{\cdot}421862 \\ \hline 30{\cdot}741006 \end{array}$$

Let us now work out the example by the ordinary method.

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$
24	30·421862	1·608438	32168	644
25	32·030300	1·640606	32812	656
26	33·670906	1·673418	33468	
27	35·344324	1·706886		
28	37·051210			

$$y_{x+n}=y_x+n\Delta y_x+n\frac{n-1}{2}\cdot\Delta^2 y_x+n\frac{n-1}{2}\cdot\frac{n-2}{3}\Delta^3 y_x+\ldots$$

$$\begin{array}{r} 644 \times -{\cdot}6 \\ \hline -386 \\ +32168 \\ \hline \times\, 31782 \times -{\cdot}4 \\ \hline -12713 \\ +1{\cdot}608438 \\ \hline 1{\cdot}595725 \times \frac{1}{5} \\ \hline +{\cdot}319145 \\ 30{\cdot}421862 \\ \hline 30{\cdot}741007 \end{array}$$

$$\frac{\frac{1}{5}-1}{2}=\frac{1-5}{10}=-{\cdot}4$$

$$\frac{\frac{1}{5}-2}{3}=\frac{1-10}{15}=-\frac{9}{15}=-{\cdot}6$$

So that not only is the preliminary calculation of the values of $\frac{n-1}{2}, \frac{n-2}{3}$, &c., avoided by the first process, but a term of the series is saved by the more rapid convergency of the expansion. If the fourth term were omitted in the second process the error would be $644 \times \cdot 6 \times \cdot 4 \times \cdot 2 = 31$.

If we change the sign of n in the formula

$$y_{n+n} = y_x + n \,.\, y'_x + n^2 \,.\, \frac{y''_x}{2} + n^3 \,.\, \frac{y'''_x}{6} + \ldots$$

we have

$$y_{x-n} = y_x - n \,.\, y'_x + n^2 \,.\, \frac{y''_x}{2} - n^3 \,.\, \frac{y'''_x}{6} + \ldots$$

the terms being the same as before except that every alternate term is negative. This property will be found convenient in interpolating a series of values in the given Table. The value of n should always be taken between $+\frac{1}{2}$ and $-\frac{1}{2}$ in order to secure the highest possible degree of convergency.

For reasons which will afterwards appear it will be convenient to put the Table in the following form:—

x	y_x
24	30·421862 1·592564 15769 104 1
25	32·030300 1·624416 16084 106 0
26	33·670906

The development of y_{x+n} in terms of y_x and its several orders of differences may be expressed by the equation

$$y_{x+m} = y_x (1+\Delta)^m$$

and if the binomial $(1+\Delta)^m$ be expanded in powers of Δ and the factor y_x annexed to each term we shall have the well known formula

$$y_{x+m} = y_x + m\Delta y_x + m\,\frac{m-1}{2}\,\Delta^2 y_x + \ldots$$

Similarly if ${}_xy_z$ denote a function of two variables, x and z, we shall have

$$ {}_{x+m}y_{z+n} = {}_xy_z(1+\Delta)^m \, . \, (1+\Delta')^n $$

the symbol Δ representing the process of differencing with respect to x, and Δ' with respect to z. Expanding as before we get (assuming third differences to vanish)

$$ {}_{x+m}y_{z+n} = {}_xy_z + m\Delta_xy_z + n\Delta'_xy_z $$
$$ + \left(m\frac{m-1}{2}\,\Delta^2{}_xy_z + mn\,.\,\Delta\Delta'{}_xy_z + n\frac{n-1}{2}\,\Delta'^2{}_xy_z \right) $$
$$ + \left(mn\frac{n-1}{2}\,\Delta\Delta'^2{}_xy_z + nm\frac{m-1}{2}\,\Delta'\Delta^2{}_xy_z \right) + mn\frac{m-1}{2}\cdot\frac{n-1}{2}\,\Delta^2\Delta'^2{}_xy_z $$

$\Delta^2\Delta'$ denoting twice differencing with respect to x and once with respect to z. Again if we suppose second differences to vanish the formula becomes

$$ {}_{x+m}y_{z+n} = {}_xy_z + m\,.\,\Delta_xy_z + n\,.\,\Delta'_xy_z + mn\,.\,\Delta\Delta'_xy_z $$

The arrangement of the Table for this case will be as follows:

x	z							
	0		1		2		3	
0	24·30 +1·48	+1·24 + ·16	25·54 +1·64	+1·33 + ·17	26·87 +1·81	+1·41 + ·18	28·28 +1 99	+1·50 + ·20
1	25·78 +1·52	+1·40 + ·16	27·18 +1·68	+1·50 + ·18	28·68 +1·86	+1·59 + ·20	30·27 +2·06	+1·70 + ·22
2	27·30 +1·55	+1·56 + ·17	28·86 +1·72	+1·68 + ·19	30·54 +1·91	+1 79 + ·22	32·33 +2·13	+1·92 + ·24

In this Table the values of the function are printed in ordinary, and the differences in old type. The difference placed beneath the function is with respect to x and that placed to the right with respect to z,—the remaining space in each square being occupied by the result of differencing once with respect to x and once with respect to z.

Example.—Let it be required to determine ${}_{\frac{1}{2}}y_{\frac{1}{2}}$.

Substituting in the last formula we find

$$ {}_{\frac{1}{2}}y_{\frac{1}{2}} = 24{\cdot}30 + \frac{1}{2} \times 1{\cdot}48 + \frac{1}{2} \times 1{\cdot}24 + \frac{1}{4} \times {\cdot}16. $$

$$ \begin{array}{r} 24{\cdot}30 \\ {\cdot}74 \\ {\cdot}62 \\ {\cdot}04 \\ \hline 25{\cdot}70 \end{array} $$

I now give a specimen of the case of two orders of differences for each variable.

x	z						
	0	Δ'	Δ'^2	1	Δ'	Δ'^2	2
0	24·297	+1·248	+ 77	25·545	+1·325	+ 85	26·780
Δ	+1·486	+ ·152	+ 16	+1·638	+ ·168	+ 15	+1·806
Δ^2	+ 30	+ 12	+ 1	+ 42	+ 13	+ 3	+ 55
1	25·783	+1·400	+ 93	27·183	+1·493	+ ·100	28·676
Δ	+1·516	+ ·164	+ 17	+1·680	+ ·181	+ 18	+1·861
Δ^2	+ 30	+ 11	+ 3	+ 41	+ 14	+ 2	+ 55
2	27·299	+1·564	+ ·110	28·863	+1·674	+ ·118	30·537

This Table is arranged upon the same principle as the last, and therefore needs no explanation. As an example of its use let it be required to determine the value of ${}_xy_z$ for $x=\frac{1}{2}$ $z=\frac{1}{3}$.

Neglecting insignificant terms we have

$$ {}_{\frac{1}{2}}y_{\frac{1}{3}} = {}_0y_0 + \left(\frac{1}{2}\Delta_0y_0 + \frac{1}{3}\Delta'{}_0y_0\right) + \left(\frac{1}{6}\Delta\Delta'{}_0y_0 - \frac{1}{8}\Delta^2{}_0y_0 - \frac{1}{9}\Delta'^2{}_0y_0\right) $$

$$ \begin{array}{r|cr} & & 24{\cdot}297 \\ & & {\cdot}743 \\ & & {\cdot}416 \\ -4 & & 25 \\ -9 & = & \bar{2}7 \\ \cline{3-3} & & 25{\cdot}468 \end{array} $$

The above is a specimen of the process of direct *double* interpolation with differences of the second degree. I now solve the

same case in the ordinary way,—that is by successive *single* interpolations.

24·297		25·545		26·870		25·036
·743		·819		·903		·441
$\bar{4}$		$\bar{5}$		$\bar{7}$		$\bar{9}$
25·036		26·359		27·766		25·468
	1·323		1·407			
		84				

The coefficients in the general formula for direct interpolation with two variables are somewhat complicated,—but this disadvantage is considerably reduced by the substitution of differential coefficients for finite differences. This substitution may be effected thus:

$${}_{x+m}y_{z+n} = {}_xy_z \cdot \varepsilon^{\frac{d}{dx}m + \frac{d}{dz}n}$$

Expanding the second factor

$$\varepsilon^{\frac{d}{dx}m + \frac{d}{dz}n} = 1 + \left(\frac{d}{dx}m + \frac{d}{dz}n\right) + \frac{1}{2}\left(\frac{d}{dx}m + \frac{d}{dz}n\right)^2 + \dots$$

Again expanding the several powers of the binomial we obtain a succession of differential coefficients of the form $\left(\frac{d}{dx}\right)^p \cdot \left(\frac{d}{dz}\right)^q \cdot$ which denotes p differentiations with respect to x and q with respect to z,—the general term being

$$\frac{m^p . n^q.}{1.2 \dots p \times 1.2 \dots q} \cdot \left(\frac{d}{dx}\right)^p \cdot \left(\frac{d}{dz}\right)^q$$

Hence if ${}_{x''}y_{z'}$ denote two differentiations with respect to x and one with respect to z, and so on, we have

$${}_{x+m}y_{z+n} = {}_xy_z + (m \cdot {}_{x'}y_z + n \cdot {}_xy_{z'}) + \left(m^2 \cdot \frac{{}_{x''}y_z}{2} + mn \cdot {}_{x'}y_{z'} + n^2 \cdot \frac{{}_xy_{z''}}{2}\right) + \dots$$

These differential coefficients may be easily computed by the methods previously described. Dividing each by the factor $1.2 \dots p \times 1.2 \dots q$ and then tabulating the results in the way adopted in the Table last given, we shall have the means of interpolating with great facility; the process being reduced to the multiplication of each tabulated value by the extremely simple expression $m^p . n^q$. As m and n are always less than $\frac{1}{2}$, and the tabulated quantities into which they are to be multiplied diminish much more rapidly than simple differences, a very few terms of the series will generally suffice.

If m and n are each equal to unity the series will consist simply of the several tabulated values. Hence if we add together the whole contents of each division the sum should be equal to the next tabular value of the function in a diagonal direction,—*i.e.* to the value corresponding to the next higher values of the two variables.

I conclude this part of my subject by giving a specimen of the tabulation of a function of two variables with four significant orders of differences. The arrangement is precisely the same as in the last Table, but the divided differential coefficients are substituted for the finite differences. Bearing in mind that in interpolating the values of m and n need never exceed $\frac{1}{2}$, it will be seen that one half of the tabulated differential values become insignificant.

	41·689196	1·353489	25763	340	3	43·068791	
	2·813181	158133	4254	73	1	2·975642	
	61914	6845	279	7	0	69045	
	909	150	10	0	0	1069	
	10	2	0	0	0	12	
	44·565210	1·518619	30306	420	4	46·114559	

We now come to the case of functions of three variables, which we shall see admits of being treated with as much simplicity as the two preceding cases.

Let the function to be tabulated be represented by ${}_{x}\overset{\omega}{y}_{z}$, and let the process of differentiation be noted as before by accents placed over the variables employed. Then

$$ {}_{x+m}\overset{\omega+k}{y}_{z+n} = {}_{x}\overset{\omega}{y}_{z}\,\varepsilon^{\frac{d}{d\omega}k+\frac{d}{dx}m+\frac{d}{dz}n} $$

and

$$ \varepsilon^{\frac{d}{d\omega}k+\frac{d}{dx}m+\frac{d}{dz}n} = 1 + \left(\frac{d}{d\omega}k + \frac{d}{dx}m + \frac{d}{dz}n\right) $$

$$ + \frac{1}{2}\left(\frac{d}{d\omega}k + \frac{d}{dx}m + \frac{d}{dz}n\right)^2 + \ldots $$

Expanding the several powers of the trinomial, we find the general term of this development to be

$$ \frac{k^p \,.\, m^q \,.\, n^r}{1.2\,..\,p \times 1.2\,..\,q \times 1.2\,..\,r}\left(\frac{d}{d\omega}\right)^p \cdot \left(\frac{d}{dx}\right)^q \cdot \left(\frac{d}{dz}\right)^r $$

so that for the expansion of ${}_{x+m}\overset{\omega+k}{y}_{z+n}$ we have

$$
\begin{aligned}
{}_{x}\overset{\omega}{y}_{z} &+ \left(k\,.\underset{x\;z}{\overset{\omega'}{y}} + m\,.\underset{x'\,z}{\overset{\omega}{y}} + n\,.\underset{x\;z'}{\overset{\omega}{y}}\right) + \left(k^2\cdot\frac{{}_{x}\overset{\omega''}{y}_{z}}{2} + km\,.\underset{x'\,z}{\overset{\omega'}{y}} + kn\,.\underset{x\;z'}{\overset{\omega'}{y}} + m^2\cdot\frac{{}^{\prime\prime}_{x}\overset{\omega}{y}_{z}}{2}\right.\\
&\left. + mn\underset{x'\,z'}{\overset{\omega}{y}} + n^2\cdot\frac{{}_{x}\overset{\omega}{y}{}^{\prime\prime}_{z}}{2}\right) + \left(k^3\frac{{}_{x}\overset{\omega'''}{y}_{z}}{2.3} + k^2m\frac{{}^{\prime}_{x}\overset{\omega''}{y}_{z}}{2} + k^2n\frac{{}_{x}\overset{\omega''}{y}{}^{\prime}_{z}}{2} + km^2\frac{{}^{\prime\prime}_{x}\overset{\omega'}{y}_{z}}{2}\right.\\
&+ kn^2\frac{{}_{x}\overset{\omega'}{y}{}^{\prime\prime}_{z}}{2} + kmn\cdot{}^{\prime}_{x}\overset{\omega'}{y}{}^{\prime}_{z} + m^2n\frac{{}^{\prime\prime}_{x}\overset{\omega}{y}{}^{\prime}_{z}}{2} + mn^2\cdot\frac{{}^{\prime}_{x}\overset{\omega}{y}{}^{\prime\prime}_{z}}{2} + m^3\cdot\frac{{}^{\prime\prime\prime}_{x}\overset{\omega}{y}_{z}}{2.3}\\
&\left. + n^3\cdot\frac{{}_{x}\overset{\omega}{y}{}^{\prime\prime\prime}_{z}}{2.3}\right) + \ldots
\end{aligned}
$$

As an example of the arrangement of the Table for three variables I will take a case with two significant orders of differences for each variable. The following extracts are supposed to form the first portions of four consecutive *right hand* pages of the book,—the left hand pages being occupied by a separate series proceeding in the reverse order:

1. *Functional Values.* $\omega = 0$.

x	z				
	0			1	
0	28·542 1·742 22	1·519 174 7	49 9 0	30·110 1·925 29	
1	30·306	1·700	58	32·064	

2. *First Differential Values.* $\omega = 0$.

x	z				
	0			1	
0	1·393 85 1	75 8 0	2 1 0	1·470 94 1	
1	1·479	83	3	1·565	

3. *Second Differential Values.* $\omega = 0$.

x	z				
	0			1	
0	34 2 0	2 0 0	0 0 0	36 2 0	
1	36	2	0	38	

4. *Functional Values.* $\omega = 1$.

x	z				
	0			1	
0	29·969 1·828 24	1·596 183 6	51 10 0	31·616 2·021 30	
1	31·821	1·785	61	33·667	

The arrangement of (1) is the same as in the last case.

Turning over the leaf we have (on the right-hand page) the results of the differentiation, with respect to ω, of each value in (1). Again turning over the leaf we have (also on the right-hand page) the divided second differentiations of the same values. The next right-hand page contains the continuation of (1) for the next higher value of ω,—and is succeeded in the same manner with the results of the two corresponding differentiations.

Example. Let it be required to determine the value of ${}_{\frac{1}{3}}y_{\frac{1}{4}}^{\frac{1}{5}}$.

$$
\begin{array}{rcr}
 & & 28{\cdot}542 \\
1{\cdot}742 \times \frac{1}{3} & = & {\cdot}581 \\
22 \times \frac{1}{9} & = & 2 \\
1{\cdot}519 \times \frac{1}{4} & = & {\cdot}380 \\
{\cdot}174 \times \frac{1}{12} & = & 14 \\
49 \times \frac{1}{16} & = & 3 \\
1{\cdot}393 \times \frac{1}{5} & = & {\cdot}279 \\
85 \times \frac{1}{15} & = & 6 \\
75 \times \frac{1}{20} & = & 4 \\
34 \times \frac{1}{25} & = & 1 \\
\hline
 & & 29{\cdot}812
\end{array}
$$

I now solve the same case by the ordinary method of successive simple interpolations.

x, z				x, z				x, z			
					$\omega=0$						
0, 0 1, 0 2, 0	28·542 30·306 32·113	+1·764 +1·807	+43	0, 1 1, 1 2, 1	30·110 32·064 34·076	+1·954 +2·012	+58	0, 2 1, 2 2, 2	31·782 33·945 36·183	+2·163 +2·238	+75
					$\omega=1$						
0, 0 1, 0 2, 0	29·969 31·821 33·718	+1·852 +1·897	+45	0, 1 1, 1 2, 1	31·616 33·667 35·779	+2·051 +2·112	+61	0, 2 1, 2 2, 2	33·372 35·643 37·992	+2·271 +2·349	+78
					$\omega=2$						
0, 0 1, 0 2, 0	31·467 33·412 35·404	+1·945 +1·992	+47	0, 1 1, 1 2, 1	33·197 35·350 37·568	+2·153 +2·218	+65	0, 2 1, 2 2, 2	35·041 37·425 39·892	+2·384 +2·467	+83

The above functional values are the same as those contained in the last Table, but without the differential values. They are here differenced in the ordinary way with respect to x,—the first step in the process being to determine the values of ${}_{\frac13}\overset{0}{y}_0$, ${}_{\frac13}\overset{0}{y}_1$, ${}_{\frac13}\overset{0}{y}_2$, which enable us to find the value of ${}_{\frac13}\overset{0}{y}_{\frac14}$; next to determine the values of ${}_{\frac13}\overset{1}{y}_0$, ${}_{\frac13}\overset{1}{y}_1$, ${}_{\frac13}\overset{1}{y}_2$, by which ${}_{\frac13}\overset{1}{y}_{\frac14}$ is found; then by computing ${}_{\frac13}\overset{2}{y}_0$, ${}_{\frac13}\overset{2}{y}_1$, ${}_{\frac13}\overset{2}{y}_2$, to get the value of ${}_{\frac13}\overset{2}{y}_{\frac14}$. The three values ${}_{\frac13}\overset{0}{y}_{\frac14}$, ${}_{\frac13}\overset{1}{y}_{\frac14}$, and ${}_{\frac13}\overset{2}{y}_{\frac14}$ suffice to determine the final value ${}_{\frac13}\overset{\frac15}{y}_{\frac14}$. The following is the process in question worked out at length.

$\frac{\frac13-1}{2}=\frac{1-3}{6}=-\frac{2}{6}=-\frac{1}{3}$ | $\frac{\frac14-1}{2}=\frac{1-4}{8}=-\frac{3}{8}$ | $\frac{\frac15-1}{2}=\frac{1-5}{10}=-\frac{4}{10}=-\frac{2}{5}$

$+43\times-\frac13$	$+58\times-\frac13$	$+75\times-\frac13$	$+110\times-\frac38$
-14	-19	-25	330
$+1·764$	$+1·954$	$+2·163$	-41
$+1·750\times\frac13$	$+1·935\times\frac13$	$+2·138\times\frac13$	$+1·630$
$·533$	$·645$	$·713$	$+1·589\times\frac14$
$28·542$	$30·110$	$31·782$	$·397$
$29·125$	$30·755$	$32·495$	$29·125$
			$29·522$

$\underbrace{29·125 \quad 30·755}_{1·630} \quad \underbrace{30·755 \quad 32·495}_{1·740}$

$\underbrace{1·630 \quad 1·740}_{110}$

$+45\times-\frac13$	$+61\times-\frac13$	$+78\times-\frac13$	$+115\times-\frac38$
-15	-20	-26	345
$+1·852$	$+2·051$	$+2·271$	-43
$+1·837\times\frac13$	$+2·031\times\frac13$	$+2·245\times\frac13$	$+1·712$
$+·612$	$+·677$	$+·748$	$+1·669\times\frac14$
$29·969$	$31·616$	$33·372$	$+·417$
$30·581$	$32·293$	$34·120$	$30·581$
			$30·998$

$\underbrace{30·581 \quad 32·293}_{1·712} \quad \underbrace{32·293 \quad 34·120}_{1·827}$

$\underbrace{1·712 \quad 1·827}_{115}$

$+47\times-\frac13$	$+65\times-\frac13$	$+83\times-\frac13$	$+122\times-\frac38$
-16	-22	-28	366
$+1·945$	$+2·153$	$+2·384$	-46
$+1·929\times\frac13$	$2·131\times\frac13$	$2·356\times\frac13$	$+1·797$
$·643$	$·710$	$·785$	$1·751\times\frac14$
$31·467$	$33·197$	$35·041$	$·438$
$32·110$	$33·907$	$35·826$	$32·110$
			$32·548$

$\underbrace{32·110 \quad 33·907}_{1·797} \quad \underbrace{33·907 \quad 35·826}_{1·919}$

$\underbrace{1·797 \quad 1·919}_{122}$

$$\begin{array}{lll} 29{\cdot}522 & & \\ & +1{\cdot}476 & \\ 30{\cdot}998 & & +74 \\ & +1{\cdot}550 & \\ 32{\cdot}548 & & \end{array} \qquad\qquad \begin{array}{r} +74 \times -{\cdot}4 \\ \hline -30 \\ +1{\cdot}476 \\ \hline 1{\cdot}446 \times \frac{1}{5} \\ \hline +{\cdot}289 \\ 29{\cdot}522 \\ \hline 29{\cdot}811 \end{array}$$

Mr. Sprague's demonstration, above referred to, of the formula

$$\frac{d}{dx}u_x = \Delta u_{x-\frac{1}{2}} - \frac{1}{24}\Delta^3 u_{x-\frac{3}{2}} + \frac{3}{640}\Delta^5 u_{x-\frac{5}{2}} - \ldots$$

is as follows:—

By the principles of the calculus of operations,

$$u_{x-\frac{1}{2}} = D^{-\frac{1}{2}}u_x, \qquad u_{x-\frac{3}{2}} = D^{-\frac{3}{2}}u_x, \text{ \&c.}$$

Hence, separating the symbols of operation,

$$\frac{d}{dx}u_x = \left(\Delta D^{-\frac{1}{2}} - \frac{1}{24}\Delta^3 D^{-\frac{3}{2}} + \frac{3}{640}\Delta^5 D^{-\frac{5}{2}} - \ldots\right)u_x$$

The problem therefore is to expand $\frac{d}{dx}$ in a series of powers of $\Delta D^{-\frac{1}{2}}$, or $\frac{\Delta}{D^{\frac{1}{2}}}$.

Now it is well known that $D = e^{\frac{d}{dx}}$, so that $\frac{d}{dx} = \log D$. Also since $\Delta = D - 1$, $\frac{\Delta}{D^{\frac{1}{2}}} = \frac{D-1}{D^{\frac{1}{2}}}$; so that the problem resolves itself into the expansion of $\log D$ in a series proceeding by powers of $\frac{D-1}{D^{\frac{1}{2}}}$, $=y$, suppose. Solving the quadratic equation,

$$D - yD^{\frac{1}{2}} - 1 = 0,$$

we get

$$D^{\frac{1}{2}} = \frac{y}{2} \pm \sqrt{\frac{y^2}{4} + 1}$$

whence

$$\log D = 2\log\left(\frac{y}{2} \pm \sqrt{\frac{y^2}{4} + 1}\right)$$
$$= 2\log\left(m + \sqrt{m^2 + 1}\right),$$

putting m for $\frac{y}{2}$, and taking the $+$ sign to the square root, since the $-$ sign would lead to the log of a negative quantity, which is imaginary. We have now to expand the above log in a series proceeding by powers of m. For this purpose, we observe that

$$\begin{aligned}\log(m+\sqrt{m^2+1}) &= \log\{m+(1+m^2)^{\frac{1}{2}}\}\\ &= \log\{m+1+\tfrac{1}{2}m^2-\tfrac{1}{8}m^4+\dots\}\\ &= \log\{1+(m+\tfrac{1}{2}m^2-\tfrac{1}{8}m^4+\dots)\}\\ &= (m+\tfrac{1}{2}m^2-\tfrac{1}{8}m^4-\dots)\\ &\quad -\tfrac{1}{2}(m+\tfrac{1}{2}m^2-\dots)^2\\ &\quad +\tfrac{1}{3}(m+\dots)^3\\ &\quad -\dots\dots\\ &= m-\tfrac{1}{6}m^3+\dots\end{aligned}$$

Assume then that

$$\log(m+\sqrt{m^2+1})=m+a_2m^2+a_3m^3+\dots+a_nm^n+\dots$$

Differentiating both sides

$$\frac{1}{\sqrt{m^2+1}}=1+2a_2m+3a_3m^2+\dots+na_nm^{n-1}+\dots$$

But expansion by the binomial theorem gives

$$\frac{1}{\sqrt{m^2+1}}=1-\frac{1}{2}m^2+\frac{1\cdot 3}{2\cdot 4}m^4-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}m^6+\dots$$
$$+(-)^p\frac{1\cdot 3\cdot 5\dots\overline{2p-1}}{2\cdot 4\cdot 6\dots 2p}\cdot m^{2p}+\dots$$

A comparison of these two expansions of $(m^2+1)^{-\frac{1}{2}}$ shows that

$$a_2=a_4=\dots=a_{2p}=\dots=0.$$

Also that

$$a_{2p+1}=(-)^p\frac{1}{2p+1}\cdot\frac{1\cdot 3\cdot 5\dots\overline{2p-1}}{2\cdot 4\cdot 6\dots 2p}.$$

We therefore have

$$\log \mathrm{D}=2\left\{m-\frac{m^3}{6}+\frac{3}{40}m^5-\frac{5}{112}m^7+\dots\right.$$
$$\left.+(-)^p\frac{1}{2p+1}\cdot\frac{1\cdot 3\cdot 5\dots(2p-1)}{2\cdot 4\cdot 6\dots 2p}\cdot m^{2p+1}+\dots\right\}$$

and putting $\frac{y}{2}$ for m, we get

$$\log \mathrm{D}=y-\frac{y^3}{24}+\frac{3}{640}y^5-\dots$$

and finally, since $y=\frac{\Delta}{\mathrm{D}^{\frac{1}{2}}}$

$$\frac{d}{dx}=\frac{\Delta}{\mathrm{D}^{\frac{1}{2}}}-\frac{1}{24}\frac{\Delta^3}{\mathrm{D}^{\frac{3}{2}}}+\frac{3}{640}\frac{\Delta^5}{\mathrm{D}^{\frac{5}{2}}}-\dots$$
$$+(-)^p\frac{1}{2p+1}\cdot\frac{1\cdot 3\cdot 5\dots(2p-1)}{2^{3p}\underline{|p}}\cdot\frac{\Delta^{2p+1}}{\mathrm{D}^{\frac{2p+1}{2}}}+\dots$$

Adjusted Mortality of the British Peerage Females.

Dr. T. N. Thiele makes the following remarks in reply to those we prefixed to his adjusted table of mortality given in the last number of this *Journal*:—

I must consider the *constant* difference of about 0·1 after the age of 50 between my adjustment and the numbers of living, which follow from the observations, but are not directly observed, and therefore *not independent of each other*, as the probabilities of living are, as an agreement completely satisfactory. The apparent disagreement I consider to be the result of one observation (52–53), where the observed probability of life disagrees strongly with the numbers of the other adjacent ages, and is not compensated by the following numbers disagreeing sufficiently in the opposite direction. Whence in comparing the calculated probabilities with the observed no systematical disagreement appears in these ages, as the following table shows; but if the numbers of living are compared, every other adjustment must show a disagreement of the same kind at the age of 50 as mine.

Table of Comparison.

Age.	Error of Probability of Living.	Mean Error.	Age.	Error of Probability of Living.	Mean Error.
0 to 1	+0·00021	±0·00513	33 to 34	− 322	
1 „ 2	− 6	283	34 „ 35	+ 261	± 264
2 „ 3	− 2	188	35 „ 36	− 447	
3 „ 4	− 43	161	36 „ 37	+ 301	
4 „ 5	− 80	156	37 „ 38	+ 162	
5 „ 6	+ 217		38 „ 39	+ 255	
6 „ 7	− 87		39 „ 40	+ 175	295
7 „ 8	− 84		40 „ 41	− 475	
8 „ 9	+ 84		41 „ 42	+ 70	
9 „ 10	+ 149	166	42 „ 43	+ 476	
10 „ 11	− 318		43 „ 44	− 223	
11 „ 12	− 416		44 „ 45	− 178	325
12 „ 13	− 20		45 „ 46	− 14	
13 „ 14	+ 397		46 „ 47	− 125	
14 „ 15	− 15	189	47 „ 48	+ 369	
15 „ 16	+ 242		48 „ 49	+ 485	
16 „ 17	+ 162		49 „ 50	− 25	377
17 „ 18	− 380		50 „ 51	− 354	
18 „ 19	+ 144		51 „ 52	+ 316	
19 „ 20	− 257	216	52 „ 53	+ 950	423
20 „ 21	+ 99		53 „ 54	+ 295	
21 „ 22	+ 94		54 „ 55	− 277	460
22 „ 23	+ 148		55 „ 56	− 382	
23 „ 24	− 161		56 „ 57	− 779	
24 „ 25	− 53	225	57 „ 58	+ 652	
25 „ 26	− 56		58 „ 59	− 30	
26 „ 27	+ 128		59 „ 60	+ 780	607
27 „ 28	+ 9		60 „ 61	+ 13	
28 „ 29	− 318		61 „ 62	− 470	
29 „ 30	+ 140	240	62 „ 63	− 454	
30 „ 31	+ 278		63 „ 64	+ 905	
31 „ 32	+ 84		64 „ 65	− 804	812
32 „ 33	+ 24		65 „ 66	− 1240	

Table of Comparison—(continued.)

Age.	Error of Probability of Living.	Mean Error.	Age.	Error of Probability of Living.	Mean Error.
66 to 67	+ 786		86 to 87	+ 5466	
67 „ 68	+ 1130		87 „ 88	+ 5307	
68 „ 69	− 395		88 „ 89	+ 1780	
69 „ 70	+ 627	± 1146	89 „ 90	+ 4803	± 7595
70 „ 71	+ 873		90 „ 91	− 817	
71 „ 72	− 621		91 „ 92	+ 7583	
72 „ 73	+ 1317		92 „ 93	+ 5727	
73 „ 74	+ 488		93 „ 94	+ 13251	
74 „ 75	− 2743	1611	94 „ 95	− 246	16107
75 „ 76	+ 1539		95 „ 96	+ 8500	
76 „ 77	+ 1545		96 „ 97	+ 30599	
77 „ 78	− 2078		97 „ 98	+ 32626	
78 „ 79	+ 2558		98 „ 99	+ 34584	
79 „ 80	+ 1798	2435	99 „ 100	+ 37065	34160
80 „ 81	− 2086		100 „ 101	+ 38828	
81 „ 82	− 3746		101 „ 102	+ 40869	
82 „ 83	− 2542		102 „ 103	+ 43028	
83 „ 84	+ 1483		103 „ 104	+ 44475	
84 „ 85	+ 6136	4236	104 „ 105	+0·46600	0·49889
85 „ 86	− 9134				

In reply to our inquiry whether he considers that the difference between the observed and the calculated probabilities of living a year at the age 52–53, viz. ·00950, is so much larger than the mean error ·00423 as to prove that the observation at that age is untrustworthy and to justify its rejection, he adds:—

I have used this number in the same way as all the other numbers, and with its due weight; and it is not my meaning to infer the existence of a systematical error from the excess of the error above the mean error. Quite the contrary. Since the difference is not greater, and appears isolated in comparing the probabilities, I conclude that there is no reason to suppose a systematical error in my calculation. But we need not look far to find other instances of the same kind. At the age 11–12 the ratio between the error and the mean error is nearly the same as at the age 52–53. But it will be observed that in the former case there is an error at the age 13–14 almost as large but of opposite sign. Now in the comparison of the numbers living, the original difference at the age 11–12 produces corresponding differences at the ages 12 and 13; in other words, until the difference at the age 13–14 can show its influence. The same occurs in the other instance, only it is much longer before the antidote begins to work, not until the point where my adjustment has its greatest errors, and negative differences appear of sufficient magnitude to balance the positive difference of the probability of living a year at the age 52–53.

Let us suppose that two persons play a game in which their chances of winning are equal, and let us also suppose that at one period of the play one of them wins a considerable sum, and that afterwards the play continues with nearly equal success on each side; so that the gain of the successful player at the termination of the play is nearly the same as it was at the

period in question. I think we have no right to infer that the play was unfair; on the contrary, the fact that the play has lasted a considerable time with about equal success to each player proves that the play was fair and honest. Now the excess of the unadjusted numbers living over the adjusted may be compared to the gains of the one player during the course of the play. In the 12th round a sum is won which is lost again in the 14th; and then in the 53rd round a sum is won which is neither sensibly increased or diminished until the 80th.

In examining my adjusted table of mortality by a comparison of the adjusted and unadjusted numbers of living, the fact must not be overlooked that the unadjusted number living at any age is the result of a calculation on which the observations for all ages between that age and the youngest observed have influence; and that therefore a long series of numbers all differing on the same side is no proof against the correctness of the adjustment, as is the case where each of the numbers compared depends upon its own particular observation, as, for instance, the probabilities of living a year.

He further says:—

On the other side I will not maintain, that my adjustment is not affected by theoretical errors, but these must be found in the first year and the last years. Though the observation of the first year agrees with my adjustment, I have reason to believe that my formula would not have answered to the observations for the beginning of life if these had been given for months and not for years. This I had hardly noticed without the sharp but kind criticism of Prof. Oppermann.

* * * * *

I have reason to believe that I shall be able to construct another formula with less constants, and allowing an easier, more direct way of calculation. But my latest formula has hitherto only been tried on one instance, and my duty calls me in these days to the calculation of the orbit of a comet and some other astronomical tasks. I can therefore not yet follow your kind summons to me to publish my formula. Of the first which I have tried I feel already ashamed, and the new one I dare not yet consider as sure.

HOME AND FOREIGN INTELLIGENCE.

ECONOMIC LIFE ASSURANCE SOCIETY.

Established 1823.

NINTH QUINQUENNIAL REPORT.

"The Directors of the Economic Life Assurance Society present to its Members a Report of the affairs of the Society, as they appeared on the 31st December last, after a full investigation of the business during the preceding five years, which complete the Ninth Quinquennium.

"Since the last Division, in 1864, 2,266 Policies, assuring £2,062,573, have been issued, giving an annual average of 453 Policies of £910 each.

"During the last five years the sum of £66,630 has been received in new Premiums, being at the rate of £13,326 a year.

"The total income from Premiums, which in 1864 was £214,104, now amounts to £228,833, indicating an average annual increase of

£2,946, after allowing for loss of income from discontinued Policies; while the gross income from all sources has increased at the rate of £6,100 per annum.

"Claims have arisen during the five years on 1,037 Policies, assuring £841,303, and carrying Bonuses to the amount of £198,089. These claims are £41,134 in excess of the expectation.

"In addition to the Bonuses on Policies upon which claims have arisen, the sum of £83,864 has been paid as Bonus in other ways, such as in reduction of Bonus liability by cash payment, reduction of Premium, purchase, &c., making a total of £281,953.

"In the valuation of the Assets, a sufficient margin has been allowed, as on former occasions, for a possible fluctuation of the Public Securities: and in regard to the Liabilities, so much only of the Premiums has been taken into account as represents the actual risk on Life. The remaining portion provides for the expenses of management, and accumulates to form the Surplus to be dealt with at the future Divisions of Profit. By this arrangement, old and new Assurers contribute rateably to the expenses of management, and no profit is declared by anticipation.

"The Assets, consisting of Funded Property, Mortgages, Life Interests, and Reversions, the Office Premises, Premiums due on 31st December (since paid), Interest accrued on Investments, Balance at Bankers and in hand, amount to £2,665,544. 12*s.* 3*d.*

"The Liabilities, consisting of the values of Policies and the Bonuses already declared, Claims accrued in 1868 but due in 1869, commission, taxes, and sundry small accounts, amount to £2,336,870. 17*s.* 6*d.* There is, therefore, after making provision for every known liability, a surplus of £328,673. 14*s.* 9*d.*

"The Directors recommend that £308,673. 14*s.* 9*d.* of this surplus be appropriated as absolute Bonus in reversion; and that the remaining sum of £20,000 be retained for the payment of Annual, Contingent, and Conditional Bonuses.

"It is further recommended, that out of this sum of £20,000, an Annual Contingent Bonus of £1 per cent. per annum be added to the absolute Bonus on such Policies, now entitled to participate, as shall become Claims during the current Quinquennial period, viz.:—

On Policies	which become claims in	1869, £1	per cent. on	sum assured.
„	„	1870, £2	„	„
„	„	1871, £3	„	„
„	„	1872, £4	„	„
„	„	1873, £5	„	„

"To the Policies which are not entitled to participate in the present Bonus, by reason of five annual Premiums not having been paid upon them, but on which claims may arise after the payment of the fifth annual Premium and before the next quinquennial investigation, the Directors propose to add a Bonus of like amount as if five annual Premiums had been paid prior to the present Division.

"The sum of £308,673. 14*s.* 9*d.* will produce reversionary Bonuses amounting to £464,816, yielding a percentage ranging from 4 to 31 on the sums assured; and a percentage ranging from 22 to 189 on the Premiums received in respect of which the Bonus is allotted.

"The same options are offered as at the last Division, and so the Bonus at present added may be commuted either

"To a present money payment;

"To a reduction of the Premiums for five years only; or

"To a reduction of the Premiums for the remainder of life.

"The Society now assures by 9,576 Policies the sum of £8,670,625, and has an Assurance Fund amounting to £2,618,531. 15*s.* 6*d.*, and an Annual Income of £338,000."

[It appears from the Chairman's address that the investigation of the Assets and Liabilities has been made upon a basis laid down by the deed of settlement.]

The following Statements illustrate the

PROGRESS AND POSITION OF THE SOCIETY.

I. *New Business.*

Years.	Policies Issued.	Sums Assured.	Average per Annum. Policies.	Per Policy.	Sum Assured.
1823–1843	6001	£5,292,512	286	£882	£252,000
*1844–1848	2384	1,778,945	477	750	357,789
1849–1853	2606	1,883,732	521	723	376,800
1854–1858	2578	1,875,434	516	727	375,087
1859–1863	2641	2,050,788	528	777	410,138
1864–1868	2266	2,062,573	453	910	412,515

* Since the above date the Society has been wholly Mutual.

II. *Position.*

Period of Division.	No. of Policies in Force.	Sums Assured and Bonuses.	Annual Income. From Premiums.	From Interest.	Total Assets.
1843	3628	£3,243,796	£90,500	£27,788	£746,482
1848	5097	4,382,804	124,321	37,634	908,060
1853	6572	5,568,320	159,861	51,658	1,357,736
1858	7818	6.608,406	176,673	73,889	1,840,212
1863	9022	7,795,574	214,104	93,371	2,315,130
1868	9576	8,670,625	228,833	115,180	2,665,544

Bonus.

The Society has declared Bonuses amounting in the aggregate to the sum of £2,336,884; the total cash Profits divided since the commencement of the Society having amounted to £1,705,188.

The following Table exhibits the Bonus declared at the several Divisions since the commencement of the Society.

Years of Division.	Absolute Bonuses declared.	Cash Divided. Amount.	Per cent. on Premiums paid since last division.
	£	£	
1834	33,000	18,000	8.7
1839	77,000	42,000	16.9
1844*	109,293	64,770	19.6
1849	274,121	168,795	38.5
1854	397,000	255,854	43.2
1859	475,354	304,710	41.7
1864	506,300	329,894	39.0
1869	464,816	308,674	32.6
	2,336,884	1,492,697	34.3

* The sum returned by the Society to its Members since the Year 1844, when it became purely Mutual, has averaged £36 7*s.* 6*d.* per cent. on the Premiums paid by them.

Balance Sheet for the Ninth Quinquennial Division of Profits, to the 31st December, 1868.

LIABILITIES.

	£	s.	d.
Values of Policies	1,791,031	6	1
Value of Bonuses	435,674	10	0
Value of Annuities payable	9,676	13	3
Reserve for Current Year's Reduction under the third option	3,500	0	1
Value of Possible Reduction in Two Reversions	473	0	0
Claims proved, but not due	83,461	6	1
Claims announced, but not proved	13,054	2	0
	2,336,870	17	6
SURPLUS—PROFIT FOR THE FIVE YEARS ENDING 31ST DECEMBER, 1868	328,673	14	9
Total	2,665,544	12	3

ASSETS.

	Amount of Stock.			Value.		
FUNDED PROPERTY, consisting of—	£	s.	d.	£	s.	d.
New 3 Per Cent. Annuities	131,810	0	0	117,311	0	0
3 Per Cent. Reduced Annuities	102,818	2	7	91,508	5	0
3 Per Cent. Consolidated Annuities	42,426	7	8	37,759	5	0
New 5 Per Cent. Annuities	11,100	0	0	11,100	0	0
Terminable Annuities	(362	10	0 per An)	4,043	10	0
India 10½ Per Cent. Stock	17,000	0	0	34,864	12	0
India Loan Debentures	30,000	0	0	30,182	8	0
Loans Guaranteed by the British Government	168,600	0	0	171,357	0	0
4 Per Cent. India Rupee Loan	30,000	0	0	26,627	8	0
	533,754	10	3	524,753	8	0
Mortgages on Landed Estates and on County and other Rates				753,126	9	7
Railway Debentures				436,321	6	0
Loans to Boards of Health and Burial Boards				554,737	4	9
Loans on Society's Policies within their surrender value				134,650	10	0
Life Interests and Reversions				177,914	17	6
Office Premises				13,921	0	0
				2,595,424	15	10
Interest accrued to 31st December from date of last payment				31,804	16	8
Interest in arrear at 31st December (since paid)				3,004	12	10
Premiums on which the days of grace had not expired				11,483	0	2
Agents' Balances (since paid)				10,209	4	0
Cash at Bankers				13,182	9	11
Cash in Office				114	4	4
Stamps in hand				121	8	6
Reversion in course of realization				200	0	0
Total				2,665,544	12	3

*Abstract of Assurances existing on the 31st December, 1869.**

	No. of Policies.	Sum Assured and Bonus Additions.	Reductions of Premium. For Life.	Reductions of Premium. Until 1874.	Annual Premiums Payable. For Life.	Annual Premiums Payable. For Limited Terms.	Average No. of Limited Annual Premiums unexpired.	Annual Life Premiums Commuted by Single or Limited Payments.
		£	£ s. d.	£ s. d.	£ s. d.	£ s. d.		£ s. d.
Table I. *Single Lives, Whole Term.*								
Ages under 20	26	15,520	2 3 6		191 16 3	89 15 10	12.00	47 10 10
20........25	140	85,860		1 8 8	1,424 15 5	144 13 7	8.17	73 13 0
25........30	483	390.790		31 3 11	6,903 14 2	1,347 12 0	4.47	561 13 5
30........35	823	667,070	8 13 9	89 3 1	13,046 18 6	1,290 2 3	5.61	604 1 9
35........40	1,129	883,987	38 10 4	159 6 1	17,696 0 2	1,872 2 11	6.37	1,462 11 3
40........45	1,318	1,168,468	48 8 2	391 8 11	25,846 3 11	1,661 1 2	5.21	1,211 1 6
45........50	1,359	1,230,286	55 2 3	376 5 4	28,914 7 8	953 12 1	5.53	1,506 7 0
50........55	1,249	1,189.954	115 14 10	545 6 10	29,539 19 2	877 18 0	3.16	1,693 16 8
55........60	904	880,855	131 0 3	428 18 9	22,518 10 7	230 16 1	2.49	837 17 5
60........65	689	695,604	119 10 4	651 13 9	19,478 14 0	129 1 2	5.06	455 16 6
65........70	618	725,050	239 3 0	892 10 9	20,363 11 8	90 10 10	1.00	646 8 4
70........75	327	374,735	61 4 5	338 10 0	11,893 8 4			98 10 10
75........80	154	209.419	14 4 11	348 1 10	6,717 6 11			
80........85	55	80,077	134 2 4	27 2 2	2,459 19 6			
85........90	7	10,391	8 15 5		286 9 7			
90 and upwards.	4	6,939			226 1 8			
	9,285	8,615,005	976 13 6	4,281 0 1	207,507 17 6	8,687 5 11		9,199 8 6
					8,687 5 11			
					216,195 3 5			
Joint Lives	46	31,732	22 0 6	89 7 9	1,270 9 5			
Last Survivor	42	59,145	1 12 5	2 16 0	359 17 4	Annual Premiums.		
Contingent Survivorships	21	20,900			178 14 10			
Contingent Survivorship Annuities	9	(£1,060 per An.)			827 0 1			
Table II.	245	193,251			5,529 9 0	Increasing Premiums.		
Short Periods	27	27,410			462 16 2			
Combined Period and Endowment	6	3,050			87 4 2	Payable during remainder of terms.		
Deferred Annuities	1	(£12 per An)			2 2 10			
	9,682	8,950,493 and (£1,072 per An)	1,000 6 5	4,373 3 10	224,912 17 3			

* This Table is extracted from the Annual Report for 1869, no such Table having been given with the Bonus Report.—Ed. *J. I. A.*

UNION ASSURANCE SOCIETY.

Established 1714.

Assets and Liabilities of the Society, at the Quinquennial Division of Profits, January, 1868.

Statement of the General Position of the Society submitted to the General Meeting held on 3rd January, 1868.

Assets.

	£	s.	d.
To Government Funds,	83,702	3	10
„ East Indian Securities,	84,949	19	10
„ Loans on Policies,	22,250	9	4
„ Loans on Mortgage of Freehold and Leasehold Property,	305,528	18	0
„ Loans on Bonds of Public Companies,	111,407	10	0
„ Guaranteed Railway Stock,	15,090	0	0
„ Balances in hands of Agents,	4,832	19	7
„ Cash in Office and at Bankers,	4,402	10	9
„ Interest due and accruing,	10,514	12	6
„ Renewal premiums unpaid,	1,286	15	8
	643,965	19	6
Less outstanding claims and bonuses,	31,160	14	10
	£612,805	4	8

Liabilities.

	£	s.	d.
By liability under the Company's Policies as shown below,	506,602	10	11
By surplus,	106,202	13	9
	£612,805	4	8

Statement shewing the Company's Liability under its Policies.

Dr.

	£	s.	d.
To Present value of £81,275 15s. 7d. being the annual premiums, as per Schedule, receivable under whole Life Policies, less one-fifth reserved for future expenses and profits,	892,425	11	8
To present value of £3452 11s. 10d. being the annual premiums receivable under special policies, less one-fifth, as above,	26,862	18	9
To amount due to the office on two-thirds and half-credit Policies,	3,270	11	3
To present value of re-assurances effected with other offices,	10,946	2	4
To Balance, forming the Reserve required to meet future Claims,	506,602	10	11
	£1,440,107	14	11

Cr.

	£	s.	d.
By present value of £2,480,400 15s. insured under 4230 Policies for the whole of life, including the bonuses added, as per schedule attached,	1,398,541	2	6
By present value £114,530 17s. 10d. insured under 220 Special Policies,	41,566	12	5
	£1,440,107	14	11

The Carlisle Table of Mortality has been adopted as the basis of the above calculations, and the rate of interest has been taken at Three per Cent.

Table shewing the whole Amount Assured and the Bonuses added thereto under 4230 *Whole Life Policies existing on* 30*th June*, 1867.

Present Age of the Lives Assured.	Total Sum Assured, and Bonus.			The Premiums payable Annually during the continuance of the Policies.			Present Age of the Lives Assured.	Total Sum Assured, and Bonus.			The Premiums payable Annually during the continuance of the Policies.		
	£	s.	d.	£	s.	d.		£	s.	d.	£	s.	d.
7	200	0	0	3	13	4	Forward	1,359,438	9	6	38,688	9	4
8	300	0	0	5	10	0	50	70,787	19	0	2,372	12	2
9	2,400	0	0	42	18	0	51	51,095	0	0	1,647	3	7
11	100	0	0	1	15	9	52	63,521	19	6	2,328	7	7
13	450	0	0	8	4	9	53	62,352	18	6	2,178	8	1
14	5,465	0	0	97	8	5	54	47,989	0	9	1,739	13	11
15	6,000	0	0	109	10	10	55	53,639	3	0	1,987	8	6
16	1,800	0	0	34	8	2	56	26,910	2	4	1,034	7	6
17	8,062	0	0	151	0	6	57	74,414	14	3	2,798	19	1
18	602	0	0	10	19	1	58	36,341	14	9	1,316	12	8
19	5,300	0	0	106	9	10	59	27,543	4	8	1,037	7	8
20	600	0	0	12	9	3	60	64,508	16	9	2,391	6	9
21	1,700	0	0	30	10	6	61	29,340	15	0	1,007	3	9
22	2,594	0	0	50	7	0	62	42,628	8	6	1,572	13	11
23	4,157	0	0	89	9	4	63	24,981	1	6	1,091	18	2
24	8,120	0	0	171	9	4	64	49,732	15	0	1,934	9	6
25	7,702	0	0	177	2	3	65	39,125	14	0	1,526	15	6
26	8,789	0	0	185	4	9	66	31,309	18	9	1,203	5	2
27	16,002	0	0	372	17	2	67	33,937	15	6	1,433	7	0
28	23,900	0	0	549	18	0	68	19,922	1	0	672	0	4
29	30,017	0	0	712	3	5	69	15,761	10	6	581	0	5
30	28,408	0	0	687	3	11	70	23,692	1	0	1,044	7	8
31	40,471	0	0	956	14	11	71	12,246	3	9	515	2	2
32	50,090	0	0	1,260	13	5	72	11,172	1	0	386	3	2
33	49,100	16	0	1,192	2	2	73	22,815	16	11	667	10	11
34	46,398	0	0	1,204	8	0	74	25,987	0	6	809	11	8
35	39,747	0	0	1,039	3	6	75	31,290	18	5	903	8	5
36	49,227	0	0	1,307	15	9	76	13,438	18	8	413	9	11
37	60,220	4	0	1,657	7	9	77	42,944	13	9	3,894	6	4
38	75,655	6	0	2,117	1	3	78	12,723	4	9	382	9	6
39	58,229	8	0	1,573	0	0	79	6,892	10	0	190	10	2
40	54,019	3	6	1,481	11	11	80	6,963	18	6	221	14	8
41	73,552	10	0	2,055	7	6	81	5,748	7	8	147	17	4
42	64,209	1	6	1,833	19	6	82	7,640	19	9	172	13	8
43	69,070	8	0	2,010	15	9	83	7,583	3	8	218	6	8
44	62,253	9	6	1,801	1	5	84	2,376	1	0	78	8	9
45	69,658	2	6	2,267	18	2	85	10,389	11	6	319	19	6
46	78,010	8	0	2,528	9	9	86	1,386	15	9	39	4	2
47	85,880	5	0	2,928	13	1	87	393	4	6	11	7	3
48	67,252	19	0	2,265	1	3	90	3,073	3	6	112	19	3
49	103,725	8	6	3,596	10	8	91	6,358	17	8	202	13	10
Forward	1,359,438	9	6	38,688	9	4		2,480,400	15	0	81,275	15	7

We find from the prospectus of this Office that the paid-up capital of the Society exceeds £500,000; that the income of the Fire department is £60,000; that four-fifths of the Office profits are appropriated to the benefit of the assured every five years; also that at the last investigation there was declared a bonus addition of £1 10*s.* percent per annum on the

sum assured for the past five years, the additions to the sums assured varying from thirty-five to sixty-five percent of the amount of premiums paid during the five years, and the cash bonuses ranging between twenty and twenty-five percent.

It is further stated that the calculations showing the position of the Society, were first prepared by their own Officers, and then examined by the Actuary of another leading Life Office, and fully confirmed by him.

THE PROVIDENT LIFE OFFICE.

Established 1806.

Extracts from the Report of the Directors upon the Tenth Declaration of Bonuses, December 1867.

The Directors have given at each periodical *Division of Profits* a statement of *Assets* and *Liabilities.* They now repeat that statement with the addition of *a solemn declaration* of its accuracy and truthfulness. The Table of whole Life Policies is given for the first time. It has been a work of considerable labour, but it is the foundation of the entire account, and without it any statement of Assets and Liabilities, and indeed all accounts, are worthless.

* * * * *

If the Government shall be disposed to pass a short Act to compel the production by each Life Office of intelligible Tables and accounts, supported by *a solemn declaration*, we may venture to assert that no further action on their part will be necessary. The materials thus afforded for a complete investigation of the affairs of a Life Office will be within the reach of all, and the condition of every such Institution in the United Kingdom may be effectually tested.

* * * * *

Table shewing the whole amount assured and the Bonuses added thereto under 9163 *Whole Life Policies existing on the* 31*st December*, 1867.

Present Age of the Lives Assured.	No. 1. Total Sum assured and Bonuses added thereto. £. s. d.	No. 2. The Premiums payable *annually* during the continuance of the Policy. £. s. d.
5	600 0 0	10 6 0
9	100 0 0	1 14 4
11	100 0 0	1 14 4
12	1,050 0 0	18 0 6
13	1,320 0 0	22 9 0
14	1,687 8 0	28 18 0
15	750 16 0	12 18 4
16	2,400 0 0	41 1 3
17	4,293 18 0	77 1 4
18	3,213 19 0	59 0 9
19	4,612 15 0	84 16 5
20	4,834 8 0	88 7 10
21	4,628 12 0	85 16 6
22	20,209 13 0	407 5 11
23	13,400 13 0	271 5 6

Table of amount assured and Bonuses added, &c.—(continued).

Present Age of the Lives Assured.	No. 1. Total Sum assured and Bonuses added thereto.			No. 2. The Premiums payable *annually* during the continuance of the Policy.		
	£.	s.	d.	£	s.	d.
24	20,065	18	11	410	16	1
25	34,120	12	0	1,021	14	3
26	44,053	3	0	707	2	10
27	46,095	0	6	945	0	3
28	61,446	13	0	1,385	4	8
29	45,580	13	8	1,039	12	4
30	69,162	19	8	1,644	4	0
31	61,243	9	8	1,482	2	2
32	82,374	3	7	1,887	12	5
33	64,602	5	8	1,596	2	1
34	77,560	9	8	1,890	7	11
35	85,717	19	10	2,226	10	6
36	88,010	15	10	2,312	9	8
37	80,232	13	10	2,116	2	1
38	103,007	10	2	2,783	7	5
39	92,135	16	0	2,446	10	1
40	76,875	14	9	2,106	9	4
41	102,617	1	2	2,818	19	3
42	88,101	3	4	2,488	11	5
43	96,666	4	11	2,823	18	0
44	124,713	9	6	3,575	18	11
45	124,809	4	10	3,738	19	4
46	81,955	9	3	2,470	19	3
47	119,053	10	1	3,376	17	2
48	138,268	15	6	3,952	7	2
49	100,766	8	2	3,117	3	5
50	104,869	15	2	3,316	13	6
51	125,793	2	5	3,689	8	4
52	104,942	3	10	3,168	9	3
53	102,490	13	5	3,317	3	6
54	133,477	8	5	4,335	4	1
55	75,268	0	9	2,478	12	11
56	118,491	13	7	3,857	12	5
57	87,110	1	1	2,996	9	1
58	75,898	4	7	2,374	11	5
59	98,012	2	8	3,717	12	4
60	103,479	9	3	4,151	13	7
61	104,386	4	11	3,944	10	3
62	92,618	11	1	2,889	3	1
63	92,384	16	11	3,209	15	11
64	82.487	11	10	2,950	5	4
65	86,317	0	11	3,234	12	4
66	70,643	3	8	2,189	0	8
67	90,503	13	3	3,060	7	7
68	107,971	14	4	4,237	18	7
69	76,741	2	6	2,467	1	11
70	65,250	12	0	2,501	18	8
71	55,299	3	5	2,079	4	2
72	45,311	2	10	1,215	17	5
73	49,202	4	3	1,497	15	4
74	52,415	9	7	1,462	8	10
75	56,803	2	1	1,569	1	11
76	31,561	5	5	1,143	15	0
77	34,407	7	8	1,011	14	11
78	48,862	7	3	1,312	11	3

Table of amount assured and Bonuses added, &c.—(continued).

Present Age of the Lives Assured.	No. 1. Total Sum assured and Bonuses added thereto.			No. 2 The Premiums payable *annually* during the continuance of the Policy.		
	£.	*s.*	*d.*	£	*s.*	*d.*
79	15,699	5	0	456	19	8
80	41,087	4	7	1,152	19	5
81	31,463	0	5	828	0	11
82	37,129	4	8	1,039	2	9
83	4,635	0	4	154	16	7
84	6,574	11	8	205	6	6
85	3,164	1	8	71	17	11
86	2,977	4	3	60	18	10
87	4,328	9	0	225	2	1
88	8,888	11	4	229	5	2
89	2,379	13	4	78	9	1
90	10,817	14	6	200	9	10
92	1,087	10	7	22	13	4
TOTAL, ...	4,715,640	8	11	141,682	13	8
*333 Short Term, Contingent and other Policies .	196,159	0	0	4,877	19	7
TOTAL	4,911,799	8	11	146,560	13	3

* The details of these policies are withheld for the sake of brevity.

Assets and Liabilities of the Provident Life Office, as shewn by the Valuation and Account taken at the Tenth (Quinquennial) Division, on the 31*st December*, 1867.

LIABILITIES.	£.	*s.*	*d.*
Present Value of Liability under 9163 Whole Life Policies, insuring £.4,715,640, as shown in the preceding Table, Col. No. 1, ...	1,265,287	11	1
The same under 333 Short Term, Contingent, and other Policies, insuring £.196,159, ...	26,106	10	9
The same under 7 Policies or Grants of Annuities, ..	2,194	8	4
Claims admitted and in course of payment, ...	49,947	13	4
Amount of Guarantee Fund, ...	61,368	15	6
	£.1,404,904	19	0

ASSETS.	£.	*s.*	*d.*
Invested in Mortgages, Annuities and Life Estates,	1,280,292	7	1
„ in the Public Funds, ...	232,091	10	0
„ in Loans on Policies, ...	87,179	15	10
„ in Loans on Bonds, ...	90,389	3	11
Interest due and accruing, ...	34,418	2	3
Value of Re-assurance Policies, Eight Policies Assuring £.27,000, ...	3,860	5	6
Balance in the hands of Agents, ...	32,935	9	5
„ at Bankers, ...	7,947	0	9
„ of Bills undue, ...	689	2	3
„ of Renewal Premiums unpaid, ...	2,561	2	0
	£.1,772,363	19	0

Result.	£.	s.	d.
Total Assets,	1,772,363	19	0
Total Liabilities,	1,404,904	19	0
Surplus applicable to the payment of Bonuses,	£.367,459	0	0

* * * * *

The two Statements of *Assets* and *Liabilities* are not new to the public, they have been continually published by the worst as well as by the best Offices, but they are unworthy of credit in many instances, and they cannot answer any good purpose unless accompanied by *a solemn declaration.* The Statement of Assets especially calls for this confirmation, because an unscrupulous Manager may draw out a statement entirely fictitious; but he would not have sufficient courage to incur *periodically* the danger of a *false declaration.* The fraud, it is true, ultimately exposes itself, and then follows amalgamation or flight, but not until many thousands of pounds have been obtained from the insuring Public.

Life Insurance and the granting Annuities, being governed by the same principles and calculated by the same Tables, the one being the converse of the other, are properly included within the practice of one and the same Institution.*

* * * * *

There are some *Life* Offices which combine with their legitimate operations business of a speculative character, the most frequent being that of *Fire* Insurance. Where such a practice prevails all control is at an end —any account of assets and liabilities is simply impossible. The risks *of a Fire Office* are indefinite and constantly varying, and the losses upon that branch of business may frequently cause heavy demands upon the savings of those who have sought the protection of the *Life* Office as a means of providing for their families.

This is perhaps the single point upon which legislation is absolutely requisite. The obligation to keep the prudential savings of Life policy-holders free from the risks incidental to speculation can only be enforced by the power of the Legislature. *Separate Capitals* and *Separate Accounts* are indispensable.

* We have not found Annuities a profitable class of business, and therefore we now confine our operations strictly to Life Assurance.

EQUITY AND LAW LIFE ASSURANCE SOCIETY.

Established 1844.

BONUS REPORT, 1870.

Fifth Quinquennial Division of Profits.

The following table shows the amount of the New Business transacted in each of the last three periods:—

	No. of Policies issued.	Sums Assured.	New Premiums.
		£	£
1855 .. 1859	725	792,485	24,776
1860 .. 1864	805	1,159,619	41,083
1865 .. 1869	977	1,735,775	57,211

The average amount of the policies issued has increased, having been £1,777 during the last five years, against £1,440 in the period from 1860 to 1864, and £1,093 in the preceding period.

On 31st December 1864, there were in force 1,756 policies, insuring £2,178,766; and adding to these the new policies issued, as shown above, we have a total of 2,733 policies, which are accounted for as follows:—

	No. of Policies.	Sums Assured.
		£
Claims	188	216,947
Surrendered	113	140,420
Lapsed	178	210,973
Void, Expired, etc.	64	128,656
In force on 31st December 1869 ..	2,190	3,217,545
	2,733	£3,914,541

Table showing the Number and Amount of the Policies in Force on 31st December 1869.

Class of Assurance.	Participating Policies.				Non-Participating Policies.		
	No. of Policies.	Sums Assured.	Existing Bonus.	Annual Premiums.	No. of Policies.	Sums Assured.	Annual Premiums.
		£	£	£		£	£
Whole Life	1,590	2,117,952	128,305·3	66,250·713	402	663,330	22,319·048
Limited Payments .	7	14,200	1,239·0	671·905	1	3,000	195·000
Ascending Scale	2	9,000	—	175·208	9	25,300	552·163
Endowment Assurances ..	13	9,000	266·5	384·305	6	11,750	439·855
Descending Scale ..	1	3,000	—	89·250	—	—	—
Joint Lives	8	16,800	693·5	966·309	5	11,900	586·788
Last Survivor	12	17,800	2,059·5	335·192	4	10,200	202·750
Contingent	—	—	—	—	59	114,296	1,330·800
Reversionary Annuities ..	—	—	—	—	6	(£969 per annum.)	124·183
					492	839,776*	25,750·587
Endowment	—	—	—	—	1	100	—
Term	—	—	—	—	32	81,348	657·972
Issue	—	—	—	—	29	96,969	—
Against failure of Issue ..	—	—	—	—	3	11,600	131·750
Extra Premiums	—	—	—	—	—	—	823·921
Total	1,633	2,187,752	132,563·8	68,872·882	557	1,029,793*	27,364·230

	No. of Policies.	Sums Assured.	Existing Bonus.	Annual Premiums.
		£	£	£
Total Participating and Non-Participating Policies ..	2,190	3,217,545*	132,563·8	96,237·112
Reassurances	—	397,352	5,435·8	11,031·441

* And Annuities of £969.

It will be noticed that the Immediate Annuities payable by the Office, amounting on 31st December 1869, to £3,104 2s. 6d. per annum, are not included in the above table; their value, £22,241 13s. 4d., having been entered as a separate liability in the General Balance Sheet of 31st December last.

Almost exactly one third of the existing insurances are effected on the non-participating scale of premiums, being a rather larger proportion than at the Division of Profits to December 1864.

The value of the liabilities under existing policies has been calculated by the same table and the same processes as on the last occasion. The Table of Mortality used has been that known as the "Experience of the Seventeen Offices'" Table, which gives a considerably larger reserve than any other of the tables commonly employed. The rate of interest assumed in the calculations has been three per-cent, a rate dictated by prudence in dealing with contracts that extend over so long a term as life insurance policies, and one that is fully justified by the practice of the leading and most successful Insurance Companies. In estimating the value of the sums assured, a correction has been applied on account of the claims being payable at an earlier date than that assumed in the ordinary tables, namely six months after death. On the other side of the account, while the full premium actually payable has been valued and brought into account, which is essential for giving a correct view of the position of the Society, there has subsequently been deducted the whole of what is technically known as the "loading," being the addition made to the net risk premium, for expenses, profit, and contingencies.

The results of the valuation are shown in the following Balance Sheet:—

Valuation Balance Sheet, 31*st December* 1869.

LIABILITIES.

	£	s.	d.
Value of £2,187,752 assured under 1,633 Policies with profits	1,163,318	0	0
Value of £132,563 16s. Bonuses thereon	81,140	8	0
Value of £839,776 assured under 492 Policies without profits	415,109	16	0
Reserve for Short Term Insurances, Extra Risks, Special Cases, &c.	11,063	8	0
Claims announced, and other Liabilities	5,488	14	0
Balance—being the excess of Assets over Liabilities	144,933	7	6
	£1,821,053	13	6

ASSETS.

	£	s.	d.	£	s.	d.
Amount of Assurance Fund as per printed account				654,373	5	6
Value of £68,872 17s. 8d. Annual Premiums on policies with profits	1,005,774	14	0			
Less reserve for expenses, future bonuses, and contingencies	203,730	10	0			
				802,044	4	0
Value of £25,750 11s. 9d. Annual Premiums on policies without profits	354,099	2	0			
Less reserve for expenses, &c.	46,955	12	0			
				307,143	10	0
Value of Reassurances for £402,787 16s. 0d.				57,492	14	0
				£1,821,053	13	6

The balance of £144,933 here shown is greatly in excess of what might fairly have been anticipated. This is explained by the circumstance that the Society has realized, in the course of the last five years, an exceptional profit by the early falling in of some large reversions. The profit from this source has, in fact, been no less than £18,568 in excess of the six per-cent interest which the reversions are calculated to produce. There has also been realised, in the same period, a profit of £7,622 on the Chancery Lane property. Bearing in mind that exceptional profits of this kind cannot be expected to recur, the Directors recommend that, with a view to equalize the rate of bonus, a portion only of this exceptional profit be divided on the present occasion, and that the remainder be carried forward for future division. They recommend, therefore, that the sum of £130,440 be divided, of which, according to the provisions of the Deed of Settlement, ten per-cent, or £13,044, will be appropriated to the Shareholders, and will allow of the declaration of a dividend of 12s. per Share for the next five years. The remaining 90 per-cent, or £117,396, will be divided among the participating Policyholders of more than one year's standing, that is to say, among holders of policies for the amount of £2,011,933, the annual premiums on which amount to £63,180. At the last Division of Profits, the sum of £62,961 was divided among the holders of policies for £1,339,608; and if the same proportion had now held, the sum divided would have been £94,561.

It is proposed to divide the surplus among the Assured according to the same principles as on the last occasion. Those policies which were in existence on 31st December 1864, and were still in force at the close of last year, will first have such a share of the profits allotted to them, as has been realized upon them in consequence of the average rate of interest on the investments during the last five years having been over five per-cent, instead of three per-cent, as assumed in the calculations; and the remaining divisible surplus will then be appropriated among all the Policyholders entitled to participate, in proportion to the premiums they have paid during the last five years.

The general results of the five Divisions of Profits which have been made, are shown in the following table:—

Table of the total additions made up to 31*st December* 1869, *to Policies of* £1,000 *each.*

Age at Entry.	Number of Premiums Paid.				
	Twenty-five.	Twenty.	Fifteen.	Ten.	Five.
	£ s.	£ s.	£ s.	£ s.	£ s.
20	448 0	354 10	271 0	161 10	71 10
30	507 0	392 0	297 0	175 0	77 0
40	575 0	444 0	336 0	195 10	85 0
45	617 10	481 0	360 10	210 0	90 10
50	663 0	530 10	397 0	231 0	99 10
55	—	605 0	454 10	263 10	114 0
60	—	707 10	537 0	308 10	134 0

* * * * *

The Progress of the Society during the last five years is exhibited in the following tables, extracted from the Annual Report for the year 1869:—

Abstract of the Receipts and Expenditure of the Equity and Law Life Assurance Society, for each of the five years ending 31st December 1869.

RECEIPTS.

	1865.			1866.			1867.			1868.			1869.			Total.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
New Premiums	10,850	6	1	9,981	2	2	11,137	13	6	11,318	6	11	13,923	11	9	57,211	0	5
Renewal Premiums.. ..	66,465	17	11	68,873	6	4	75,264	19	2	81,320	9	6	87,617	14	11	379,542	7	10
Dividends and Interest (less Income Tax)	20,603	2	9	23,622	8	0	24,785	0	4	28,527	9	5	30,148	10	3	127,776	10	9
Sundry Profits (including Reversions fallen into possession)	19,842	14	6	14,134	9	6	2,077	4	3	4.549	0	2	1,971	5	11	42,574	14	4
Annuity Purchase Money..	1,277	5	0	603	1	10	11,275	19	0	6,657	0	6	2,352	10	0	22,165	16	4
Total Income	119,039	6	3	117,214	7	10	124,630	16	3	132,372	6	6	136,013	12	10	629,270	9	8

EXPENDITURE.

	1865.			1866.			1867.			1868.			1869.			Total.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
Claims (less Reassured) ..	34,009	4	0	44,096	3	9	26,345	10	0	34,686	0	0	51,901	6	10	191,038	4	7
Surrenders (less Reassured)	456	16	6	1,297	18	3	1,748	11	2	2,545	16	4	2,658	4	4	8,707	6	7
Bonus in Cash	7,717	9	7	667	11	0	670	10	0	115	15	0	—			9.171	5	7
Annuities	779	13	5	806	12	11	1,067	9	4	2,583	13	10	2,827	16	10	8,065	6	4
Reassurances (less Commission)	8,718	13	1	9,269	1	9	10,077	19	6	10,950	13	6	11,728	0	8	50.744	8	6
Proprietors' Dividends ..	4,250	0	0	4,250	0	0	4,250	0	0	4.250	0	0	4,250	0	0	21.250	0	0
Expenses of Management ..	4,583	17	10	4,936	17	8	4,813	17	9	4,649	3	3	4,933	15	4	23,917	11	10
Written off Cost of House ..	2,400	0	0	261	18	2	300	0	0	300	0	0	300	0	0	3,561	18	2
Commission	3,389	8	7	3,677	16	7	4,186	7	3	4,338	19	7	4,573	2	10	20,165	14	10
Total Outgo	69,305	3	0	69,264	0	1	53,460	5	0	64,420	1	6	83,172	6	10	336,621	16	5
Increase of Assets during the Year	52,734	3	3	47,950	7	9	71,170	11	3	67,952	5	0	52,841	6	0	292,648	13	3

In the same period the Assets of the Society have increased from £443,966 to £736,615, and the total sums assured from £2,178,766 to £3,217,545, exclusive of Bonuses, the several amounts in each year being as follows:—

	Assets.	Sum Assured.
On 31st December, 1864	£443,966	£2,178,766
,, ,, 1865	496,700	2,373,983
,, ,, 1866	544,651	2,568,495
,, ,, 1867	615,821	2,803,652
,, ,, 1868	683,774	3,027,386
,, ,, 1869	736,615	3,217,545

In compliance with a wish expressed in several quarters, and in anticipation of the returns to be made in future under the "Life Assurance Companies Act," the Directors have decided to publish such particulars of the Whole Life Policies, which form the great bulk of the business, as will enable the liability of the Office under those policies to be ascertained by any person desirous of doing so. They are shown in the following table.

Table giving particulars of the Whole Life Policies in force as at 31st December 1869, *after deduction of Reassurances.*

Age on 31st December 1869.	Sums Assured.	Bonuses Declared.			Annual Premiums.		
	£	£	*s.*	*d.*	£	*s.*	*d.*
Under 20	5,500	..			83	2	1
20 „ 24	39,750	269	0	0	751	2	5
25 „ 29	148,086	705	5	0	3,082	13	11
30 „ 34	298,860	3,484	0	0	6,961	0	9
35 „ 39	344,150	9,352	0	0	8,696	6	4
40 „ 44	324,904	11,703	17	0	9,231	14	2
45 „ 49	403,963	19,874	7	4	12,550	17	6
50 „ 54	362,308	25,214	0	0	12,695	15	8
55 „ 59	213,950	16,468	7	5	8,305	2	3
60 „ 64	145,639	16,314	11	4	6,994	10	9
65 „ 69	72,980	10,886	10	0	3,981	10	4
70 „ 74	59,369	7,308	10	0	3,903	0	7
75 „ 79	11,150	1,052	0	0	894	9	0
80 and upwards.	900	318	0	0	100	5	5
Total Whole Life Policies ..	£2,431,509	£122,950	8	1	£78,231	11	2
Whole Life Reassurances ..	349,773*	5,354	16	11	10,338	4	1
Special Cases	436,263	4,258	10	0	7,667	7	0
Total Participating and Non-Participating Policies	£3,217,545	£132,563	15	0	£96,237	2	3

* There were also Reassurances on Special Cases, amounting to £47,579.

At the Fifth Quinquennial Division of Profits to 31st December 1869, the sum of £117,396 was divided among the holders of Policies for £2,011,933, and produced additions to the sums assured, which amounted to £202,009, and averaged upwards of Two per-cent per annum on the sums assured, and no less than Seventy per-cent on the Premiums received during the five years on the Policies which participate.

ROYAL INSURANCE COMPANY.

Established 1845.

Valuation and Report, 31st December, 1869, (by Mr. S. Brown).

I have completed the Valuation of the Life Branch of the Royal Insurance Company to 31st December last in the same form as is proposed in the Schedules of the "Life Assurance Companies Act," now passing through Parliament.

As to the Policies for the whole duration of Single Lives, all Policies on the Lives of persons born in the same year have been brought together, and taken at their mean age on 31st December, 1869, distinguished into Policies "with Profit" and "without Profit" at every age.

The Table used for the Valuation has been Mr. Griffith Davies' Equitable Experience, and the rate of Interest assumed has been 3 per cent.

The net premiums only, as computed upon each Policy, have been valued so as not to bring into account any portion of the future Profits.

In all other classes of Assurance, except those for the whole duration of Single Lives, so brought together in ages, each Policy has been valued separately, the total amounting in number to 1584, and the value computed by the same Table as above, except the Survivorships and a few cases of complicated contingencies for which the Carlisle Table afforded greater facilities of computation and would lead to nearly the same results.

As to the Annuities, those for the whole duration of Single Lives have also been brought together under existing ages, and valued by the $3\frac{1}{2}$ per cent. Equitable Experience Table, that being the rate on which they are based. Immediate Annuities on two or more Lives have been valued at the same rate. All the others, comprising Contingent and Deferred Annuities, have been valued separately by the 3 per cent. Table.

The Schedules comprise the values of the Assurances "with" and "without" Profits, at each existing age, and the total values of each of the other classes of Assurances and Annuities, of which a brief summary may be thus given, compared with the accumulated Funds in each branch:—

Estimated Liabilities and Assets in the Life Department of the Royal Insurance Company on the 31st December, 1869.

	Number of Policies.	Sums Assured and Existing Bonuses.	Net Premiums after deducting the Loading and Bonus Reductions.	Net Value of Assurances and Annuities.	Accumulated Funds.	Surplus.
		£	£	£	£	£
General Life Assurances, less Re-assurances....	14,292	6,672,504	166,994	835,568	1,068,516	232,948
Children's Endowments	222	48,998	1,824	22,795	25,589	2,794
Immediate Annuities....	447	16,228 Ans.	—	144,079	156,427	12,348
Contingent and Deferred Annuities	16	2,126 Ans.	492	3,235	4,803	1,568
Total..	14,977	6,721,502 18,354 Ans.	169,310	1,005,677	1,255,335	249,658

The amount of the Accumulated Funds in each branch of the Life Department has been furnished me by a Certificate from one of the Auditors, and I assume, of course, that all the Investments are of a first-class character, and would on an average be realised at the full value at which they are estimated.

The total Surplus in the Life Assurance Fund appears to be £232,948.

But there is a considerable item which is not included in the Funds, as exhibited in the Balance-Sheet, namely, Debts on Policies which have been effected on the *partial-credit plan*, amounting to £17,008. It may be at the option of the Policy-holders to drop any of these Policies and not pay up the debt; but in that case the reserved value would constitute an item of Profit.

Another item, half-yearly and quarterly Premiums on Policies, taking date between the 1st July and 31st December in each year, amounting to

£13,762, might also be brought to the credit of the Funds in a Valuation made separately for each Policy, in which the Policies are valued as if the full Annual Premium had been paid at the commencement of the current year of Assurance. But in the method of bringing to account in a classification under ages the unpaid portion of the current year's Premiums, these payments are valued as a credit, and cannot therefore be included again.

It is satisfactory to find that by a valuation of each Policy separately, computed by monthly intervals of the Premium, which has been made in the Office, the total results, with the correction of this item as above, almost exactly agree with the Valuation now presented by grouping the Policies under ages.

Taking, then the Surplus in the Life Assurance Fund, as above given, at	£232,948
And adding thereto the Debts due under the *partial-credit plan*	17,008
The Total Surplus appears to be......	£249,956

By an estimate made in the Office, it appears that the sum required in cash to give a Reversionary Bonus of One-and-a-half per cent. per annum on the Sum Assured by the Policies entitled to participate in the Bonus for their respective periods of existence during the last Five years, would be

	£163,844
Proprietors' Share of Profits	81,922
Total Profits to be Distributed	£245,766

Summary of the Valuation in each Branch of the Life Department.

GENERAL LIFE ASSURANCES.—WITH PROFITS.

	No.	Sums Assured and Bonuses.	Net Premiums.	Value of Sums Assured and Bonus.	Value of Net Premiums.	Net Value of Assurance.
		£	£	£	£	£
Single Lives—						
Ordinary Policies	11,109	5,299,554.096	130,712.259	2,665,480.4	2,073,981.2	682,476.8
Bonus on Do. ..		163,735.464		90,977.6		
Policies with increasing Pms...	12	9,888.	224.803	4,996.8	4,434.1	562.7
Limited No. of Premiums	48	33,164.	1,008.608	16,061.5	7,179.	8,882.5
Premiums paid up	3	1,293.029		672.1		672.1
Endowment Assurances	791	194,897.266	6,657.897	121,311.7	77,222.8	44,088·9
On Two or more Joint Lives	74	29,950.	1,069.262	18,332.1	13,288.5	5,043.6
Survivor of Two or more Lives......	33	13,326.5	175.751	4881.6	3,275.5	1,606.1
Survivorships	7	962.2	12.376	114.7	96.8	17.9
	12,077	5,746,770.555	139,860.956	2,922,828.5	2,179,477.9	743,350.6

Summary of the Valuation—(*continued*).

GENERAL LIFE ASSURANCES.—WITHOUT PROFITS.

	No.	Sums Assured.	Net Premiums.	Value of Sums Assured.	Value of Net Premiums.	Net Value of Assurance.
		£	£	£	£	£
Single Lives—						
Ordinary Policies	1,599	917,764.817	27,287.913	491,299.4	389,242.7	102,056.7
Policies with increasing Premiums	10	15,360.	353.835	8,128.1	7,153.8	974.3
Limited Number of Premiums	10	9,750.	335.919	4,287.6	1,968.	2,319.6
Complete Policies	26	11,800.	477.972	5,341.1	5,096.9	244.2
Paid-up Policies	25	5,447.122		3,419.		3,419.
Endowment Assurances	301	68,117.163	2,793.414	42,626.	31,476.	11,150.
On Joint Lives.........	42	19,551.354	912.208	12,587.1	9,108.3	3,478.8
On Survivor of Two or more Lives..........	35	22,086,667	356.352	9,303.4	6,135.5	3,167.9
On Survivorships	39	33,500.	378.361	3,731.8	2,739.7	992.1
On Temporary Assurances	126	85,502.714	1,669.176	1,374.5		1,374.5
On sums payable at fixed term	2	3,000.	49.	1,511.9	748.9	763.
	2,215	1,191,879.837	34,614.150	583,609.9	453,669.8	129,940.1
Total	14,292	6,938,650.392	174,475.106	3,506,438.4	2,633,147.7	873,290.7
Re-assurances......	66	266,146.326	7,480.975	143,062.2	105,339.8	37,722.4
	£	6,672.504·066	166,994.131	3,363,376.2	2,527,807.9	835.568.3

CHILDREN'S ENDOWMENTS.

	No.	Sums Assured.	Net Premiums.	Value of Sums.	Value of Net Premiums.	Net Value of Assurance.
		£	£	£	£	£
Endowments without return of Premiums	49	11,263.635	188.891	7.838.8	984.8	6,854.0
Endowments with return of Premiums	173	37,734.475	1,634.834	15,941.2		15,941.2
	222	48,998.110	1,823.725	23,780.	984.8	22,795.2

SUMMARY OF THE VALUATION—(*continued*).

ANNUITIES.

	No.	Annuities.	Net Premiums.	Value of Annuities.	Value of Net Premiums.	Net Value of Annuities
		£	£	£	£	£
Annuities on Single Lives	385	13,440.417		115,283.4	·	115,283·4
Temporary Annuities . .	2	30.8		243.3		243.3
Annuities on more Lives than one............	60	2,756.533		28,552.1		28,552.1
	447	16,227.750		144,078,8		144,078.8
Contingent and Deferred Annuities	16	2,126.113	492.417	5,395.9	2.161.0	3,234.9
Total Annuities	463	18,353.863	492.417	149,474.7	2,161.0	147,313.7

REPORT OF THE MANAGER ON THE QUINQUENNIAL LIFE VALUATION, AT 31ST DECEMBER, 1869.

* * * * *

Two independent Valuations have been made, one by the Officers of the Company, and another by Mr. Samuel Brown. In the one case, each Policy was valued separately; in the other, they were grouped in classes according to age.

The results of the Valuations were nearly identical; but Mr. Brown's figures have in every instance been adopted.

* * * * *

Annexed are Tables showing the Number of Whole-Life Policies in existence at the various Ages, the Sums Assured thereon, and other information, which will enable any Policy-holder to test for himself the correctness of the Valuation. These Policies comprise by far the largest portion of the Liability; but should any one desire to carry his calculations further, the more complex statements for the remaining Liabilities will be at once supplied to him.

The amount you have to deal with on the present occasion is the Profit during the last Five Years on Life Policies, amounting to £249,956.

Taking, as on the last occasion, an even sum, say £245,800, the proportion of Profit which falls to the Shareholders is £81,933 6s. 8d., and the amount to be appropriated to the Assured for Bonus purposes is £163,866 13s. 4d. This sum will provide a Reversionary Bonus of £1 10s. per cent. per annum to be added to the original sum assured of every Participating Policy effected previously to the 1st January, 1868, and existing on the 1st January, 1870, for each entire year that it had been in existence since the last Appropriation of Bonus.

I would recommend that the Profit shown on the Annuity and Endowment Branches be allowed to accumulate for at least another five

years. The Surplus Balance on the accounts is now considerable; but, looking to the experience of other Offices and the almost proverbial longevity of Annuitants, I think it more prudent to leave the amount until such time as it is unmistakeably a realised Profit.

Progress.

	No. of New Policies.	Sum Assured.	Premium.
		£	£
For the Five Years, 1860 to 1864, the results were..	7402	3,439,215	110,820
For the Five Years, 1865 to 1869, they are	7913	3,671,654	116,618
Showing an increase of	511	£232,439	£5,798

During the Five Years ending 1869, 2017 Proposals, for £962,603, were declined as not reaching the high standard of eligibility adopted by the Company, while during the previous Five Years the number of declined cases was 1508, for £776,519.

Life Funds.

After the last Division of Profits in 1865 the entire Funds, including Annuity and Endowment Accumulations and Debts on Partial-Credit Policies, amounted to	£642,866
The entire Life Fund, including Debts on Partial-Credit Policies, after deduction of Shareholders' Profit, now amounts to	1,190,410
Showing an Increase of	£547,544

or an increase of 85 per cent. during the last Five Years.

Expenses.

The Expenses continue to show a diminishing ratio. During the Five Years ending 1864 the entire expenditure, including Commission and every other charge, was at the rate of 10.40 per cent. upon the Life Income. During the last Five Years the ratio has only been 8.70 per cent. on the Income for the period.

Mortality.

In the Quinquennial Period now under review, the total amount of the Premiums received on Life Assurance Policies has been £975,414 5s. 5d.; and the Claims paid, including Reversionary Bonuses, £437,244 16s. 2d., being not quite 45 per cent. of the Premiums received. An estimate of the Rate of Mortality experienced during the same time has been made, from which it appears that whilst about 921 Deaths might have been expected, according to the Table of Mortality by which the Company's Premiums are calculated, the actual number of the Deaths which have taken place is only 776, or 84 per cent. of the expectation. Another estimate, based on the average amount at risk on each Life, shows that Claims might have been anticipated to the extent of £494,880, a sum which exceeds the actual total paid by upwards of £57,000, an excess which indicates that a considerable profit has been made from the favourable Mortality experienced.

CLASSIFICATION OF POLICIES AS AT 31ST DECEMBER, 1869.

ORDINARY WHOLE-LIFE POLICIES.—WITH PROFITS.

Present Age of the Assured.	Number of Policies.	Original Amounts Assured. £	s.	d.	Declared Bonus Additions to Amounts Assured. £	s.	d.	Annual Premiums Payable. £	s.	d.
16 to 17	1	400	0	0				7	2	0
17 „ 18	11	6,096	13	4				114	5	10
18 „ 19	5	1,500	0	0				28	1	0
19 „ 20	12	5,600	0	0				116	0	6
20 „ 21	15	5,500	0	0				106	9	4
21 „ 22	34	11,800	0	0				229	9	11
22 „ 23	55	26,850	0	0				580	18	0
23 „ 24	65	22,908	6	8	19	0	0	569	17	1
24 „ 25	100	38,492	16	8	84	16	8	888	2	4
25 „ 26	130	40,965	12	6	74	0	0	957	3	1
26 „ 27	142	50,415	11	1	72	13	4	1,217	10	5
27 „ 28	216	80,630	14	7	88	0	0	1,869	18	4
28 „ 29	227	83,458	17	1	235	0	0	1,960	8	1
29 „ 30	266	119,275	19	6	1,019	16	8	2,974	5	9
30 „ 31	317	142,042	14	2	852	0	0	3,745	18	5
31 „ 32	324	148,867	10	0	1,151	8	4	3,913	7	5
32 „ 33	361	157,991	13	5	1,295	0	0	4,135	13	3
33 „ 34	362	162,645	5	7	1,520	11	8	4,397	3	3
34 „ 35	351	140,748	19	2	1,821	3	4	3,796	10	6
35 „ 36	446	205,460	8	4	3,072	10	0	5,738	18	2
36 „ 37	390	181,077	8	7	3,296	5	0	5,006	12	1
37 „ 38	410	176,188	12	7	2,968	10	0	4,950	19	7
38 „ 39	457	217,099	2	4	3,694	11	8	6,164	3	5
39 „ 40	437	199,583	17	1	4,807	10	0	5,699	8	2
40 „ 41	433	222,922	0	9	6,894	3	4	6,411	15	8
41 „ 42	460	222,176	14	6	6,734	3	4	6,511	2	2
42 „ 43	416	211,087	17	4	5,233	0	0	6,446	4	5
43 „ 44	447	217,358	5	4	6,988	6	8	6,816	4	10
44 „ 45	407	209,645	9	6	8,663	10	0	6,451	13	10
45 „ 46	347	222,287	19	5	6,592	11	8	6,984	1	1
46 „ 47	293	140,128	0	4	5,945	10	0	4,601	1	9
47 „ 48	331	169,021	11	11	8,257	14	8	5,657	19	7
48 „ 49	289	130,691	12	4	5,004	10	0	4,419	19	0
49 „ 50	261	131,276	11	4	6,214	13	4	4,523	13	7
50 „ 51	243	126,754	3	4	5,366	10	8	4,508	6	6
51 „ 52	241	141,092	19	0	5,846	0	0	5,024	5	10
52 „ 53	228	102,089	13	10	7,595	13	4	3,772	12	4
53 „ 54	201	99,676	15	11	4,605	18	8	4,105	16	9
54 „ 55	155	100,975	17	1	5,457	4	3	4,224	5	5
55 „ 56	173	96,722	2	6	5,502	10	0	3,859	12	9
56 „ 57	129	85,162	13	8	4,116	17	4	3,255	14	9
57 „ 58	134	61,868	5	11	4,978	3	4	2,550	3	3
58 „ 59	134	80,574	12	4	4,910	14	8	3,685	0	5
59 „ 60	110	43,173	13	4	3,142	9	4	1,997	11	0
60 „ 61	87	44,492	0	0	2,646	13	4	2,080	14	10
61 „ 62	86	37,868	7	0	3,072	0	0	1,926	19	5
62 „ 63	56	30,131	12	4	2,199	0	0	1,562	16	11
63 „ 64	61	28,617	0	3	1,613	0	0	1,510	15	10
64 „ 65	57	26,521	13	4	1,925	8	0	1,792	6	3
65 „ 66	32	21,775	0	0	1,306	3	4	1,092	9	5
66 „ 67	36	11,150	0	0	1,561	3	4	599	13	7
67 „ 68	27	10,720	13	4	487	13	4	634	4	11
68 „ 69	11	8,900	0	0	1,051	0	0	557	13	6
69 „ 70	21	7,866	13	4	309	0	0	492	3	10

CLASSIFICATION OF POLICIES—(*continued*).

ORDINARY WHOLE-LIFE POLICIES.—WITH PROFITS.

Present Age of the Assured.	Number of Policies.	Original Amounts Assured.			Declared Bonus Additions to Amounts Assured.			Annual Premiums Payable.		
		£	s.	d.	£	s.	d.	£	s.	d.
70 „ 71	11	4,157	6	8	597	6	8	264	12	6
71 „ 72	12	8,266	13	4	1,104	0	0	508	5	9
72 „ 73	10	2,700	0	0	296	0	0	187	9	10
73 „ 74	5	1,500	0	0	142	0	0	116	16	4
74 „ 75	5	5,000	0	0	412	0	0	216	15	7
75 „ 76	10	3,500	0	0	410	0	0	264	16	4
76 „ 77	3	1,700	0	0	72	0	0	120	5	11
77 „ 78	5	1,200	0	0	66	0	0	103	1	0
78 „ 79	5	2,600	0	0	204	0	0	145	3	1
80 „ 81	1	100	0	0	30	0	0	6	18	4
83 „ 84	1	200	0	0				26	3	6
84 „ 85	1	300	0	0	108	0	0	21	1	3
	11,109	5,299,554	1	11	163,735	9	3	169,206	19	6
Deduct Re-Assus.		191,499	0	0	974	6	6	5,410	7	4
	11,109	5,108,055	1	11	162,761	2	9	163,796	12	2

CLASSIFICATION OF POLICIES AS AT 31ST DECEMBER, 1869.

ORDINARY WHOLE-LIFE POLICIES.—WITHOUT PROFITS.

Present Age of the Assured.	Number of Policies.	Amount Assured.			Annual Premiums Payable.		
		£	s.	d.	£	s.	d.
19 to 20	1	350	0	0	5	3	10
20 „ 21	1	100	0	0	1	14	2
21 „ 22	1	500	0	0	15	2	6
22 „ 23	4	1,500	0	0	39	13	8
23 „ 24	3	1,000	0	0	24	8	6
24 „ 25	15	5,150	0	0	131	11	6
25 „ 26	7	2,150	0	0	45	16	3
26 „ 27	16	4,972	7	11	133	4	0
27 „ 28	24	11,250	0	0	292	1	5
28 „ 29	26	9,718	15	0	240	5	3
29 „ 30	45	15,876	19	6	417	18	6
30 „ 31	23	9,759	7	6	222	19	2
31 „ 32	41	17,135	13	11	389	12	1
32 „ 33	36	12,858	17	1	371	17	0
33 „ 34	41	26,808	12	11	643	10	11
34 „ 35	47	25,891	17	6	682	0	4
35 „ 36	57	35,325	2	1	872	8	3
36 „ 37	47	20,108	19	2	566	5	9
37 „ 38	54	30,524	0	11	886	7	0
38 „ 39	45	29,842	18	4	804	3	3
39 „ 40	41	26,218	19	2	697	2	10
40 „ 41	41	21,799	4	5	612	14	3
41 „ 42	56	39,922	4	5	1,133	10	6
42 „ 43	46	20,735	4	2	566	5	8
43 „ 44	58	62,178	17	1	2,075	5	4

CLASSIFICATION OF POLICIES—(*continued*).

ORDINARY WHOLE-LIFE POLICIES.—WITHOUT PROFITS.

Present Age of the Assured.	Number of Policies.	Amount Assured. £ s. d.	Annual Premiums Payable. £ s. d.
44 „ 45	52	37,203 12 11	1,111 16 11
45 „ 46	48	29,758 1 2	938 12 11
46 „ 47	55	26,855 17 1	801 19 1
47 „ 48	45	21,858 4 7	760 5 2
48 „ 49	42	28,685 15 8	1,162 11 3
49 „ 50	45	30,751 7 1	1,908 14 10
50 „ 51	34	25,379 12 9	954 19 8
51 „ 52	43	26,118 9 7	998 17 2
52 „ 53	46	24,867 12 1	897 19 2
53 „ 54	33	18,449 5 5	634 11 1
54 „ 55	29	14,465 18 9	516 17 6
55 „ 56	37	18,622 7 11	793 4 6
56 „ 57	25	14,039 2 4	575 8 1
57 „ 58	23	15,225 10 0	661 7 5
58 „ 59	26	18,358 6 8	928 12 10
59 „ 60	26	14,767 10 0	721 4 4
60 „ 61	16	10,440 0 0	585 4 0
61 „ 62	22	17,116 3 9	770 12 6
62 „ 63	17	5,006 12 4	250 12 0
63 „ 64	24	12,612 10 0	642 15 2
64 „ 65	19	7,772 9 7	402 16 0
65 „ 66	16	9,411 9 2	555 19 4
66 „ 67	11	9,787 10 0	674 16 8
67 „ 68	7	2,450 0 0	184 19 8
68 „ 69	18	5,600 0 0	373 8 7
69 „ 70	16	11,291 12 4	832 9 1
70 „ 71	8	3,232 6 8	200 8 4
71 „ 72	8	2,709 7 6	201 2 0
72 „ 73	8	9,000 0 0	639 13 3
73 „ 74	6	1,100 0 0	76 19 3
74 „ 75	5	6,400 0 0	548 4 7
75 „ 76	1	200 0 0	15 15 0
76 „ 77	2	1,350 0 0	120 16 0
77 „ 78	5	1,200 0 0	105 18 1
78 „ 79	4	2,800 0 0	232 17 11
83 „ 84	1	1,200 0 0	103 10 0
	1,599	917,764 16 5	32,857 1 3
Deduct Reassurances	. .	61,673 0 0	2,473 6 6
	1,599	856,091 16 5	30,383 14 9
From p. 142 . .	11,109	5,108,055 1 11	163,796 12 2
Bonus „	. .	162,761 2 9	
All other Life Assurances	1,584	545,596 0 3	19,066 18 4
Children's Endowments. .	222	48,998 2 2	1,823 14 6
Annuities	463	18,353 17 3 Ans	557 14 1
Total	14,977	6,721,502 3 6 18,353 17 3 Ans	215,628 13 10

Investments of the Company, at 31st December, 1869.

Freehold Property owned by the Company and Mortgages on Freehold Property	£254,550	19	5			
£30,000 Reduced 3 per Cent. Annuities	28,012	17	6			
Indian Government 5 per Cent. Debentures	100,000	0	0			
First-class English Railway Preference and Guaranteed Stocks	361,309	11	4			
Loans to Local Authorities of various towns in Great Britain who have obtained the sanction of the Secretary of State to borrow the amounts	141,383	18	3			
Bonds of the Mersey Docks and Harbour Boards	62,272	14	2			
Bonds of the British and Irish Magnetic Telegraph Company	10,000	0	0			
Bonds of the Liverpool Corporation	5,160	0	0			
Bonds of the London and St. Katherine Docks Company	40,000	0	0			
English Railway Debenture Bonds	55,455	0	0			
United States Government Stocks	106,008	9	2			
Canadian Consolidated Stock and Canada Dominion Stock	31,244	7	3			
Short Loans on First-class English Dividend Paying Stocks, with margins from 20 to 50 per cent. on market values	504,953	19	3			
Loans on Security of Life Policies	64,425	2	10			
				£1,764,776	19	2
Cash in hand, in various Banks and in hands of Agents				86,092	8	4
				£1,850,869	7	6

Certified Life Balance-Sheet, 31st December, 1869.

Dr.

To General Life Assurance and Annuity Account	£1,264,084	0	4
	£1,264,084	0	4

Cr.

By Life Assurance Fund	£1,068,515	16	3
„ Annuities Granted Account	156,426	11	6
„ Endowments	25,589	3	0
„ Contingent Annuities	4,803	5	0
„ Unsettled Life Claims	8,749	4	7
	£1,264,084	0	4

CITY OF GLASGOW LIFE ASSURANCE COMPANY.

Established 1838.

During the five years (ending 20th January 1869) since last valuation of the Company's affairs, 4075 Policies have been issued, assuring £2,171,274 at Premiums amounting to £69,555.

During the same period, Claims in connection with 361 deaths have been met by payments amounting to £291,004, including Bonus Additions.

The number of Policies, Total Amounts Assured, and Premium

Revenue, at the commencement and end of the Quinquennial period, were—

1864. 4092 Policies—Assuring £2,636,170—Premium Revenue £85,656.
1869. 6662 Policies—Assuring £3,837,720—Premium Revenue £120,858.

During the period the Accumulated Funds have been increased by upwards of Fifty per cent, and now amount, exclusive of paid-up Capital, to fully £600,000. The average return on investments is £4:10*s.* per cent.

The Company hold £24,250 Five per cent Preference Stock of the Great North of Scotland Railway, acquired at par; a small Stock, understood to have a preference over all the other Stocks for dividend and arrears of dividend, until by a recent decision the Court of Session found that the Four-and-a-half per cent Preference—a Stock of double amount, subsequently issued—had right to a *pari passu* share of the Profits of each Year. In the present circumstances of the Railway this decision has so depressed the value of the Five per cent Stock, that, in connection with their holding, the Directors considered it proper to write off £11,590 before striking a balance for Profit.

The Committee of Directors appointed by the Board to examine into the state of the Company's other investments, report favourably of their position.

The Actuary reports the results of a careful valuation of the Company's business, showing a surplus of £48,983—which the Directors recommend should be applied to provide a Bonus of £1 per cent per annum on the Sums Assured and Existing Bonuses under Policies entitled to participate, with the ordinary dividend of Eight per cent to Proprietors.

While the Directors regret that, owing to the depreciation in the value of the Railway Stock before referred to, the result of this valuation is less satisfactory than it otherwise would have been, they at the same time desire to direct attention to the circumstance that, on this occasion, Bonus is for the first time given on existing Bonus, as well as on the Sum Assured; and, consequently, that the *actual* rate exceeds the *nominal.* In the circumstances, they have no hesitation in recommending an intermediate Bonus on Sums Assured of £1 per cent per annum as formerly.

The following is an Abstract of the Investigation Balance-Sheet:—

Dr.

Paid-up Capital		£60,000 0 0
Value of Policies, Sum Assured, and Bonus	£1,752,121 0 0	
Less value of future Premiums .	1,239,446 0 0	
		512,675 0 0
Note.—The *pure* premium alone is valued. Total amount of Annual Premium unvalued, £21,893, less £3995 payable to Re-assuring Offices.		
Value of Annuities		38,958 0 0
Claims by Death, outstanding at 20th January 1869 .		15,206 13 0
Balance—excess of Assets over Liabilities . . .		48,983 8 3
		£675,823 1 3

Cr.

Loans on Heritable Security		£157,415 2 1
Loans on Life Interests		86,234 8 4
Loans on the Company's Policies		40,563 8 3
Loans on Debentures—Railways		20,000 0 0
Loans on Personal Security		22,250 0 0
Loans on other Securities		16,903 19 6
Indian and Colonial Stocks		24,642 7 5
Railway and other Stocks		72,599 5 9
House Property and Ground Rents		39,108 6 9
Office Furniture and Stamps		847 19 1
Reversions and Annuities purchased		12,381 0 0
Re-assurances—		
Policies	£138,949 0 0	
Annuities	873 0 0	
		139,822 0 0
Premiums due at Head Office, less Re-assurance Premiums		2,045 14 1
Interest accrued since last payment		5,165 1 6
Agents' Balances		21,534 12 11
In Bank at Head Office and Branches		14,309 15 7
		£675,823 1 3

CORRESPONDENCE.

ON A TABLE OF MORTALITY DEDUCED FROM THE NEW EXPERIENCE OBSERVATIONS, H^{MF}.

To the Editor of the Assurance Magazine.

SIR,—Having recently constructed a Table of Mortality, based upon the Healthy Male and Female Observations published by the Institute of Actuaries, from which the number exposed to risk and the deaths during the first *three* years of Assurance—*i.e.*, years 0, 1, and 2—have been eliminated, I beg to place the same at your disposal for insertion in the *Journal* of the Institute, should you consider the Table sufficiently interesting and useful for publication. It has been very carefully prepared, and has been graduated by the "New Method of Adjusting Mortality Tables," proposed and explained by Mr. Woolhouse in the last volume of the *Journal.*

In explanation of my reason for excluding years of Assurance 0, 1, and 2 only, I may point out that the mortality amongst Male lives, which form 88 percent of the whole of my Table, during those three years taken together can be shown, by reference to Mr. Sprague's exhaustive paper "On the rate of Mortality amongst Assured Lives as influenced by the Duration of the Assurance" (vol. xv., p. 338), to be 68·87 percent of the expectation by the 17 Offices' Experience, and 72·34 percent of the expectation by the New H^M Experience itself, whereas the two succeeding years, 3 and 4, give 99·13 percent of actual to expected deaths by the 17 Offices' Table, and 102·84 percent by the New Experience Table. The Experience Committee have excluded years of Assurance 0 to 4 from the H^M Observations, and, I understand from Mr. Woolhouse's paper,

recommend the resulting table "for the general purposes of valuations," although the actual deaths during the two years 3 and 4 thus thrown out are quite cent percent of the expected mortality. In this way 166,166 years of risk and 1188 deaths are rejected; and therefore I have thought it worth while to construct a Table, based upon the H^{MF} Observations, and commencing at year of Assurance 3. Several eminent medical men express the opinion that the value of selection is practically lost after three years, and their opinion, though founded upon individual experience only, is confirmed by Mr. Sprague's investigations upon the subject, although there is no doubt that its effect is traceable for several years after. I venture however to suggest that the Table now produced may be assumed to indicate with considerable accuracy the mortality which may be expected to prevail amongst the assurers in a Life Office, such assurers consisting of male and female lives in fair average proportions, and from whom the effect of medical selection has passed away.

I have computed the probability of dying in a year at each age by the entire adjusted H^{MF} Table given by Mr. Woolhouse on page 396 of the last volume of the *Journal*, and have placed them side by side with the corresponding probabilities deduced from the partial H^{MF} Table to facilitate comparison. The diminished mortality at ages 10 to 18 by the partial experience, as compared with the total experience, is chiefly attributable to the small number of facts observed upon in both cases; but it is also due to the heavier mortality which prevailed at those ages in the two years immediately succeeding entry. It might therefore be advisable, in constructing monetary values, to disregard the probabilities of dying at ages 10 to 18 inclusive in favour of those obtained from the total experience. If this be done, the following would be the adjusted numbers-living and decrements at ages 10 to 24, to be substituted for those in the Table.

Age.	l_x	d_x	Age.	l_x	d_x	Age.	l_x	d_x
10	100,410	455	15	98,427	389	20	95,947	727
11	99,955	410.	16	98,038	425	21	95,220	808
12	99,545	380	17	97,613	477	22	94,412	834
13	99,165	367	18	97,136	545	23	93,578	850
14	98,798	371	19	96,591	644	24	92,728	846

Taking however the two Tables as they stand, it will be noticed that the exclusion of the first three years of Assurance reveals an increased rate of mortality commencing at age 19, and that such increase progresses rapidly until it reaches a maximum at age 24, at which age the mortality is 32 percent more by the partial than by the total experience. The difference then begins to decrease with more or less regularity until the age of 80, after which a change occurs, the mortality at ages 81 to 85 being slightly more by the total experience than by the partial experience, which fact is due to the few admissions at those ages and to the superior vitality of the female lives included. The mortality by the partial experience then increases until the end of the Table. The following Table shows very clearly the comparative influence of medical selection upon the mortality at different ages.

H^{MF} (adjusted).—*Number of Deaths compared.*

Ages.	Total Experience.	Partial Experience.	Ages.	Total Experience.	Partial Experience.
10–19	100·00	93·54	60–69	100·00	102·89
20–29	100·00	124·21	70–79	100·00	100·72
30–39	100·00	112·23	80–89	100·00	100·39
40–49	100·00	107·09	90–99	100·00	103·12
50–59	100·00	104·15			

It will be observed that the adjusted partial experience Table presents a secondary maximum of mortality at the age of 24, the probability of dying at that age being greater than at the ages immediately preceding and succeeding it. Mr. Sprague has pointed out, in his paper above referred to, that this peculiarity occurs at the age of 22 in the adjusted H^M Table, and that the same fact is noticeable at the same age in Mr. Berridge's adjustment of the Peerage Table. If, however, we examine the complete unadjusted H^M Table, as well as the partial unadjusted H^{MF} Table, we shall find that this maximum of mortality occurs at age 23. The increased mortality at this particular age is still more clearly defined in the H^M Table, from which the first *five* years of Assurance have been excluded; and it is worthy of note that the observations of the Mortality of the Government Male Life Annuitants by the late and present Actuaries to the National Debt show the same increase at that age, and further, that in the original Peerage Tables this maximum of mortality is indicated at age 23 in both the Male and Female observations. In reference to this point I would call attention to the following figures, and would remark that the facts upon which the percentages are computed are, I think, sufficiently numerous to command confidence in the results.

H^M (unadjusted).—*Mortality Percent.*

		Age 22.	Age 23.	Age 24.
Total Experience . .		·625	·773	·686
Excluding years of Assurance.	0	·951	·898	·779
	0 and 1	·835	1·060	·764
	0 to 2	·861	1·267	·765
	0 „ 3	1·055	1·660	·643
	0 „ 4	1·108	1·705	·460
	0 „ 5	1·044	1·728	·560

These figures are very remarkable, and, with one trifling exception which ascribes the maximum to age 22, tend to confirm the opinion that it is at age 23 that the climax is reached. The increase of mortality at this age appears to be very decided and uniformly progressive in proportion to the length of time elapsed since selection, whereas at ages 22 and 24 the chance of dying fluctuates considerably when similarly examined in reference to the period when selection took place.

In conclusion, I give a few specimens of annuities and premiums computed at 4 percent interest, and based upon the adjusted H^{MF} experience

excluding the first three years of Assurance; and beg to add, that I shall be happy to furnish you with the complete 4 percent Table if desired.

Age.	Value of Annuity of £1.	Value of Reversion of £1.	Annual Premium for Assurance of £1.
10	19·7416	·202244	·009751
15	18·9855	·231325	·011575
20	18·1930	·261806	·013641
25	17·5818	·285314	·015355
30	16·8711	·312650	·017495
35	16·0145	·345600	·020312
40	15·0099	·384234	·024000
45	13·8265	·429750	·028985
50	12·4985	·480828	·035621
55	11·0439	·536773	·044567
60	9·4777	·597010	·056978
65	7·9032	·657570	·073858
70	6·3508	·717274	·097578
75	4·9025	·772980	·130958
80	3·7397	·817700	·172522
85	2·7026	·857592	·231619
90	1·8861	·888996	·308027
95	1·1431	·917574	·428156

I am, Sir,
Your obedient servant,

WILFRED A. BOWSER.

Cleveland House,
Lower Clapton, 17th October, 1870.

New Mortality Experience. H^{MF}, *excluding the first Three Years of Assurance.*

Age.	Unadjusted.		Adjusted.			Probability of Dying in a Year.	
	Number-living.	Decrement.	Number-living.	Decrement.	Expectation.	Partial Experience Adjusted.	Total Experience Adjusted.
10	100000	702	100000	354	48·91	·00354	·00442
11	99298	0	99646	340	48·08	·00341	·00409
12	99298	218	99306	337	47·24	·00339	·00388
13	99080	190	98969	341	46·40	·00344	·00381
14	98890	658	98628	358	45·56	·00363	·00385
15	98232	602	98270	382	44·72	·00389	·00404
16	97630	252	97888	416	43·90	·00425	·00436
17	97378	314	97472	461	43·08	·00473	·00482
18	97064	356	97011	515	42·28	·00531	·00543
19	96708	831	96496	603	41·50	·00625	·00604
20	95877	570	95893	701	40·76	·00731	·00649
21	95307	854	95192	783	40·06	·00822	·00679
22	94453	820	94409	826	39·39	·00875	·00691
23	93633	1113	93583	850	38·73	·00908	·00695

New Mortality Experience. H^{MF}, *&c.*—(continued.)

Age.	Unadjusted.		Adjusted.			Probability of Dying in a Year.	
	Number-living.	Decrement.	Number-living.	Decrement.	Expectation.	Partial Experience Adjusted	Total Experience Adjusted.
24	92520	783	92733	851	38·08	·00918	·00695
25	91737	610	91882	840	37·43	·00914	·00700
26	91127	863	91042	802	36·77	·00881	·00709
27	90264	826	90240	811	36·10	·00899	·00733
28	89438	775	89429	830	35·42	·00928	·00758
29	88663	898	88599	828	34·75	·00934	·00783
30	87765	888	87771	830	34·07	·00946	·00806
31	86877	761	86941	839	33·39	·00965	·00823
32	86116	866	86102	830	32·71	·00964	·00837
33	85250	812	85272	813	32·02	·00953	·00855
34	84438	838	84459	825	31·33	·00977	·00875
35	83600	740	83634	833	30 63	·00996	·00900
36	82860	870	82801	850	29·93	·01026	·00933
37	81990	885	81951	870	29·24	·01062	·00967
38	81105	921	81081	893	28·55	·01101	·01000
39	80184	935	80188	901	27·86	·01135	·01027
40	79249	898	79287	905	27·17	·01141	·01050
41	78351	893	78382	907	26·48	·01157	·01068
42	77458	903	77475	911	25·78	·01176	·01091
43	76555	899	76564	920	25·08	·01202	·01127
44	75656	939	75644	938	24·38	·01240	·01173
45	74717	981	74706	969	23·68	·01297	·01232
46	73736	979	73737	1007	22·99	·01366	·01301
47	72757	1084	72730	1050	22·30	·01444	·01372
48	71673	1061	71680	1087	21·62	·01516	·01442
49	70612	1112	70593	1128	20·94	·01598	·01511
50	69500	1224	69465	1163	20·27	·01674	·01577
51	68276	1218	68302	1189	19·61	·01741	·01651
52	67058	1241	67113	1227	19·38	·01828	·01732
53	65817	1142	65886	1268	18·29	·01924	·01831
54	64675	1265	64618	1314	17·64	·02033	·01945
55	63410	1483	63304	1362	16·99	·02151	·02065
56	61927	1457	61942	1413	16·36	·02281	·02196
57	60470	1436	60529	1465	15·72	·02420	·02336
58	59034	1540	59064	1523	15·10	·02578	·02489
59	57494	1449	57541	1583	14·49	·02751	·02669
60	56045	1750	55958	1661	13·88	·02968	·02873
61	54295	1693	54297	1737	13·29	·03199	·03104
62	52502	1841	52560	1824	12·71	·03470	·03366
63	50761	1952	50736	1911	12·15	·03766	·03647
64	48809	1949	48825	1992	11·61	·04080	·03937
65	46860	2094	46833	2047	11·08	·04371	·04233
66	44766	2065	44786	2100	10·57	·04689	·04543
67	42701	2124	42686	2137	10·06	·05006	·04866
68	40577	2257	40549	2157	9·566	·05319	·05204
69	38320	2304	38392	2189	9·076	·05702	·05599
70	36016	2003	36203	2229	8·594	·06157	·06095
71	34013	2087	33974	2281	8·125	·06714	·06686
72	31926	2466	31693	2354	7·674	·07427	·07368
73	29470	2393	29339	2404	7·250	·08194	·08154
74	27077	2802	26935	2438	6·852	·09051	·09004
75	24275	2222	24497	2404	6·484	·09813	·09799
76	22053	2366	22093	2365	6·135	·10705	·10581

New Mortality Experience. H^{MF}, *&c.*—(continued).

Age.	Unadjusted.		Adjusted.			Probability of Dying in a Year.	
	Number-living.	Decre-ment.	Number-living.	Decre-ment.	Expectation.	Partial Experience Adjusted.	Total Experience Adjusted.
77	19687	2113	19728	2257	5·811	·11441	·11322
78	17574	2302	17471	2131	5·497	·12197	·12110
79	15272	2017	15340	2001	5·191	·13044	·12938
80	13255	1776	13339	1863	4·895	·13966	·13868
81	11479	1762	11476	1710	4·609	·14901	·14907
82	9717	1432	9766	1568	4·328	·16055	·16068
83	8285	1536	8198	1426	4·060	·17394	·17426
84	6749	1214	6772	1270	3·810	·18753	·18857
85	5535	1211	5502	1115	3·574	·20265	·20267
86	4324	952	4387	955	3·355	·21768	·21732
87	3372	751	3432	805	3·150	·23455	·23248
88	2621	691	2627	651	2·962	·24781	·24581
89	1930	454	1976	520	2·773	·26316	·25923
90	1476	394	1456	411	2·585	·28228	·27778
91	1082	309	1045	319	2·405	·30526	·29708
92	773	304	726	233	2·242	·32093	·31069
93	469	235	493	168	2·066	·34077	·33029
94	234	0	325	124	1·875	·38154	·35694
95	234	26	201	79	1·724	·38806	·36441
96	208	130	122	48	1·516	·39837	·37334
97	78	39	74	36	1·207	·48648	·46809
98	39	0	38	26	·815	·68420	·65999
99	0	0	12	12	·500	1·00000	1·00000
100	0	0	0	0			

ON HERR LAZARUS'S PAPER ON THE THEORY OF PROBABILITIES.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In the July number of the *Journal* you inserted a letter from me, having for its object the elucidation of a passage in Herr Lazarus's paper "On some problems in the Theory of Probabilities." I have since received a very courteous communication from Herr Lazarus in reference to the subject of my letter; and I beg to send you the substance of that communication out of fairness to Herr Lazarus, at the same time feeling confident that it will greatly interest some of your readers.

He says, in explanation of the passage upon which my remarks were based, "The simplest way to find the sum $\Omega_0+\Omega_1+\Omega_2$ would be to extend "one of the equations (28) or (29), so as to include Ω_0. I think it is self-"evident from (28) that

$$\text{“}\ \Omega_0+\Omega_1=\frac{\int_0^p x^{m-1}(1-x)^n dx}{\int_0^1 x^{m-1}(1-x)^n dx}-\frac{\int_0^p x^{m+z}(1-x)^{n-z-1}dx}{\int_0^1 x^{m+z}(1-x)^{n-z-1}dx}$$

"and as by (29) $\Omega_2 = \dfrac{\int_0^p x^{m-z-1}(1-x)^{n+z}dx}{\int_0^1 x^{m-z-1}(1-x)^{n+z}dx} - \dfrac{\int_0^p x^{m-1}(1-x)^n dx}{\int_0^1 x^{m-1}(1-x)^n dx}$;

"it follows directly by mere addition that

$$\text{“}\ \Omega_0+\Omega_1+\Omega_2 = \frac{\int_0^p x^{m-z-1}(1-x)^{n+z}dx}{\int_0^1 x^{m-z-1}(1-x)^{n+z}dx} - \frac{\int_0^p x^{m+z}(1-x)^{n-z-1}dx}{\int_0^1 x^{m+z}(1-x)^{n-z-1}dx}$$

"and from this equation I derive

$$\text{“}\ \Omega_0+\Omega_1+\Omega_2 = \frac{1}{\sqrt{\pi}}\int_0^{k_2}\varepsilon^{-t^2}dt + \frac{1}{\sqrt{\pi}}\int_0^{k_3}\varepsilon^{-t^2}dt + \frac{B_2}{A_2\sqrt{\pi}}\varepsilon^{-k_2^2} - \frac{B_3}{A_3\sqrt{\pi}}\varepsilon^{-k_3^2} \quad . \quad (50)$$

With regard to the signs of the first two terms in this expression, Herr Lazarus says, "On page 246, at the bottom, we found the inequalities

$$\text{“}\ m<p(\mu+1), \qquad m>p(\mu+1)-1.$$

"It follows that

$$\text{“}\ \frac{m}{\mu+1}<p, \qquad \frac{m+1}{\mu+1}>p;$$

"and in consequence thereof,

"$\dfrac{m+z}{\mu-1}>p$, the + sign of the first term is fixed;

"$\dfrac{m-z-1}{\mu-1}<p$, the + sign of the second term is fixed."

There is thus, then, no necessity for the double sign which I prefixed to these terms. At the same time I think it would have been as well had this step in the demonstration been inserted in Herr Lazarus's paper.

Herr Lazarus kindly points out a misprint in my letter. In the expression for $\Omega_0+\Omega_1+\Omega_2$, on page 454, the factor $\dfrac{1}{\sqrt{\pi}}$ has been omitted from the first two terms.

I am, Sir,
Your obedient servant,

WILLIAM SUTTON.

Dec. 1, 1870,
18, *Lincoln's Inn Fields.*

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On the effect of Migrations in disturbing Local Rates of Mortality, as exemplified in the Statistics of London and the surrounding country, for the Years 1851–1860. *By* THOMAS A. WELTON, F.S.S.

[Read before the Institute, 19th December, 1870.]

Introduction.

I PROPOSE in this paper to deal with a question which has an important bearing upon those calculations as to deathrates, which are now so widely circulated and so generally felt to possess interest.

It has occurred to the Registrar-General, and, as I believe, to other inquirers, to remark that the mortality happening in London is diminished, "because domestic servants, shopwomen and milliners, " who have come from the country, retire when health fails them " to their native air."

It is obvious that the resort of sick persons to country districts would produce an effect on the mortality in such districts of the opposite nature. And I suppose that few would be surprised if it were found that places on the south coast, to which persons suffering from consumption and other ailments are apt to remove, exhibited heavy deathrates. But I do not think that the aggrava-

tion of mortality at ages 15–35, which happens in the most obscure districts and amongst the humblest families, has ever been thoroughly appreciated, or more distinctly pointed to than it is in the paragraph just quoted.

Researches into the comparative mortality occurring in different parts of Lancashire and Cheshire led me to perceive

(1) that a striking depression in the mortality of Liverpool (with its environs*) at ages 10–35 coincided with the circumstance that enormous numbers of immigrants, both from Ireland and from parts of England and Wales, served to sustain the rapid increase of population, for which that unhealthy town is remarkable; and

(2) that in the rural districts of North Lancashire, which lose a portion of the natural increase of their inhabitants by emigration, there was a slight rise in the relative rates of mortality at the same ages:—

Ages.	Deathrates, 1851–60, per 100 Persons.						Per Cent. on English Rates.			
	England and Wales.		Liverpool.		North Lancashire.		Liverpool.		North Lancashire.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 – 5	7·24	6·27	9·93	9·02	5·26	4·50	137	144	73	72
5 –	·85	·84	1·15	1·11	·82	·87	135	131	97	103
10 –	·49	·51	·54	·52	·54	·52	111	102	110	102
15 –	·67	·74	·72	·64	·69	·81	107	86	103	110
20 –	·88	·85	·98	·79	·96	·99	111	92	109	117
25 –	·96	·99	1·22	1·09	1·01	1·09	128	110	105	110
35 –	1·25	1·21	1·76	1·56	1·24	1·26	141	128	99	104
45 –	1·80	1·52	2·58	2·12	1·66	1·59	144	140	92	105
55 and under 65	3·08	2·70	4·32	3·57	2·84	2·58	140	132	92	96

It will be seen that at ages 0–10 and 45–65 Liverpool is very unhealthy for both males and females. At the same ages North Lancashire is not unhealthy; indeed, at ages 0–5, the mortality there, of both boys and girls, is little more than one-half that observed in Liverpool.

But at ages 15–25, the female deathrates in North Lancashire are much higher than in Liverpool. And at ages 15–65, the female deathrates in Liverpool are depressed as compared with the male deathrates; the contrary being the case in North Lancashire.

* The Districts of Liverpool, West Derby, Birkenhead, Wirral, Prescot, and Ormskirk.

The general aspect of the figures is consistent with an idea, that the normal rates of mortality in Liverpool are from 30 to 40 per cent. higher than the English rates; but that they are depressed, through the operation of some special causes, between ages 10 and 35, and that such causes have a greater influence over the female than the male sex.

There are so many diverse attractions influencing the migrations of natives of Lancashire and Cheshire, that to identify any large district as the principal "recruiting ground" of the population of Liverpool, is out of the question. But the problem as to the effect of migrations on rates of mortality having once been seen to possess great importance, I was led to consider the question, where a better example could be found. And I concluded that the most important instance, and the one which promised to repay the most careful investigation, was that of the Metropolis and the surrounding counties.

The figures for the Metropolitan Division correspond in a remarkable manner with those just given for Liverpool and its neighbourhood:—

Ages.	Per Cent. on English Rates.				Liverpool more than London.	
	London.		Liverpool.			
	Males.	Females.	Males.	Females.	Males.	Females.
0 – 5	115	116	137	144	22	28
5 –	114	109	135	131	21	22
10 –	93	82	111	102	18	20
15 –	93	74	107	86	14	12
20 –	94	76	111	92	17	16
25 –	110	88	128	110	18	22
35 –	131	105	141	128	10	23
45 and under 55	137	119	144	140	7	21

With hardly an exception, these ratios rise and fall together, London being throughout less insalubrious than Liverpool, and both places being apparently most healthful for persons at the age 15–20.

On referring to the observed mortality in the Eastern Counties, whence so many of the working class in London are derived, and where comparatively few persons choose to reside for the benefit of their health, we have the following ratios:—

Ages.	Per Cent. on English Rates.				Eastern Counties compared with London.	
	London.		Eastern Counties.			
	Males.	Females.	Males.	Females.	Males.	Females.
0 – 5	115	116	84	82	– 31	– 34
5 –	114	109	86	92	– 28	– 17
10 –	93	82	94	111	+ 1	+ 29
15 –	93	74	96	115	+ 3	+ 41
20 –	94	76	104	113	+ 10	+ 37
25 –	110	88	92	101	– 18	+ 13
35 –	131	105	81	93	– 50	– 12
45 and under 55	137	119	74	84	– 63	– 35

Anything more thoroughly contrasted cannot well be conceived, and it is no wonder if such results, so near the surface, have put me upon making a more extended inquiry.

Characteristics of the Area of this Inquiry.

The country extending about seventy or eighty miles round London is extremely free from peculiarities in the circumstances or occupations of its people. No manufactures of striking magnitude, no mines, no great seaports are to be met with in that wide circuit. The most remarkable features are the dockyard towns, Portsmouth and Chatham, the camp at Aldershot, the huge watering place, Brighton, and the two Universities. As for Ipswich, Reading, Maidstone, and places of the like character, such third or fourth-rate towns are naturally met with at a moderate distance from one another and from the capital, in fully peopled agricultural districts. Southampton, like Gravesend and Folkestone, owes much of its importance to the Metropolis, which may be said to have, within the district indicated, many dependencies but no rivals.

The principal industry in the outlying parts of this great district is beyond comparison agriculture, and it is a matter of common observation, not only that agriculture offers employment for but limited numbers of the female sex, but that recent improvements tend to diminish the number of male labourers required.

It was therefore natural that in the ten years 1851–61, during which period the number of adult persons employed in agriculture in this country absolutely decreased, there should be a considerable migration from the agricultural districts surrounding the Metropolis; in the Eastern Counties, for example, where the

natural increase by births in excess of deaths would have been about 140,000 persons, only 27,472 were found to have been added to the population enumerated in 1851, in the succeeding 10 years.

The Registration Divisions, numbered II., III. and IV., comprehend the rural districts which surround the Metropolis, and also the County of Norfolk.

The following tables exhibit in a summary form, (1) the birthplaces of the inhabitants of London, the surrounding counties, and the remainder of England and Wales; (2) the computed numbers of persons who removed from the one of these three divisions to either of the others or abroad, during the ten years 1851–61:—

Born in	Living (in 1861) in			
	London.	Divisions II., III. and IV.	The other Divisions.	Totals.
London	1,741,177	205,996	113,920	2,061,093
Divisions II., III. and IV...	526,043	3,754,400	211,015	4,491,458
The other Divisions........	326,951	230,473	12,009,975	12,567,399
England and Wales ..	2,594,171	4,190,869	12,334,910	19,119,950
Scotland, Ireland, and other Countries	209,818	90,606	645,850	946,274
Totals	2,803,989	4,281,475	12,980,760	20,066,224

Migrations in 1851–1861 :—

Birthplaces of those who Removed.	Where they Removed to.				
	London.	Divisions II., III. and IV.	The other Divisions.	Other Countries.	Totals.
London	..	94,782	43,291	31,267	169,340
Divisions II., III. and IV.	160,545	..	82,404	199,097	442,046
The other Divisions	90,993	93,897	..	432,214	617,104
England and Wales ..	251,538	188,679	125,695	662,578	1,228,490
Scotland	10,952	8,375	45,958		
Ireland	22,529	19,887	164,539		
Islands in the British Seas	1,698	1,623	4,204		
Other Countries	31,486	12,968	36,039		
Totals....	318,203	231,532	376,435		

The natural increase in London being about 292,890 persons, was raised by these migrations to 441,753. The natural increase in the surrounding counties, being about 519,381, was reduced by migration to 308,867.

Partly through the immigration of rather greater numbers of women than of men, but chiefly through the emigration of more men than women, and the excessive mortality among males, the numbers of the sexes in London are very unequal, the females exceeding the males, in 1861, by more than 188,000. But there is no great excess of female population in the surrounding counties; in fact, there is in many places rather a deficiency.

The following were the numbers of each sex enumerated in the divisions mentioned, in 1861:—

Ages.	London.		Divisions II., III. and IV.	
	Males.	Females.	Males.	Females.
0 – 10	330,228	332,327	537,689	532,667
10 – 20	250,748	272,756	447,126	432,665
20 – 35	336,971	417,220	457,951	499,304
35 – 45	170,434	195,983	238,947	254,339
45 – 55	114,279	132,639	183,609	191,221
55 and upwards	105,121	145,283	247,692	262,528
Totals. .	1,307,781	1,496,208	2,113,014	2,172,724

The disparity in numbers at particular ages will appear on examination of the following statement:—

Ages.	Females to 100 Males.		
	London.	Divisions II., III. and IV.	England and Wales.
0 – 10	100·6	99·1	99·6
10 – 20	108·8	96·8	100·1
20 – 35	123·8	109·0	112·1
35 – 45	115·0	106·4	106·7
45 – 55	116·1	104·1	105·5
55 and upwards	138·2	106·0	112·9

The proportions returned at several ages are shown in the next table:—

Ages.	To 1,000 Inhabitants.					
	London.		Divisions II., III. and IV.		England and Wales.	
	Males.	Females.	Males.	Females.	Males.	Females.
0 – 10	118	119	125	124	126	125
10 – 20	89	97	104	101	101	101
20 – 35	120	149	107	117	112	126
35 – 45	61	70	56	59	57	61
45 – 55	41	47	43	45	42	44
55 and upwards	37	52	58	61	49	56

It thus appears that in London there are relatively few persons under 20 years of age, and many aged 20 to 45. At 55 and upwards there is a deficiency of males, and in a less degree, of females. In the surrounding counties the numbers aged 20–35 are small, but there is an excess of persons aged 55 and upwards.

The ages of the immigrants into London can only be matter of conjecture. But the age and sex of inhabitants not natives enumerated in 1861 being known, viz.:—

Natives of	Males.		Females.	
	Under 20.	20 and upwards.	Under 20.	20 and upwards.
Other English Divisions ..	74,652	309,051	84,900	384,391
Scotland	2,708	17,399	2,799	12,827
Ireland................	6,845	40,740	7,195	52,097
Other Countries	8,940	30,219	8,224	19,825

and the computed *net* immigration in the preceding ten years being—

From other English Divisions . .	251,538
,, Scotland	10,952
,, Ireland	22,529
,, other Countries . . .	33,184
In all	318,203 persons,

it may fairly be inferred that a considerable majority of the immigrants were under 20 years of age. If, for example, 2,000 persons had annually immigrated at each year of age between 10 and 20, this would have amounted to an immigration of 200,000 persons under 20 within the 10 years, which number, at the end of that term, would have been represented by not more than 108,000 persons, under 20 years of age, and not less than 85,000 persons,

aged 20 years and upwards. But there were as many as 196,263 immigrants enumerated in 1861 at ages under 20. And although I do not forget the circumstance that a number of children under 10 are brought to London by their parents, I cannot but think that the remaining 88,000 should be deemed to be more than sufficient to represent the survivors of such children, and that we are consequently at liberty to infer a net immigration of at least 200,000, but more probably 250,000 persons, at ages under 20, within the 10 years.

It is not to be lost sight of, that, through the reflux of some of the immigrants (of course at rather higher ages than those of the bulk of persons who arrive) the *net* result comprises a greater proportion of young persons than does the gross immigration. For instance, the immigrants in a particular year might number

25,000, aged 0–20,
10,000, aged 20 and upwards;

but if there were a reflux of

5,000, aged 0–20,
5,000, aged 20 and upwards;

the net result would show, instead of 71 per cent., 80 per cent. under 20 years of age.

On referring to the Census of Occupations, I find that the principal employments for inhabitants of London aged 10–20 and 20–35, are as under :—

	Males.	
	Aged 10–20.	Aged 20–35.
Domestic Servant	7,946	15,447
Messenger, Porter, Errand Boy	15,844	6,714
Warehouseman	1,144	2,776
Commercial Clerk	6,268	10,200
Law Clerk	1,764	2,728
Seaman (Merchant Service)	2,488	7,218
Dock Servant, Labourer	1,139	4,470
Railway Official, Servant	,984	3,543
Carman, Carrier, Carter, Drayman	1,748	6,791
Labourer	6,555	18,300
Building Trades (Sub. Order 14)	9,078	33,752
Blacksmith	1,517	4,079
Engine and Machine Maker	1,869	5,171
Printer	3,852	5,393
Baker	1,747	4,801
Butcher	2,357	4,383
Grocer	1,758	3,850
Draper, Mercer	2,341	5,059
Tailor	2,003	7,842
Shoemaker, Bootmaker	4,573	12,005
Cabinet Maker, Upholsterer	1,610	4,747
Soldier	1,222	8,083

	Females.	
	Aged 10–20.	Aged 20–35.
Domestic Servant	60,328	92,776
Governess	,674	3,433
General Teacher	1,413	,728
Milliner, Dressmaker	10,648	28,179
Shirtmaker, Seamstress	4,126	9,741
Tailoress	2,376	5,426
Shoemaker, Bootmaker	1,794	3,813
Artificial Flower-maker	1,782	1,780
Bookbinder	1,532	1,682
Laundress	2,646	11,753
Silk manufacture	1,028	1,859

The numbers of women employed as domestic servants being so large, and a very considerable proportion of the female immigrants being unskilled poor persons, it is likely that many of the 153,000 female servants aged 10–35 were immigrants. Girls brought up in London are, I understand, for the most part inclined to betake themselves to other occupations, and for doing so they have many opportunities. At all events, a great proportion of the female immigrants, whether employed as servants or as milliners, are so circumstanced that in case of serious illness they have almost no choice but to return to the home of their parents. They are as little attached to the place as it is possible they could be. Such a remark, I believe, applies to a much smaller proportion of the male immigrants; indeed, it is probable that of those above 25 years of age, the majority have married and established themselves in town.

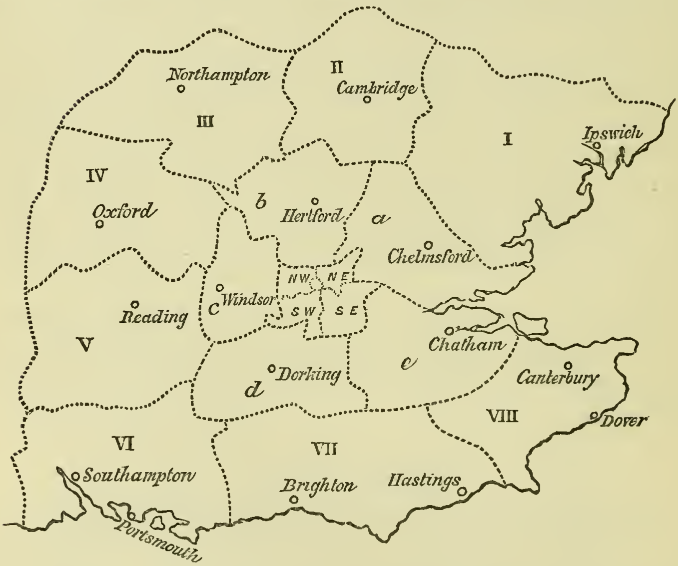
By a little manipulation of the facts which are ascertained at the Census, and without asking an additional question, the occupations and civil condition of immigrants into London might be ascertained.

Subdivisions adopted.

In the preceding remarks I have been content to refer to the three Registration Divisions numbered II., III. and IV., as representing with sufficient accuracy the country round London. But on approaching the subject of deathrates it becomes necessary to discriminate more carefully.

The following map will show, better than anything else, the plan I have adopted, for the ascertainment of the facts most interesting and important in connection with this inquiry, viz. :—

1. What are the rates of mortality amongst males and females at different ages in the great district having London for its centre and extending 70 or 80 miles around?
2. What are the rates in the central parts of London* which are built over or otherwise occupied so that no material increase of the existing house accommodation is possible?
3. What are the rates in the remainder of London and its suburbs, dividing the same into four quarters for the sake of discovering local variations?
4. What are the rates in a belt of country, just outside the suburbs, in which many families connected with London reside, such belt being again divided into several parts?
5. What are the rates in the outer belt of country, where not so many natives of London reside, and where towns of some magnitude are met with? By allotting separate areas to certain important towns, eight subdivisions of this Outer Belt have been made.



The names of the Registration Districts comprised in each subdivision are shown in Appendix A.

* In determining this question I have been obliged to exclude many districts whereof part but not the whole would answer this description: the data being given for entire Registration Districts only.

Mortality: Principal Results.

The average rate of mortality within the great circle delineated on the map appears to have been in 1851–60, 21·5 per thousand. The national rate was 22·2 per thousand, and the average rate in all parts of England beyond the circle in question was 22·5 per thousand.*

The rates of mortality at particular ages, distinguishing the sexes, were as under:—

Ages.	London (Great Circle).		Rest of England and Wales.		England and Wales.	
	Males.	Females.	Males.	Females.	Males.	Females.
0 –	68·0	58·8	74·4	64·6	72·4	62·7
5 –	8·1	8·0	8·7	8·6	8·5	8·4
10 –	4·3	4·8	5·1	5·2	4·9	5·1
15 –	6·1	6·8	7·0	7·7	6·7	7·4
20 –	8·6	7·9	8·9	8·9	8·8	8·5
25 –	10·0	9·3	9·4	10·2	9·6	9·9
35 –	13·7	12·1	11·9	12·2	12·5	12·1
45 –	19·2	15·7	17·4	15·0	18·0	15·2
55 –	32·1	28·0	30·3	26·5	30·8	27·0
65 –	67·2	60·9	64·5	57·6	65·3	58·7
75 and upwards	168·8	158·6	163·9	154·0	165·4	155·5
All Ages..	22·6	20·5	23·3	21·7	23·0	21·3

From the above table we gather, first, that the absolute deathrates were lower in the London Circle than in the remainder of the country at ages

0–25 among males,
0–45 „ females,

but that the reverse was the case at all higher ages. Next, that the deathrates among females were comparatively depressed in the London Circle at ages 20–45, and in a less degree at ages 45–55; thus:—

Ages.	Female Deathrate compared with Male Rate.	
	London Circle.	Rest of England.
20 –	91 per cent.	100 per cent.
25 –	93 „	109 „
35 –	88 „	103 „
45 –	81 „	86 „
55 –	87 „	87 „

* In all the tables, and throughout the rest of this paper, the rates of mortality shown are averages for the ten years 1851–60 per thousand persons of the age and sex mentioned.

Having regard to facts which are yet to be explained in detail, I am inclined to believe that the female deathrates in the London Circle are depressed through the departure of sick persons who eventually die beyond the borders of that circle, as well as by the constant immigration of healthy persons. But I by no means assert that if all the facts could be ascertained and proper allowances made, the true loss by death (in London and the surrounding districts) would appear to be at ages 20–45 as considerable amongst females as amongst males. The national averages at ages 20–25 and 35–45 shew lower deathrates amongst females than males, and although I anticipate that it would be necessary to transfer part of the deaths which happen in counties beyond the circular limit to the account of the Metropolis, thus abating the female deathrates in those counties, I see no reason to imagine that the London rates would be so much altered thereby, as to bring them up to the level of the mortality there observed amongst males. This remark applies with especial force to the ages 35–45. At ages 25–35 the national averages shew that females are less healthy than males; and although the London mortality would require to be augmented by fully seven per cent., in order to bring the female deaths up to the male standard, I am unable to say that such an augmentation is altogether unlikely to be justified by facts, when I consider that some invalids from the London Districts die abroad.

In any case, we may rest assured that the female mortality at ages 25–45 is really much heavier (as compared with that amongst males) in the further parts of the country than in the London Circle. Thus:—

	Female Mortality as compared with Male.	
	Age 25–35.	Age 35–45.
London Circle	93 per cent.	88 per cent.
Norfolk and adjacent Districts	110 „	110 „
South-Western Counties	95 „	93 „
West Midland Counties	106 „	99 „
North Midland Counties	133 „	121 „
Yorkshire	122 „	107 „
North-Western Counties	108 „	99 „
Northern Counties	113 „	103 „
Wales	99 „	105 „

It may be that the female mortality at these ages is aggravated in the manufacturing districts by reason of the employment of

women in unhealthy occupations. In the Manchester District (comprehending a very large part of the North-western Counties) the ratios were (in the same period 1851–60):—

Age 25–35 . 115 per cent.
„ 35–45 . 104 „

although there was a large immigration of females into that district. But to pursue this investigation would carry me far beyond the limits of the present inquiry. It is enough that I have pointed to the want of regularity in the proportions of the deathrates of the sexes at the ages mentioned, which is the more remarkable inasmuch as the like proportions at ages 0–5 are wonderfully steady.

	Deathrate, Age 0–5.		Female compared with Male Deathrate.
	Males.	Females.	
London (Great Circle)	68·0	58·8	86 per cent.
Rest of England and Wales	74·4	64·6	87 „
England and Wales	72·4	62·7	87 „
Norfolk and adjacent Districts	67·3	56·7	84 „
South-Western Counties	56·3	48·7	86 „
West Midland Counties	75·6	65·5	87 „
North Midland Counties	68·3	57·8	85 „
Yorkshire	79·6	68·4	86 „
North-Western Counties	93·7	82·0	88 „
Northern Counties	69·4	61·6	89 „
Wales	61·9	54·2	87 „
Manchester District	98·7	85·4	87 „

Turning to the larger subdivisions of that great circle round London which is the subject of this paper, we find that the rates of mortality are as shown in Table III. annexed, and that if we take the mortality in England and Wales as a standard, notwithstanding any objection which might be urged against its use for that purpose, we obtain the proportional figures shown in Table IV.

It will be observed that the ratios at ages under 10 and over 45 show the country districts to be salubrious and London itself to be unhealthy. Such results being in accordance with probability, need no explanation. But the mortality at ages 10–35 indicates something very different. And when I consider that between those ages the population of London is very much disturbed by migrations, and that in particular there are great numbers of females of those ages resident in London merely for the sake of employment,

who naturally leave town in case of serious illness, I cannot contemplate the tables without the conviction being forced upon me, that to this cause, rather than to the migrations of the rich, we owe the apparently low mortality in London, and the apparently high mortality in the surrounding rural districts, at those ages.

It will be noticed that the London (Centre and Suburbs) rate is lower than that observed in either the Inner or the Outer Belt of rural districts

at ages 20–25 among males,
at 10–35 „ females,

but is higher than either at all other ages.

It is evident that the immigration of healthy young people from the country must tend to lower the standard of mortality in London at the ages 10–35, whilst such immigrants are still in their prime. But when we take into account the poverty and misery which disfigure so many parts of the Metropolis, it is quite incredible that by reason of such immigration the average deathrates could be reduced not only below the national standard, but even below the deathrates which prevail in salubrious country districts. And if we again consider the moderate mortality in early youth and at ages above 35 which is observed to take place in the surrounding districts, to what can we attribute the heavy deathrates in those very districts at ages 10–35 unless to such an influx of sick persons as I have suggested? When we get rid of a great many such migrations, by taking the average within the large circuit round London shown on the map, we find we have almost got rid of the irregularities in question at the same time.

The comparative mortality among males and females respectively, is, under the circumstances, disturbed to a surprising extent. Even after the influence of migrations has ceased—I mean such migrations as result from the necessities of the wage-receiving class—female mortality continues to be comparatively depressed in London, I suppose through the departure of invalids, the daughters and wives of persons in fair circumstances, who could not themselves, if equally unwell, afford to quit their business engagements.

The following figures represent the comparative deathrates amongst females at the ages mentioned, the male rate being assumed to be 100:—

Ages.	Female Deathrate (from all Causes) compared with Male Rate.				
	Centre.	Suburbs.	Centre and Suburbs.	Inner Belt.	Outer Belt.
0	88	87	88	85	85
5	92	96	95	100	102
10	93	94	93	113	130
15	93	86	88	125	137
20	82	77	78	94	109
25	83	84	83	93	108
35	76	82	79	88	105
45	71	75	73	85	92
55	77	80	79	87	96
65	82	83	82	93	95
75 and upwards	90	90	90	94	95

Even if the female deathrates in the Outer Belt are augmented by no more than 20 per cent. at ages 10–20, and 10 per cent. at ages 20–35, by reason of the migrations to which reference has been made, no one will call such augmentations slight or unimportant. The above table shows that it is by no means unlikely that the deathrates are modified to at least the extent mentioned.

At the ages most affected by the influences now under consideration, phthisis, or tubercular consumption, is the most fatal disease. During the 10 years 1851-60 the deaths in England and Wales were as under:—

Ages.	Males.			Females.		
	All Causes.	Phthisis.	Per Cent.	All Causes.	Phthisis.	Per Cent.
0	916,882	16,820	1·8	789,701	16,125	2·0
5	94,592	5,838	6·2	93,151	6,856	7·4
10	49,393	7,725	15·6	50,424	12,885	25·6
15	61,239	21,950	35·8	68,630	32,670	47·6
20	73,090	33,565	45·9	78,497	39,465	50·3
25	129,877	54,729	42·1	148,339	68,388	46·1
35	134,064	43,017	32·1	137,722	47,342	34·4
45	142,253	30,344	21·3	126,196	25,903	20·5
55	161,473	17,442	10·8	152,746	13,480	8·8
65	186,305	6,813	3·7	195,108	5,437	2·8
75 and upwards	189,368	1,062	·6	231,665	1,067	·5

The causes of death at high ages are often obscure, and it is obvious therefore that no safe inference could be based on the calculated mortality from phthisis at ages 75 and upwards. But if my theory be correct, the same circumstances which disturb the deathrates from all causes at ages 10–35 should affect in a greater

measure the observed mortality by this particular disease, which, as it incapacitates from work some time before death, leaves ample time for that return home of sick persons on which my theory depends.

The deathrates by phthisis are shown in Tables V. and VI.

Here again the rates of mortality in London (Centre and Suburbs) fall below those observed in both the Inner and Outer Belts at ages—

20–25 among males.
10–35 „ females.

but are higher than either at almost every other age.

Contemplating the excess of the national deathrate from phthisis above that observed in the "Great Circle" at ages 10–25, it would seem probable not only that consumptive persons from London migrate beyond the limits of that circle, but that there is actually an aggravated deathrate from this disease in the rest of the country at the ages in question. But this by the way.*

It will be admitted that whether we take deaths by all causes or by phthisis alone, the results for females are by much the most remarkable. I have not commented upon the fact that the *Central* Districts of London are apparently healthier for females at ages 10–35 than the Outer Belt, where the mortality amongst infants and aged persons is low. But this greatly strengthens my argument, because in those Central Districts there is much misery, and there are several of the largest metropolitan hospitals (viz., St. Bartholomew's, Guy's, St. Thomas's, London, Westminster, King's College, and Charing Cross Hospitals), in respect of which no correction has been made, although it is certain that many persons who die in those hospitals come from what, for want of a better term, I have designated the suburbs.†

It is in the power of the authorities to obtain statistics as to the birthplaces of the dying, when, if I am right, a most inadequate

* It may also be noted that the London deathrates by phthisis occupy the respective positions of the *minimum* of the eleven divisions for females, and the *maximum* for males.

† It would be interesting to examine into the rates of mortality by other causes severally, but to do so would involve no little labour. The contrasts would in general be less striking, the female deathrates by *all causes except phthisis* being as under:—

	Ages 0–5.	5–	10–	15–	20–	25–	35–	45–
London (Centre)....	86·36	9·78	3·71	3·82	4·24	5·97	10·22	17·19
„ (Suburbs) ..	64·37	7·94	3·17	3·17	3·41	4·67	7·98	13·52
Inner Belt	46·30	6·30	3·61	4·19	4·19	5·05	7·60	11·25
Outer Belt	47·67	6·61	4·00	4·28	4·59	5·23	7·34	10·54

The Centre is thus (apparently) healthier than the Outer Belt at ages 10–25.

number of deaths of persons born in the provinces would be found to occur in London, taking into consideration the numbers of such persons residing there, at the ages 10–35. It might also be possible, in a few rural Registration Districts, where the deathrates among females are manifestly excessive at the same ages, to institute a careful inquiry during a few years into the history of those persons who died at the ages in question, so as to determine how many of them had lost their health whilst away in London or at some other distant place, and come home because unfit for work. Until some such tests are applied, I think I am bound to infer—

(1.) That the recorded mortality at ages 10–35 is not inconsistent with what might reasonably be expected, if my theory as to migrations be correct.

(2.) That such mortality furnishes no reliable evidence as to the salubrity of the Metropolis or the unhealthiness of the surrounding rural districts for persons aged 10–35.

Mortality: Results for Subdivisions.

It is not to be expected that such a conclusion as that the observed mortality at certain ages in the country districts around London should be reduced by as much as 10 to 20 per cent., and the London mortality at the same ages increased by 15 to 30 per cent., in order to arrive at the real risk of death in the respective places, will be readily accepted. I am bound, indeed, to test it as far as I can.

One of the most effective tests would be the division of the Inner and Outer Belts into parts, and comparison of the rates of mortality within each of those fractions with the London rates. If it be found that that which is true of the whole is true also with respect to either of such parts, the proof will amount to demonstration that some general cause is acting, with greater or less force, throughout the districts in question, and is powerful enough to overbear local dissimilarities, thereby rendering obscure the question as to which are the most salubrious districts, unless it can be solved by a reference to mortality at lower and higher ages.

It should be remarked here that on my hypothesis, those districts where the greatest number of servants are employed would exhibit the most favourable rates of mortality among females at ages 10–35. Less attractive districts, where there are fewer servants and fewer immigrants from London, are the most likely to send away many girls to London, and these should show the heaviest deathrates. Table I. furnishes the means of judging which districts are of either character.

Table IX. exhibits the rates of mortality from all causes in the several subdivisions of London, amongst males and females respectively. Table XII. shows the ratios which those figures bear to the national averages.

It happens that the northern half of the Outer Belt, comprising the districts numbered I. to IV., contains the smallest proportions of female servants and of inhabitants derived from London. The average ratios for the northern and southern halves are as under:—

Ages.	Deathrates (all Causes), 1851–60.			
	Districts I. to IV.		Districts V. to VIII.	
	Males	Females.	Males.	Females.
0	61·19	51·45	54·33	46·22
5	6·97	7·06	7·15	7·41
10	4·42	5·83	4·03	5·14
15	5·99	8·99	6·11	7·49
20	8·35	10·29	9·51	9·25
25	8·62	10 60	10·35	9·93
35	10·01	11·94	12·17	11·42
45	13·67	13·70	15·90	13·56
55	24·52	24·34	26·05	24·17
65	58·07	55·63	57·76	54·20
75 and upwards	164·70	156·50	158·20	150·25

It will be seen that the mortality amongst females at ages 10–35 is highest in the Northern Districts, where also it is more excessive, in comparison with that amongst males, than in the Southern Districts.

Dividing the Inner Belt similarly, and placing Chelmsford and Maidstone Districts in the poorer group, we have:—

Ages.	Deathrates (all causes), 1851–60.			
	Districts *a* and *e*.		Districts *b*, *c* and *d*.	
	Males.	Females.	Males.	Females.
0	59·35	49·35	52·62	45·55
5	7·00	7·17	6·67	6·50
10	4·14	4·76	4·04	4·53
15	6·33	7·74	5·23	6·76
20	9·52	8·93	7·80	7·58
25	10·85	9·79	9 02	8·80
35	13·47	12·13	12·32	10·72
45	16·15	14·50	16·59	13·60
55	28·34	24·67	28·36	24·52
65	60·30	57·37	62·89	57·54
75 and upwards	164·18	151·23	167·91	159·07

Here the mortality amongst females aged 10–35 is lowest, it is true, in the richer group of districts, but then there is a still more important difference in the mortality amongst males, so that the female deathrate in both groups is lower than that amongst males at ages 20 and upwards, and there is no ground for saying that it is comparatively excessive in the poorer districts.

This anomaly, it appears to me, is caused by the existence of military hospitals at Chatham, in which there is a very heavy mortality not properly belonging to the district. If we exclude the Registration Districts of Medway, Maidstone and Gravesend, we have an almost entirely rural population in the residue of the Chelmsford and Maidstone Districts. The Registration Districts of Edmonton, Barnet and Uxbridge being excluded from the other group, it also is freed from the disturbance to its mortality caused by the great lunatic asylums of Hanwell and Colney Hatch, as well as the semi-suburban district extending northwards from Hackney, and becomes likewise in a great measure rural. The mortality in the diminished groups is found to be as under :—

Ages.	Deathrates, 1851-60 (excluding Towns, &c.).			
	Districts *a* and *e*.		Districts *b*, *c* and *d*.	
	Males.	Females.	Males.	Females.
0	56·22	46·33	52·36	45·46
5	6·71	6·97	6·63	6·50
10	4·13	4·88	4·03	4·62
15	5·77	8·11	5·11	6·96
20	8·10	8·95	7·64	7·86
25	8·73	9·57	8·48	8·67
35	11·40	11·54	10·82	10·44
45	14·79	13·76	15·09	13·05
55	26·89	23·54	27·30	24·01
65	58·79	55·99	61·12	57·61
75 and upwards	164·28	151·80	170·18	159·08

The anomaly, it will be seen, is not reproduced in the above figures. At each age from 10 to 35 the female mortality as shown above is both absolutely and relatively highest in the less wealthy of the two groups.

But, it may be urged, exceptional circumstances affect what I have termed the "Inner Belt." Let us see what would be the figures for the "Outer Belt," after excluding the principal towns, and the military camps at Aldershot and Shorncliffe.

Adhering to the division of that Belt into two parts, comprising

respectively the four northern and southern fractions, we have the understated results:—

Ages.	Deathrates, 1851–60 (excluding Towns, &c.).			
	Districts I. to IV.		Districts V. to VIII.	
	Males.	Females.	Males.	Females.
0	59·28	49·47	47·34	40·18
5	6·83	6·98	6·74	6·90
10	4·39	5·98	3·92	5·31
15	5·83	9·34	5·88	8·10
20	8·27	10·65	9·24	9·89
25	8·17	10·64	9·50	10·01
35	9·46	11·73	10·54	11·11
45	12·63	13·23	13·94	12·80
55	23·38	23·90	23·81	23·18
65	56·54	55·11	55·37	53·35
75 and upwards	163·57	155·61	156·82	149·85

Here we see an extraordinary female deathrate at each age 10 to 35 in both groups; but it is nevertheless highest both absolutely and relatively in the poorer districts, as I contend should be.

Taking, for the moment, the mortality in *London* (*centre and suburbs*) as a standard, the following would be the comparative figures for the above districts:—

Ages.	Ratios, the London Rate being 100.					
	Districts I. to IV.		Districts V. to VIII.		Towns, &c., in Outer Belt.	
	Males.	Females.	Males.	Females.	Males.	Females.
0 –	73	69	58	56	88	87
5 –	71	77	70	76	85	92
10 –	97	142	87	126	99	113
15 –	94	171	95	149	108	121
20 –	100	165	112	154	116	128
25 –	78	123	91	115	111	115
35 –	59	93	66	88	93	98
45 –	52	75	58	72	86	88
55 –	56	73	57	71	79	82
65 –	68	81	67	78	81	84

It is not pretended, of course, that these last figures show the extent to which the country mortality is aggravated by the effect of migrations; the results, extraordinary as they are, required a two-

fold cause, viz., that the London deathrates should be depressed just when the country rates are most heavy. Thus, resuming the use of the national ratios by way of standard, we have:—

Ages.	Ratios, the English Rate being 100.							
	Districts I. to IV.		Districts V. to VIII.		Towns, &c. Excluded.		London Centre and Suburbs.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 –	82	79	65	64	99	99	112	114
5 –	80	83	79	82	95	99	113	108
10 –	90	118	80	105	91	94	92	83
15 –	87	127	88	110	100	89	93	74
20 –	94	125	105	116	109	97	94	76
25 –	85	107	99	101	121	100	109	87
35 –	76	97	84	91	120	102	128	104
45 –	70	87	78	84	116	103	135	117
55 –	76	88	77	86	106	99	134	121
65 –	87	94	85	91	103	98	127	117

The figures for the towns, it will be seen, resemble the London rates in several respects. Thus:—

1. There is a high deathrate amongst males at ages 25 and upwards;
2. The female deathrates are depressed in comparison with those amongst males at ages 15 and upwards;
3. The female deathrates are lowest (comparatively) at ages 10–25;

and in all these respects the facts for the country districts are of an opposite nature.

Of course, it may be still imagined, that by a different manipulation of boundaries, other results might have been arrived at; in answer to which I can say, in the first place, that I have not altered the arrangements which were originally made (as a matter of course) before the calculations were commenced, and in the next place, suppose we take the maxima and minima of a couple of my divisions. Thus:—

Outer Belt, District I., comprises 20 Registration Districts; the highest female mortality at ages 10–35 is observed in the first four undermentioned; the lowest in the second four:—

	Deathrates, 1851-60, per 1,000.							
	Males.				Females.			
	10-	15-	20-	25-	10-	15-	20-	25-
Halstead	4·88	8·04	13·18	8·68	7·83	12·94	12·29	12·21
Samford	3·43	7·58	10·43	7·72	6·19	10·77	15·26	12·34
Risbridge	4·79	4·81	8·49	8·04	7·63	11·08	11·97	13·25
Lexden	3·05	8·29	10·06	7·51	6·50	11·36	13·40	10·79
Colchester	5·23	6·70	11·68	11·51	6·09	9·50	9·33	10·58
Bosmere	5·18	4·14	6·91	6·55	5·51	8·19	8·64	10·98
Ipswich	3·54	7·25	9·38	9·71	4·00	6·55	8·42	10·76
Bury St. Edmunds ..	5·41	5·48	11·16	14·11	4·56	5·30	7·62	9·83

It will be seen that the lowest four include the *three towns* of greatest importance within the district (and in which the infantile deathrates are highest of any), and one rural district, Bosmere, which shows the very lowest infantile deathrate, and where, in spite of the mortality among females aged 10-35 being relatively low, it is *above* the national rates, and strikingly in excess of the male deathrates. Bosmere is situate between Ipswich and Stowmarket, and includes the small towns of Debenham and Needham Market.

Outer Belt, District II., comprises 13 Registration Districts, the highest female mortality at ages 10–35 is observed in the first three undermentioned; the lowest, in the second three.

	Deathrates, 1851-60, per 1,000.							
	Males.				Females.			
	10-	15-	20-	25-	10-	15-	20-	25-
Newmarket	5·06	5·86	8·16	9·18	6·03	9·11	11·08	12·82
Saffron Walden	4·44	5·31	9·41	6·95	6·28	8·97	11·21	10·22
Chesterton	4·93	5·45	8·78	8·29	5·89	10·66	10·57	9·25
Royston	4·20	4·88	7·97	7·25	4·75	8·57	9·05	9·76
North Witchford....	3·73	6·35	8·56	8·12	3·97	7·72	7·94	8·43
Cambridge	4·65	7·58	7·95	11·99	3·71	6·94	6·86	8·16

Here, again, the only town in the district of any magnitude is at the bottom of the list, and the excess of female mortality in rural places is quite sufficiently striking.

The same influences which operate so largely to increase the apparent mortality amongst females in the "Outer Belt," extend beyond it to a considerable distance. Norfolk, for example, sends a large and constant stream of population to London, and doubtless receives back many disabled individuals. The following table

gives the result for Norfolk, with the addition of some adjacent districts :—

Ages.	Deathrates, 1851-60, per 1,000.						Ratios, the English Rate being 100.			
	Norfolk.		Three Towns.		Rest of Norfolk.		Three Towns.		Rest of Norfolk.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 –	67·3	56·7	88·2	75·4	62·0	51·9	122	120	86	83
5 –	7·8	8·4	9·3	9·8	7·5	8·0	109	116	88	95
10 –	4·8	5·5	4·7	5·0	4·9	5·7	96	99	100	113
15 –	6·5	7·9	6·8	5·8	6·4	8·6	102	79	96	117
20 –	8·9	9·3	8·4	8·3	9·1	9·7	95	97	103	114
25 –	8·7	9·6	9·8	9·6	8·3	9·5	102	97	87	96
35 –	9·7	10·8	13·0	11·5	8·8	10·5	104	95	71	86
45 –	12·5	12·0	17·7	14·4	11·2	11·3	99	95	62	74
55 –	22·8	20·5	31·9	25·9	20·6	18·9	103	96	67	70
65 –	52·5	45·7	67·8	54·9	48·9	42·9	104	94	75	73

Here we see that the three towns (Norwich, Lynn and Yarmouth) are much less healthy than the open country at ages 0–10 and 35 upwards. But at ages 10–25 the towns appear to be more healthy, for females at all events, than the villages. And as the rural population is very large, so that its peculiarities are impressed upon the average figures, it is probable that migrations to and from places beyond the boundary of this Norfolk District (whether London or elsewhere) contribute very much to produce the results just shown.

The mortality in the other bordering districts has not been investigated. In them, other influences may be supposed to affect the deathrates, besides the migrations to which our attention has been directed, and to do justice to the subject a very careful and extended inquiry would be requisite.

Table XIX. exhibits the mortality from phthisis in the several subdivisions of London amongst males and females respectively.

Dividing the Outer Belt as before, we have :—

Ages.	Deathrates (Phthisis), 1851-60.					
	Districts I. to IV.		Districts V. to VIII.		Towns, &c., Excluded.	
	Males.	Females.	Males.	Females.	Males.	Females.
0 –	1·70	1·52	·98	1·00	1·34	1·24
5 –	·53	·68	·37	·53	·52	·63
10 –	·63	1·83	·49	1·22	·62	1·23
15 –	1·95	4·69	2·03	3·84	2·24	2·92
20 –	4·12	5·72	4·39	5·41	4·25	4·06
25 –	3·72	5·25	4·62	5·05	5·23	4·63
35 –	3·01	4·52	3·56	4·03	5·59	4·46
45 –	2·71	3·24	2·94	2·98	4·93	2·98
55 –	2·28	2·42	2·49	2·08	3·51	2·09
65 –	1·72	1·38	1·61	1·35	2·09	1·38

and the Inner Belt gives the following results (excluding the same six Registration Districts as before), viz.:—

Ages.	Deathrates (Phthisis), 1851-60.					
	Districts *a* and *e*.		Districts *b*, *c* and *d*.		Towns, &c., Excluded.	
	Males.	Females.	Males.	Females.	Males.	Females.
0	1·44	1·16	1·04	1·00	1·02	·87
5	·39	·58	·35	·47	·30	·45
10	·47	1·07	·52	1·07	·44	·88
15	1·68	3·31	1·66	3·10	2·02	2·31
20	3·28	4·30	3·63	3·99	4·63	3·53
25	3·26	4·26	3·79	4·16	5·56	4·17
35	3·45	3·89	3·74	3·59	5·19	3·84
45	2·91	2·77	3·18	2·65	3·99	2·92
55	2·65	1·88	2·74	1·97	3·23	2·31
65	1·44	1·04	1·60	1·46	2·39	1·32

Lastly, Table XXIII. gives the ratios which the deathrates by all causes amongst *females* bear to the *male deathrates*, in the various subdivisions of our "great circle;" and the range of such ratios is very remarkable.

Conclusion.

The tables have barely been explained in the remarks which I have allowed myself to make. That they disclose important facts, and that if my interpretation of those facts be disputed, another must be found, I believe all will allow.

It is rather disquieting to have one's attention called to the unsoundness of the bases upon which local deathrates, tables of mortality, and I know not what else, have been calculated. But we must admit, that it is more gratifying to a scientific mind that we should discover regular principles* operating amidst the disturbances which have been noticed, than that both disturbances and principles should remain sunk in obscurity, or disregarded through apathy.

It is vain, it would seem, to look (at least in this country) for any quiet self-contained place, where population remains undisturbed by migrations, and where statistics of mortality can therefore be obtained, requiring no rectification. The quieter the place, the more certain it is that it cannot furnish employment for the natural increase of the numbers of its population, some of whom must therefore remove to the great cities and industrial hives

* That the tables show a great degree of regularity in the results deduced from the statistics of different places similarly circumstanced, will be apparent on inspection.

which act as magnets, and draw new inhabitants from solitudes unthought of by their busy workers.

Again, in speaking of population, we must always bear in mind that the humbler classes form a great majority in almost every place, and that unless we apply ourselves to the consideration of their ways we must run the risk of falling into serious errors.

It will be seen throughout that the female deathrates are most largely affected. But the phenomena, although different, are likewise important in the case of males. The tables show that for that sex, at least, towns are exceedingly insalubrious, and I cannot help fancying that the heavy mortality occurring amongst men aged 35 and upwards in London and elsewhere is due to other than "home influences." Brutality and drunkenness do not conduce to lengthen life; and the very poor, in Bethnal Green, suffer much less than the better paid inhabitants of Whitechapel and St. Giles's.

If we wish to form an idea as to the comparative salubrity or otherwise of two country districts, I think we must rely on the deathrates at the ages of 0–10 and 45–75, and abandon the thought of deducing any instructive result from their mortuary statistics at the intervening ages. And the same remark, subject to some correction, will apply to town districts. The principal rectifications there required are such as are necessitated by the existence of large hospitals and workhouses, and by the tendency of sick persons, especially ladies, to leave towns.

In quitting the subject, I need hardly remark upon the usefulness of any research which may tend to make us better acquainted with the character and value of data which are in daily use. Apparent accuracy is in general a mere matter of arithmetical labour, but it has often served as the foundation of popular fame. It is therefore the more necessary that labours which are not likely to be appreciated by the multitude should receive the attention of actuaries and men of science.

APPENDIX A.

Particulars as to the composition of groups of Registration Districts referred to in this paper.

London (Centre). London City, East London, St. George in the East, Whitechapel, West London, St. Luke Clerkenwell, Holborn, St. Giles, Strand, St. Martin-in-the-Fields Westminster, St. James' Westminster, St. Saviour, St. Olave, and St. George, Southwark.

Suburbs (North East). West Ham, Islington, Hackney, Shoreditch, Bethnal Green, Stepney, Mile End, Poplar.
„ (North West). Brentford, Kensington, Chelsea, St. George Hanover Square, Marylebone, Hampstead, Pancras.
„ (South West). Lambeth, Wandsworth, Richmond, Kingston, Croydon.
„ (South East). Bermondsey, Newington, Camberwell, Rotherhithe, Greenwich, Lewisham, Bromley.

Inner Belt (*a*). Epping, Ongar, Romford, Orsett, Billericay, Chelmsford, Rochford, Dunmow.
„ (*b*). Hendon, *Barnet*, *Edmonton*, Ware, Bishop's Stortford, Hitchin, Hertford, Hatfield, St. Albans, Watford, Hemel Hempstead, Berkhamsted, Luton.
„ (*c*). Chertsey, Staines, *Uxbridge*, Windsor, Amersham, Eton.
„ (*d*). Epsom, Guildford, Hambledon, Dorking, Reigate, Godstone, East Grinstead.
„ (*e*). Dartford, *Gravesend*, North Aylesford, Hoo, *Medway*, Malling, Sevenoaks, Tunbridge, *Maidstone*, Hollingbourn, Milton, Sheppey.

Outer Belt (I.) Maldon, Tendring, *Colchester*, Lexden, Witham, Halstead, Braintree, Risbridge, Sudbury, Cosford, Thingoe, *Bury St. Edmunds*, Mildenhall, Stow, Hartismere, Bosmere. Samford, *Ipswich*, Woodbridge, Plomesgate.
„ (II.) Huntingdon, St. Ives, St. Neots, Biggleswade, Caxton, Chesterton, *Cambridge*, Linton, Newmarket, Ely, North Witchford, Royston, Saffron Walden.
„ (III.) Banbury, Brackley, Towcester, Potterspury, Hardingstone, *Northampton*, Daventry, Brixworth, Wellingborough, Kettering, Thrapstone, Bedford, Ampthill, Woburn, Leighton Buzzard, Winslow, Newport Pagnell, Buckingham.
„ (IV.) Faringdon, Abingdon, Wycombe, Aylesbury, Thame, *Headington*, *Oxford*, Bicester, Woodstock, Witney, Chipping Norton.
„ (V.) Hartley Wintney, Basingstoke, Whitchurch, Andover, Kingsclere, Newbury, Hungerford, Wantage, Wallingford, Bradfield, *Reading*, Wokingham, Cookham, Easthampstead, Henley, *Farnborough*.
„ (VI.) *Farnham*, Westhampnett, Chichester, Midhurst, Westbourne, Havant, *Portsea Island*, *Alverstoke*, Fareham, Isle of Wight, Lymington, New Forest, *Southampton*, South Stoneham, Romsey, Stockbridge, Winchester, Droxford, Catherington, Petersfield, Alresford, Alton.
„ (VII.) Rye, *Hastings*, Battle, Eastbourne, Hailsham, Ticehurst, Uckfield, Cuckfield, Lewes, *Brighton*, Steyning, Horsham, Petworth, Thakeham, Worthing.
„ (VIII.) Cranbrook, Tenterden, West Ashford, East Ashford, Bridge, *Canterbury*, Blean, Faversham, Thanet, Eastry, *Dover*, *Elham*, Romney Marsh.

The names in *italics* are those of towns and other districts which have been treated as exceptional.

The district called in the paper that of Norfolk consists of the Registration County of Norfolk, with Wisbech, Hoxne, Blything, Wangford and Mutford added.

The following tables are selected from a much more extensive series appended to the paper.—Ed. *J. I. A.*

TABLE I.—*Showing the Population and Number of Deaths in each of the Subdivisions of the Great Circle round London, with several additional particulars.*

Districts.	Mean Population, 1851-61.	Deaths, 1851-60.		To 1,000 Inhabitants (in 1861).			
		All Causes.	Phthisis.	Number Aged 20-35.		Female Servants Aged 20 and upwards.	Natives of Metropolitan Division.
				Males.	Females.		
London—							
Centre	748,930	199,948	25,495	130	142	38·6	
Suburbs, N.E.	643,396	144,170	16,145	116	138	29·5	
„ N.W.	697,493	154,541	19,355	112	170	79·5	
„ S.W.	299,434	62,004	7,011	108	149	57·0	
„ S.E.	393,807	88,330	10,286	126	135	34·1	
Total	2,034,130	449,045	52,797	116	150	51·1	
London—							
Centre and Suburbs	2,783,060	648,993	78,292	119	148	48·1	
Inner Belt—							
a. Chelmsford	148,696	29,262	3,558	105	102	26·1	54·8
b. Hertford	264,677	50,852	6,130	100	127	37·2	105·0
c. Windsor	114,391	22,795	2,543	107	117	43·1	85·6
d. Dorking	113,684	19,814	2,410	107	115	39·6	67·7
e. Maidstone	269,908	55,678	6,696	125	117	29·0	67·1
Total	911,356	178,401	21,337	110	117	34·0	78·6
Outer Belt—							
I. Ipswich	403,333	82,311	11,788	95	110	24·4	17·9
II. Cambridge	274,691	54,816	7,334	97	108	22·8	15·8
III. Northampton	336,024	71,829	8,449	105	115	21·1	17·1
IV. Oxford	217,021	45,684	5,689	101	112	26·6	20·5
V. Reading	234,879	45,323	5,725	110	108	31·5	33·8
VI. Southampton	418,375	84,119	11,790	135	120	31·1	40·2
VII. Brighton	289,886	55,232	7,900	99	127	49·8	58·0
VIII. Canterbury	226,237	44,152	5,222	121	115	32·3	43·8
Total	2,400,446	483,466	63,897	109	115	29·8	30·8
Total (Great Circle)	6,094,862	1,310,860	163,526	114	131	39·0	
England and Wales	18,996,916	4,210,715	508,923	112	126	29·7	

Table II.—*Shewing the like particulars for the Inner and Outer Belts of Country round London, treating certain Towns and other Districts separately.*

Districts.	Mean Population, 1851-1861.	Deaths, 1851-60. All Causes.	Deaths, 1851-60. Phthisis.	To 1,000 Inhabitants (1861). Number Aged 20-35. Males.	Number Aged 20-35. Females.	Female Servants Aged 20 and upwards.	Natives of Metropolitan Division.
Inner Belt—							
a. Chelmsford	148,696	29,262	3,558	105	102	26·1	54·8
b. Hertford	195,499	37,256	4,619	98	123	30·4	55·5
c. Windsor	93,075	18,128	2,117	108	118	42·7	80·6
d. Dorking	113,684	19,814	2,410	107	115	39·6	67·7
e. Maidstone	167,517	32,274	3,525	115	116	29·8	60·7
Total	718,471	136,734	16,229	106	115	32·4	61·8
Towns, &c.	192,885	41,667	5,108	124	124	39·7	138·5
Total	911,356	178,401	21,337	110	117	34·0	78·6
Outer Belt—							
I. Ipswich	332,775	66,593	9,597	92	106	22·4	14·4
II. Cambridge ..	247,603	49,312	6,403	96	105	20·2	13·0
III. Northampton	298,516	62,597	7,364	103	113	21·1	15·6
IV. Oxford	180,438	37,709	4,565	99	108	22·2	17·3
V. Reading	199,774	38,533	4,745	97	105	30·5	28·0
VI. Southampton	255,136	46,837	6,720	103	114	35·9	28·9
VII. Brighton	194,332	35,127	4,991	101	110	39·6	32·9
VIII. Canterbury ..	158,064	30,671	3,785	98	112	30·1	39·1
Total	1,866,638	367,379	48,170	99	109	27·1	22·2
Towns, &c.	533,808	116,087	15,727	142	133	38·1	58·3
Total	2,400,446	483,466	63,897	109	115	29·8	30·8

Table III.—*Rates of Mortality in* 1851–60 *per* 1,000 *Persons of each Sex at each Age in the several parts of the Great Circle round London.*

Ages.	Centre. Males.	Centre. Females.	Suburbs. Males.	Suburbs. Females.	London (Centre and Suburbs). Males.	London (Centre and Suburbs). Females.	Inner Belt. Males.	Inner Belt. Females.	Outer Belt. Males.	Outer Belt. Females.
0 – 5	99·66	88·11	75·14	65·64	81·39	71·36	55·76	47·33	57·92	48·96
5 –	11·56	10·63	8·93	8·55	9·58	9·07	6·82	6·81	7·06	7·23
10 –	5·01	4·65	4·32	4·04	4·51	4·20	4·09	4·64	4·23	5·50
15 –	6·65	6·18	6·01	5·19	6·20	5·45	5·76	7·19	6·04	8·27
20 –	8·98	7·40	7·97	6·11	8·28	6·44	8·66	8·18	8·95	9·77
25 –	12·31	10·18	9·63	8·13	10·42	8·67	9·89	9·24	9·50	10·26
35 –	19·97	15·09	14·40	11·76	16·02	12·65	12·86	11·35	11·08	11·68
45 –	29·91	21·21	21·86	16·46	24·23	17·74	16·39	14·00	14·74	13·63
55 –	48·87	37·84	38·45	30·87	41·42	32·67	28·35	24·59	25·24	24·26
65 –	91·57	74·83	80·23	66·35	83·07	68·38	61·70	57·47	57·92	54·95
75 and upwards	193·16	173·53	181·33	164,09	183·80	166·20	166·16	155·53	161·55	153·52

TABLE IV.—*Ratios of the above to the Rates for England and Wales, the latter being considered as* 100.

Ages.	Centre.		Suburbs.		London (Centre and Suburbs).		Inner Belt.		Outer Belt.		Great Circle.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 – 5	138	140	104	105	112	114	77	75	80	78	94	94
5 –	136	126	105	102	113	108	80	81	83	86	95	94
10 –	103	92	89	80	92	83	84	92	87	109	89	95
15 –	99	84	90	70	93	74	86	97	90	112	91	92
20 –	102	87	90	72	94	76	98	96	101	115	97	92
25 –	129	103	101	82	109	87	103	93	99	103	105	94
35 –	160	124	115	97	128	104	103	93	89	96	110	100
45 –	167	140	122	108	135	117	91	92	82	90	107	103
55 –	158	140	125	114	134	121	92	91	82	90	104	104
65 –	140	128	123	113	127	117	94	98	89	94	103	104
75 and upwards	117	112	110	106	111	107	100	100	98	99	102	102

TABLE V.—*Rates of Mortality by Phthisis in* 1851–60 *per* 1,000 *Persons of each Sex at each Age in the several parts of the Great Circle round London and in England and Wales.*

Ages.	Centre.		Suburbs.		London (Centre and Suburbs).		Inner Belt.		Outer Belt.		Great Circle.		England and Wales.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 – 5	1·88	1·75	1·28	1·27	1·43	1·39	1·18	1·03	1·38	1·29	1·37	1·30	1·33	1·28
5 –	·84	·85	·48	·61	·57	·67	·35	·51	·47	·62	·50	·62	·53	·62
10 –	·70	·94	·55	·87	·59	·89	·49	1·03	·58	1·50	·57	1·17	·76	1·29
15 –	2·24	2·36	1·94	2·02	2·03	2·11	1·74	3·00	2·04	3·99	1·99	2·97	2·40	3·52
20 –	3·78	3·16	3·55	2·70	3·62	2·82	3·76	3·99	4·24	5·18	3·88	3·81	4·05	4·29
25 –	5·07	4·21	4·27	3·46	4·51	3·66	4·01	4·19	4·40	5·03	4·40	4·20	4·03	4·58
35 –	7·08	4·87	5·33	3·78	5·83	4·07	3·96	3·75	3·79	4·34	4·79	4·13	4·00	4·18
45 –	7·52	4·02	5·31	2·94	5·96	3·23	3·25	2·75	3·24	3·09	4·45	3·11	3·83	3·12
55 –	6·06	2·85	4·55	2·17	4·98	2·34	2·80	2·02	2·58	2·23	3·55	2·25	3·33	2·38
65 – 75	3·64	1·79	3·24	1·37	3·34	1·47	1·69	1·29	1·75	1·37	2·28	1·40	2·39	1·63

TABLE VI.—*Ratios of the above to the Rates for England and Wales, the latter being considered as* 100.

Ages.	Centre.		Suburbs.		London (Centre and Suburbs).		Inner Belt.		Outer Belt.		Great Circle.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 – 5	141	137	96	99	108	109	89	80	104	101	103	102
5 –	158	137	91	98	108	108	66	82	89	100	94	100
10 –	92	73	72	67	78	69	64	80	76	116	75	91
15 –	93	67	81	57	85	60	73	85	85	113	83	84
20 –	93	74	88	63	89	66	93	93	105	121	96	89
25 –	126	92	106	76	112	80	100	91	109	110	109	92
35 –	177	117	133	90	146	97	99	90	95	104	120	99
45 –	196	129	139	94	156	104	85	88	85	99	116	100
55 –	182	120	137	91	150	98	84	85	77	94	107	95
65 –	152	110	136	84	140	90	71	79	73	84	95	86

Table IX.—*Rates of Mortality in* 1851–60 *per* 1,000 *Persons of each Sex at each Age in the Suburbs of London.*

Ages.	N.-E. Suburbs.		N.-W. Suburbs.		S.-W. Suburbs.		S.-E. Suburbs.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0–	76·04	66·96	79·66	68·60	68·47	60·85	71·47	62·29
5–	8·77	8·34	9·23	9·23	8·64	7·99	8·93	8·26
10–	4·32	3·60	4·55	4·45	4·05	4·41	4·20	3·84
15–	6·11	5·28	5·61	5·29	5·77	5·03	6·65	4·97
20–	7·87	6·27	7·45	5.68	7·14	6·12	9·42	6·77
25–	9·25	8·42	10·01	7·72	8·79	7·85	10·18	8·80
35–	13·43	11·87	15·57	11·61	13·64	11·18	14·51	12·32
45–	21·00	16·55	23·59	17·07	20·71	15·14	20·99	16·15
55–	38·52	31·35	40·71	31·54	35·22	29·41	36·89	29·98
65–	81·39	67·81	82·15	67·90	70·25	62·56	82·87	64·31
75 and upwards	188·32	167·41	181·79	164·76	175·53	155·56	177·18	165·02

Table XII.—*Ratios which the respective Rates of Mortality in Table IX. bear to the Rates for England and Wales, the latter being considered as* 100.

Ages.	N.-E. Suburbs.		N.-W. Suburbs.		S.-W. Suburbs.		S.-E. Suburbs.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0–	105	107	110	109	95	97	99	99
5–	103	99	108	110	102	95	105	98
10–	89	71	93	88	83	87	86	76
15–	91	72	84	72	86	68	99	67
20–	89	74	84	67	81	72	107	79
25–	97	85	105	78	92	79	106	89
35–	108	98	125	96	109	92	116	101
45–	117	109	131	112	115	100	117	106
55–	125	116	132	117	114	109	120	111
65–	125	116	126	116	108	107	127	110
75 and upwards	114	108	110	106	106	100	107	106

Table XIX.—*Rates of Mortality by Phthisis in* 1851–60 *per* 1,000 *Persons of each Sex at each Age in the Suburbs of London.*

Ages.	N.-E. Suburbs.		N.-W. Suburbs.		S.-W. Suburbs.		S.-E. Suburbs.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0–	1·41	1·42	1·19	1·31	·98	1·01	1·40	1·16
5–	·44	·60	·47	·68	·50	·55	·55	·55
10–	·56	·72	·60	1·02	·50	·87	·50	·87
15–	1·88	1·91	2·03	2·13	1·91	1·94	1·91	2·09
20–	3·47	2·59	3·66	2·62	3·04	2·74	3·83	3·00
25–	4·06	3·63	4·69	3·35	3·86	3·20	4·21	3·65
35–	5·05	3·77	5·93	3·85	4·75	3·43	5·15	3·96
45–	5·17	3·01	5·80	3·04	4·94	2·55	4·91	2·93
55–	4·43	2·19	4·96	2·12	3·96	2·04	4·47	2·32
65–	3·05	1·42	3·10	1·32	2·71	1·29	4·05	1·45

TABLE XXII.—*Ratios which the respective Rates of Mortality in Table XIX. bear to the Rates for England and Wales, the latter being considered as* 100.

Ages.	N.-E. Suburbs.		N.-W. Suburbs.		S.-W. Suburbs.		S.-E. Suburbs.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0 –	106	111	89	102	74	79	105	91
5 –	83	97	89	110	94	89	104	89
10 –	74	56	79	79	66	67	66	67
15 –	78	54	85	61	80	55	80	59
20 –	86	60	90	61	75	64	95	70
25 –	101	79	116	73	96	70	104	80
35 –	126	90	148	92	119	82	129	95
45 –	135	96	151	97	129	82	128	94
55 –	133	92	149	89	119	86	134	97
65 –	128	87	130	81	113	79	169	89

TABLE XXIII.—*Ratios which the Female Deathrates by all Causes at each Age bear to the Male Deathrates at the corresponding Age in the same places respectively, the latter being considered as* 100.

Districts.	Ages.									
	0–	5–	10–	15–	20–	25–	35–	45–	55–	65–
Great Circle	86	98	111	112	91	93	88	81	87	91
London, Centre	88	92	93	93	82	83	76	71	77	82
Suburbs, N.E.	88	95	83	86	80	91	88	79	81	83
„ N.W.	86	100	98	94	76	77	75	72	77	83
„ S.W.	89	92	109	87	86	89	82	73	84	89
„ S.E.......	87	92	91	75	72	86	85	77	81	78
Inner Belt—										
a. Chelmsford	82	107	124	150	106	108	94	93	89	90
b. Hertford	86	102	112	137	100	105	93	84	88	89
c. Windsor	89	93	118	137	104	95	86	77	80	104
d. Dorking........	87	96	115	135	107	105	114	101	94	95
e. Maidstone	82	100	115	131	116	111	109	94	86	101
Towns, &c.........	85	97	103	88	69	69	67	74	82	87
Outer Belt—										
I. Ipswich ...	84	103	140	155	119	129	134	106	104	92
II. Cambridge ..	82	97	124	160	128	120	109	94	89	91
III. Northampton	84	111	145	171	139	147	132	113	114	107
IV. Oxford	84	97	133	147	133	118	115	104	98	99
V. Reading	86	97	137	146	121	111	112	99	99	102
VI. Southampton	84	108	131	129	100	98	101	84	95	93
VII. Brighton	85	105	154	157	111	111	112	92	100	99
VIII. Canterbury ..	83	100	118	119	101	104	99	96	96	92
Towns, &c.	86	103	107	98	86	86	83	75	82	85

The following account of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

The PRESIDENT, in inviting discussion, observed that the paper contained facts of considerable interest and importance.

Mr. Samuel Brown remarked that there was scarcely any subject more important to the actuary than the law of population and the theories on which it is based. Mr. Welton's observations implied that the principal differences which arose in the average rates of mortality in different places were attributable to the varying degrees of sanitary condition of the people at the ages at which the heaviest mortality occurs, and the varying proportions of population at those ages; the number of persons at such ages being dependent in a great measure upon the emigration from or immigration into those districts. No doubt this movement of population was one of the most important questions which the population statistics of this country presented, and it was one on which, he was afraid, they were far from possessing thorough information, so as to be able to found a sound conclusion upon it. He believed that in Sweden and Norway a minute account was kept of changes of residence, that every one coming into a new district was bound to state the time when he arrived, the place he had left, and the ages of himself and family at the date of removing. They had, therefore, the means of going very carefully into this important enquiry. In this country, on the contrary, we were not prepared to submit to that minute inspection and registration which was necessary to elucidate the subject. All we could hope was that each successive Census —and, if it were possible to have Censuses more frequently, he believed we should be able to arrive at more strict accuracy in general and more full materials for study—would enlighten us as to the sanitary state of different classes of the population.

Mr. Walford would like to ask Mr. Welton how far he had taken into acccount two disturbing causes. One was the large immigration into London in the spring of the year, coming from all parts of the kingdom. No one could walk through the numerous squares at the west-end of London without observing that for six or seven months of the year the majority of the houses were unoccupied, whereas in the season there would be as many as 15, 20, or 25 persons in each of them. Whenever the effect of this movement of the population should be taken up and considered, he believed it would be found to cause a marked result in reference to the rate of mortality. If suggestions made by the committee appointed at the Social Science Congress and by the Statistical Society had not been disregarded, the very points they were now discussing with surmise and doubt would have been thoroughly elucidated by the next Census. Then, the second disturbing cause arose from the circumstance that a large number of tradespeople in London, as soon as they had acquired a competency, retired into the rural districts, and there ended their days.

Mr. Sprague said they had that evening had a paper read before them of a character which did not often come under their notice. They had not of late years in the Institute given much attention to this question of population, but he thought they must consider that it was a subject which very properly came within their professional studies. So far as he had been able, in listening to the paper, to follow Mr. Welton's argument, it appeared to him that he had fully proved his case. He had shown that there were certain depressions in the rate of mortality in particular places and at particular ages, accompanied by a corresponding excess at the same ages in other places. He gave a hypothetical explanation of these facts, and having apparently tested that explanation in a variety of ways, it appeared that all the facts he had produced were quite consistent with that hypothesis. It

was a matter of some surprise at first to find that the simple cause which Mr. Welton had explained should produce this vast difference in the rate of mortality shown in the tables. It was a matter of notoriety to everybody who knew anything about London that almost all domestic servants came up from the country, and if they got into a state of chronic bad health there was nothing for them but to go back into the country. That that fact should have produced the effect which Mr. Welton had pointed out seemed at first sight almost incredible, and yet there could hardly be a doubt about it. To him this paper had a special and peculiar interest. It showed how the rate of mortality in country districts at certain ages was increased, not only by the migration of unhealthy persons from London to them, but also by the migration of healthy persons from them into London. It was obvious that the migration of unhealthy persons from London into the country would have the effect of lowering the mortality in London and increasing it in the country; but it was not quite so obvious at first sight that the migration of healthy persons out of the country into London would have the same effect of raising the rate of mortality in the country. But when attention was directed to that point, it became quite clear that the migration of healthy persons out of the country would leave behind a body of persons whose vitality and whose health were not up to the average, and therefore the rate of mortality amongst these persons would be higher than if there had been no migration. This was exactly parallel to a circumstance he (Mr. Sprague) developed in a paper which he read before the Institute, showing how the rate of mortality amongst assured lives was influenced by the surrender of policies on healthy lives. They had to compare the mortality of the country districts with the assured lives in an Office, after the effect of medical selection had passed away, and then came the effect produced by the surrender or lapse of policies. There was no doubt that those persons who surrendered or lapsed their policies were on the average in much better health than those who kept them in force. That was parallel to healthy persons leaving the country and coming to reside in London, which had the effect of increasing the rate of mortality in the country districts which they left. He thought that this lapsing and surrendering of policies by healthy persons accounted for the high mortality experience in the later years of an Assurance Company; and of course, the larger the rate of lapsing the greater would be this effect on the rate of mortality.

Mr. Brown asked Mr. Welton upon what basis he had computed his rates of mortality—was it the average number of lives at risk at the two Censuses?

Mr. Welton answered in the affirmative.

Mr. Walford added to his previous observations, that the Census was taken in the month of April, when London was full.

Mr. Welton observed that this tended to make London appear healthier than it otherwise would do.

The President said that the papers they had recently had before them, and which had been full of interest and very useful to them in their professional pursuits, had been of a more strictly technical character than the present, but it was not the object of the Institute to confine itself entirely to technical enquiries. It was the earnest wish of the members to obtain information upon collateral subjects. One of their most distinguished and valuable members—their esteemed ex-president—had shown his appreciation of that assertion by his very handsome gift to the Institute which was to be applied to found prizes for questions in political economy, and other

questions which they were not in the habit of considering as strictly professional. There could be no doubt whatever that all statistics required a most severe and strict examination before they proceeded to draw conclusions from them; and he thought there were no statistics that would require more severe and strict examination than those of the population of England, which was so energetic, active and moveable, and in which the changes going on in all directions were so frequent and numerous. There were many points of observation in the question under consideration, but the only one to which he wished particularly to draw Mr. Welton's attention was as to the large mortality under ten years of age in certain districts which showed a much more favourable rate of mortality at higher ages. He would suggest whether the less mortality at the higher ages might not be due in some measure to the weeding out, as it were, of the weakly and and sickly lives at the earlier ages, leaving a more healthy and stronger population to survive.

Mr. Welton, in reply, pointed out that the remark of Mr. Brown, representing him to have argued that the relative numbers at different ages varied very much in consequence of the effect of migration, so that in the country districts the number of old lives would bear a much greater proportion to the whole population than in town districts, where they were every year receiving large numbers of new inhabitants through migrations—did not represent exactly the scope of his argument. He rather sought to show that the *quality* of lives at the different ages was affected by these migrations—so much so, that the lives which remained behind in the country districts were not only deprived of those persons who were strongest and most enterprising, but that this residuum, so to speak, received back a good many invalids who had migrated to London, besides a certain number of sick and disabled who were born and bred in London itself. In point of fact, it was a difference in quality, as well as in quantity, of lives at different ages. With regard to migrations to London in the season, it was very difficult to measure their effect, but he would venture to say that whatever the fulness of the population of London might be at the time of taking the Census, it did not explain why the rate of mortality should show such peculiarities as those he had pointed out in his paper. With reference to retired tradesmen going to reside in the country, he doubted very much whether persons who had earned their living in London during the active period of their lives, and then retired into the country, were worse than the average of the lives of those amongst whom they went to live. The President had remarked that the great infantile deathrate in London would weed out a great many of the weakest lives and diminish the deathrate at higher ages, but that would apply equally to the deathrate among persons of each sex, and would not get rid of the extraordinary contrast which his tables exhibited. If they took the London deathrate as 100 at each age, the female deathrate in certain outer districts at ages under 5 was 69 percent, for the next five years 77 percent, but became aggravated enormously at and after the age 10 to 15, because, as he said, people then began to come to town to earn their living, and invalids began to return home. So greatly was the rate at the higher ages aggravated about Ipswich and Cambridge, that the comparison was 171 in those districts to 100 in London. This seemed almost incredible, and it was because the facts were so extraordinary that he had thought them worthy of the attention of the Institute.

On the Rate of Mortality amongst the Natives compared with that of Europeans in India. By SAMUEL BROWN, F.I.A.

[Read before the Institute, 30th January, 1871.]

THE various Pension Funds which have been established in the Presidencies of India in the Military, Medical, and Civil Services of the late Honourable East India Company have afforded the means of bringing together a great variety of interesting facts relating to the rates of mortality and marriage amongst Europeans in India. They were originally founded as mutual charitable schemes, giving assistance or pensions to the widows or children of such Members of the services as had not left sufficient pecuniary means for the support of their families or for putting them forward in life. The Company saw fit to encourage these laudable designs by allowing high rates of interest on their invested Funds, and making favourable arrangements in the exchanges for payments to be made in England.

But as these Funds were not based on scientific principles it was generally found that the subscriptions did not accumulate sufficiently fast to meet, with the proper reserves for prospective claims, the benefits for the families of the older Members which fell in first, and which rapidly increased. The Trustees were consequently soon obliged to have recourse to the skill of the actuary to ascertain the real position of the Funds and to take advice as to the best means of placing them on a proper footing to carry out their original objects.

Many of the points involved were of a very novel character, and several of the Funds were fortunate in having applied to the late Mr. Griffith Davies, whose conscientious discharge of duty led him to take great pains to collect original facts from the actual experience of the Funds, so as to throw light on the difficult and complicated questions frequently submitted to him.

Mr. Neison also added considerably to the amount of new materials for Valuation Tables of all kinds, and he shows very distinctly the great improvement in the rate of mortality which appears in each Presidency, both in the Military and Medical Services, since the beginning of the present century.

In the paper which I had the honour to read before this Institute on 29th December, 1862, and which was printed in the vol. xi. of our *Journal,* I gave a brief summary of the various Reports and papers which had appeared up to that time relating to mortality in India, and also a short *exposé* of the mode of

collecting the experience of the Madras Military Fund for 50 years, from 1808 to 1858, which had been printed in the recent Report on that Fund.

At the meeting of the British Association, at Bath, September, 1864, I contributed a further paper "On the Rates of Mortality and Marriage amongst Europeans in India." In this was pointed out more especially the mortality amongst the Bengal, Madras and Bombay Civil Services, deduced from the records of those respective Funds. The rates of Marriage amongst Bachelors and Widowers, and whether Military or Civil Servants, or on Active Service or Retired, were contrasted, and the observations in the Bengal Civil Fund were brought down to 1863.

Since that paper appeared, I have been called upon to make the following Reports on the Military or Civil Funds of India, in most of which a few new collections of original facts have been procured, or the former experience has been confirmed by further observations of a similar kind:—

1862,	Sept. 10.	Report on the Madras Military Fund.
1863,	Jan. 31.	Second Report on do. (on questions specially submitted to the actuary).
1865,	Feb. 27.	Report on the Bengal Civil Fund.
,,	May 6.	,, on the Bengal Medical Retiring Fund.
,,	Dec. 7.	,, on the Madras Civil Fund.
1866,	Aug. 5.	,, on the Alterations of Tables used for the Valuations of the Bombay Civil Fund.
,,	Oct. 26.	Second Report on the Bengal Civil Fund.
1868,	Feb. 29.	Report on the Madras Medical Fund.
,,	Sept. 24.	,, on the Bombay Medical Retiring Fund.
1869,	Mar. 16.	,, on the Bengal Military Fund.
1870,	June.	,, on the Bombay Military Fund.
,,	Aug. 29.	Second Report on the Madras Civil Fund.

In the First and Second of these Reports I had the pleasure of being associated with my friend Mr. Peter Hardy, so esteemed and honoured in this Institute, and by every Member of our profession. Notwithstanding his severe illness he took much interest in it, and it is with grief I remember that almost his last signature was given to this Report. Colonel J. T. Smith, also a distinguished Member of your Council, gave the experience he had gained in the working of the Fund and his general scientific skill to clear up some of the difficult and novel questions we had to examine.

Although the other Reports have not been published, I have been favoured by the kindness of the Trustees or Managers with a few copies, and one of each will be found in the library of the Institute for the use of any Member who may desire to avail himself of the original data they contain. I believe several of the Tables will be useful in inquiries relating to the statistics of marriages and families of a more general character than those for which they were specially formed.

As all these Military or Army Medical Funds have been or are in course of being transferred to the Government of India, and it is not intended to continue them when the obligations to the existing Members are fulfilled, and as it is not proposed by the Government to have any further Valuations, it seems to me desirable to put together the principal Tables of Observation so as to afford, as far as possible, the means of forming Mortality Tables for the different classes of Europeans in the whole of India as authentic and as complete as we are likely to get them for many years to come. It will take a considerable time to organise an efficient census and to keep proper registers of the arrivals and the births, marriages and deaths of Europeans in that vast country in a fit state to be of any public service.

I. *Mortality of Military Officers in India.*

Under this head I have combined the original observations for the three Presidencies; for Bengal, from 1800 to 1847, given by Mr. Neison in his Report on the Bengal Military Fund in 1849; for Madras, from 1808 to 1857, to be found with the most recent corrections in my paper printed in the eleventh volume of our *Journal*; and for Bombay, from 1816 to 1865, in my Report on the Bombay Military Fund, June, 1870.

The following is a summary of the total numbers in these tables:—

	Entered.	Lived.	Died.	Withdrew.	Living at close of Observations.
Bengal........	5,199	91,267	2,019	75	3,105
Madras	5,104	77,239	2,400	473	2,231
Bombay	3,589	57,886	1,501	431	1,446
			Bachelors	Retired	211
	13,892	226,392	5,920	979	6,993

Although the observations close at different dates, they comprise in each case about 50 years' experience in each Presidency, and being based on 13,892 entrants and 5,920 deaths are sufficient, either separately or combined, to form trustworthy Tables of Mortality.

In the original Tables the facts are given at every age of the Members, whether on Active Service or Retired, and in the two latter Presidencies for Married and Unmarried separately. In the Bengal and Madras Funds they are also distinguished into periods of years, so that the progress of improvement in the rate of mortality since the beginning of the present century may be minutely traced. Much controversy has arisen on this subject, Mr. Neison's views being to a certain extent opposed by those of Messrs. Finlaison and Lewis; but, I am gratified to say, that all the observations I have collected, whether relating to the Military or Civil Services, indicate that the improvement is still going on.

Many circumstances are favourable to this result. The much more frequent visits to Europe which the rapidity and facility of steam communications render possible, the selection of more healthy stations for the troops generally, the establishment of sanatoria in the hill stations, where even a brief sojourn restores health and strength to the exhausted constitution, and to which places railways now afford rapid and convenient access, and above all, the great change in the social habits and customs of European society in India, have effected a marked and permanent diminution in the high rate of mortality, which was formerly thought a necessary consequence of residence in tropical climates.

In Table A I have contented myself with giving the Summary of the three combined Tables in quinquennial divisions of age, referring those, who desire to carry out more minute inquiries, to the original data in each Report.

The Army Medical Fund Reports, which principally relate to the provision for Annuities on Retirement, also afford a distinct Table for Madras from the experience of the years 1800 to 1866. The number exposed to risk was 14,334 and the number of deaths 478. At the younger ages, under 45, the mortality is about 25 per cent. in excess of that of the combined table for military officers; but there is a marked diminution in the rate, if we compare only the lives exposed to risk from 1825 to 1866.

The Medical Services are of course exposed to much the same class of climate risks as officers in the army, and against the risk of being killed in warfare may be set the peculiar dangers attaching to their professional duties.

We have also the means in the other Reports of tracing the mortality amongst the Members of the Government Civil Services in India. From the positions which they occupy, the considerable incomes they enjoy, furloughs in case of illness, and other social advantages, they may be compared with the best classes in this country, in order to see the effect of climate on health. I have therefore selected the observations on the British Peerage, as given by Messrs. Bailey and Day and Farr's Healthy Life Table as a graduated Table, in order to exhibit the excess of mortality amongst Officers and Civilians in India over what we may consider to be the nearly corresponding classes in this country.

It must be noticed that these Tables include the Retired Members, and as the age of 50 to 55 may be roughly taken as about the average ages of retirement, the greater part of the observations after those ages may be considered as relating to residents in this country, who have survived the effects of exposure to service in India.

Comparing each quinquennial group of ages, it will be seen that under 50 years of age the mortality amongst Military Officers in all of India is very nearly a constant of about 2·8 per cent. on the number exposed to risk at those ages, and that it exceeds that of the healthy classes in England by about $1\frac{3}{4}$ to 2 per cent. on the number exposed to risk; from 50 to 70, the excess is more nearly 1 per cent.; after that age it fluctuates, but on the whole is below the Peerage rate, but somewhat higher than Farr's Healthy Life Table.

Both these classes of Tables indicate clearly the much smaller mortality found to prevail at the same ages amongst married as compared with unmarried members. In the Bombay Military Fund at ages 20 to 30, the former was less than 60 per cent. of the latter ages; 30 to 35, about 80 per cent., and at 35 to 40, about two-thirds only of the Unmarried rate.

A similar remark may be made on the rates in the Madras Military Fund. There the Married rates under 30 are only about 60 per cent. of the Unmarried rates. At all ages, except at 65–70 when there appears a singularly low rate amongst the Unmarried, the Married rates are only 60 to 80 per cent. of the rates amongst the Unmarried.

The combined Table from the Observations in the Madras and Bombay Military Funds fully bears out the above remarks.

Nor is this a feature peculiar to the Indian Tables. The Registrar-General of Scotland has published a Table in which

503,000 lives of Married Men were observed and compared with 243,000 Bachelors, and at all ages of life after 20, a marked difference in the mortality was shown in favour of Married life.

It is not, however, shown to the same extent in the records of the Civil Funds, and it would be very desirable to obtain some more precise data for an investigation of this character, which very much affects the condition of similar Funds, since the most important and permanent benefits are those granted to the families of the Married Members and their contributions during marriage form the principal sources of revenue.

As the Unmarried Members pay an increased contribution on marriage, the dates of marriage are generally recorded, and I have given a column in Table A(2), showing the numbers who married in quinquennial groups of ages, comparing the percentages with the British Peerage by Mr. Day; it will be noticed that the percentage amongst Indian Military Officers is less under age 35, but after that age considerably higher than in the British Peerage.

II. *Mortality amongst European Females in India.*

For the purpose of comparing the rates of mortality, European Female Lives in India may be divided into three classes—Wives, Widows and the Daughters of Members of the different Funds, as the circumstances affecting them differ considerably.

The wives generally comprise those who, at the earlier periods of adult Life, are exposed for the first time to the influence of tropical climates. The wives of Military and Medical Officers, except during temporary absences in Europe, reside at the same stations, and may be supposed subject to the same risks of climate as their husbands. As the differences between Military or Civil Service would not affect them, we may bring together such facts as can be collected from either class of the Funds.

Unfortunately the facts are not very numerous. In early days the ages were seldom registered unless they became Widows, although their names were on the books as long as the joint life subscriptions were paid. A large quantity of materials for observations of mortality was thus rendered useless.

Mr. Griffith Davies, in his Report on the Madras Military Fund to the end of 1836, collected the experience of the Fund as to the rates of death or marriage amongst the Widows and Daughters. These observations, though very few in number and

defective as to dates, there being as regards Widows only 11 deaths and 24 remarriages recorded, out of 41 deaths and 69 remarriages, and as regards the Daughters only 90 deaths or marriages, constituted for some time the only bases of the Tables on which this part of the Valuations was founded.

Mr. Neison in his Report on the Bengal Military Fund, August, 1849, states that he had been furnished with registers; but so defective, that out of 907 wives, whose ages were given, the deaths of 14 only were stated; whilst of the remainder, whose ages were not furnished, 141 deaths were recorded. It was possible by analyzing the list of Widows, whose ages were generally given, to obtain sufficiently accurate records for the ages of three-fourths of the wives, but considering the immense labour involved, he was satisfied to fall back on Mr. G. Davies' Table.

In the documents furnished to me by the Madras Military Fund down to 1858, there seemed reason to hope that much more accurate data would be obtained for this important inquiry, and some pains were taken by various methods to fill up the deficient dates of birth. In some instances the age at marriage could be obtained by near approximations, either from the average difference of age between husband and wife, according to the rank of the husband or the date of birth of the first child. Since 1840 the registers have been tolerably accurate.

The total numbers were

Entries under ages	1,737	Died	300
Entered on the Widow's List, but age at or date of marriage not given	207	Became Widows	654
Deaths recorded, but names not found in the entries	3	Withdrew	21
Widows, whose ages were not given	101	Living to 1858	1,073
	2,048		2,048

These are worthy of attention as comprising a much larger number of facts than had previously been observed.

But inasmuch as the conditions of climate would not be very different for the Wives of Civilians or for the Wives of Military Officers, we may add to the above the observations in the Bengal Civil Fund from 1850 to 1863, given in my Report on that Fund in 1865. They were 517 in number, of whom 54 died (3 being

killed in the Mutiny), 14 withdrew, 71 became Widows, and 378 were living at the close of the Observations.

The total numbers then exposed to risk were 26409·5, and the total deaths 354. The number at each quinquennial group of ages and the force of mortality per cent. will be found in Table B(1). It will be observed that the mortality at the younger ages is high, and gradually diminishes till 30. The rates at the middle periods of life are disturbed by the effect of the retirement of their Husbands, and after that the number diminishes as the number of Widows increases.

I am disposed to think therefore that the facts under the head of Wives and Widows may be combined, as I have shown in the Appendix, to form one Table. The younger Widows whose Husbands die in India probably remain, and most of them remarry there, whilst after the middle periods of life the Wives would be returning to Europe on the retirement of their Husbands, and the Widows would then more frequently return than remain in India after the death of the Husband. The combined Table may thus be considered as giving a fair view of the mortality amongst European Females in India and after their return to this country. But it is necessary to remark that after 40 years of age the mortality amongst Wives appears so light that I fear some of the deaths must have escaped registration, and this part of the Table must be dealt with cautiously, subject to revision whenever further facts can be procured.

As to the Statistics of Widows the registers in all the Funds have been much more carefully filled up. The age at entry on Widowhood is generally stated, and the date of death or remarriage accurately known by the cessation or diminution of the Annuity. I have therefore been enabled to bring together five collections of facts, which united form a reliable Table for the computation of Pensions or other Benefits to the Widows of Indian Officers or Civilians, whether continued till death or to cease on remarriage. The rate of remarriage from 15 to 35 diminishes with great regularity by one third in each quinquennial period, and then by one half till age 45, and by one quarter from age 45 to 60.

The following are the total numbers brought under observation :—

	Entered.	Died.	Re-married Widows.	With-drew.	Living at close of Obser-vations.	Exposed to Risk.
Madras Military Fund, 1808 to 1857 (S. Brown)	680	140	145	..	395	8273·5
Bengal Military Fund, 1800 to 1846 (Neison). .	635	92	119	5	419	6094·5
Bombay Military Fund, 1833 to 1849 (G. Davies)	..	27	49	..	..	2045·
Madras Medical Fund, 1807 to 1855 (Neison). .	111	24	14	1	72	1408·
Bengal Civil Fund, 1850 to 1862 (S. Brown) . .	71	18	8	3	42	1170·
	..	301	335	..	..	18991·

Mr. Davies does not give in his Table the number of entrants and survivors at each age, but as the numbers exposed to risk are compared with the deaths and remarriages, it was thought right to include these additional facts as, though few in number, they are authentic and accurate.

I have given in the appendix the groups for each quinquennial period of age, but for the purpose of comparison, and also for the better graduation of the Tables for practical use, the summary in decennial periods is more convenient, and in the following Abstract the total facts and percentages of mortality may be compared with each other and with Farr's Healthy English Life Table.

ABSTRACT I.—*Showing the Total Numbers exposed to Risk and Number of Deaths and Mortality per Cent., in Decennial Periods of Age, among European Military Officers, Civilians, and their Wives and Widows, in the three Presidencies of India combined.*

Ages.	A(1). MILITARY OFFICERS.		A(2). CIVILIANS.		B. WIVES AND WIDOWS.		FORCE OF MORTALITY PER CENT.				
	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Military Officers.	Civilians.	Farr's Healthy Life, Male.	Wives and Widows.	Farr's Healthy Life, Female.
15–25	67921·5	1,613	9092·5	155	6316·5	106	2·38	1·70	·69	1·68	·77
25–35	75915·	2,137	13489·5	226	15383·	197	2·81	1·68	·82	1·28	·89
35–45	42444·5	1,210	8849·5	184	12473·5	154	2·85	2·08	·93	1·23	1·00
45–55	20027·	572	5471·5	117	6989·5	89	2·86	2·14	1·27	1·27	1·19
55–65	7702·	243	3064·5	95	3156·5	61	3·15	3·10	2·59	1·93	2·16
65–75	1711·5	107	1355·5	70	940·	34	6·25	5·16	5·49	3·62	4·99
75–85	264·	35	321·5	49	139·5	12	13·26	15·24	12·82	8·60	11·87
85–95	11·5	3	19·5	4	2·	2	26·09	20·51	28·35	100·	
	215997·	5,920	41664·	900	45400·5	655	2·74	2·16	..	1·44	

In regard to Military Officers, allowing for a rather reduced rate of mortality from ages 15 to 25, the rate may be considered

almost a constant of 2·85 per cent. till age 55, which embraces nearly the whole period of active service in India. From ages 55 to 95 the rate of mortality as nearly as possible doubles every 10 years. As compared with Farr's Healthy Life Table it shows during the period of service an excess of from 1·6 to 2 per cent. on the number exposed to Risk.

The rate of mortality amongst Civilians indicates a constant rate of about 1·7 per cent. up to 35 years of age, then increases slowly till 55, and is then alternately higher and lower in each ten years than the Healthy Life Table. Under age 55 it exceeds the latter by about ·8 to 1 per cent. on the number exposed to Risk.

The effect of combining the mortality of the Wives and Widows into one Table is to show a higher rate in the early period of age 15 to 25; on first arrival in India, 1·68 per cent.; and then nearly a constant of about 1·25 per cent. from 25 to 55. After that the rate is lower than Farr's Healthy Life Table (Female) to the end of life. This may be accounted for by the fact that they are generally Annuitants in comfortable circumstances, and, as the survivors of those who have undergone the risks of tropical climates, may be supposed to enjoy the best constitutions.

The question of the rate of mortality amongst children born of European parents in India is a very interesting one. It might be reasonably supposed that a climate so trying to parents would be still more dangerous to the delicate constitutions of young children, but this conclusion is not borne out by the facts. It is evident from the numbers that in the year after birth many names of children have, especially in the early periods of the Funds, escaped registration either as births or deaths. But making allowance for this, the mortality is generally less up to age 15 than at the corresponding ages in Europe.

No doubt this may be partly attributed to the custom of sending children for education to England, especially if they give indications of failing health, and change of climate is recommended. The greater facilities of reaching the Hill Stations in India may lead for the future to more remaining in India through childhood; but it is not likely to have unfavourable results, since the climate in the Hills is equal to the most favoured localities of Europe. It is gratifying therefore to notice in Table C and D of the Appendix the very light rate of mortality shown to prevail amongst the children of European parents in India.

The mortality amongst the Daughters may be compared with that of the Sons up to 20 years of age, but after that age the observations on the former are continued until marriage or death.

The following are the Tables which have been combined in Table D into one for the present purpose:—

	Entered.	Died.	With-drew.	Married.	Became Annui-tants.	Living to date of Obser-vations.	Exposed to Risk.
Madras Military Fund, 1808 to 1857 (S. Brown)	2,884	445	29	194	774	1,442	27035·
Do. Annuitants, 1808 to 1857 (S. Brown)	774	54	30	202	..	488	8453·
Bengal Military Fund, 1808 to 1850 (G. Davies)	..	371	..	..	..	..	10634·
Madras Medical Fund, 1824 to 1855 (Neison)	426	71	33	41	..	281	4834·5
Bengal Civil Fund, 1850 to 1862 (S. Brown)	955	90	22	103	..	740	7711·5
	..	1,031	..	..	..	..	58668·

On examining the continuation of these Tables after 20 years of age it will be noticed that the deaths recorded in the middle periods of life are so few that there is reason to fear the returns are defective in this respect, although the rates of marriage proceed with great regularity, and are consistent with other observations. It would be advisable therefore for the present, in any practical application at the adult ages, to use the Mortality Table deduced from the combined experience of the Wives and Widows.

The mortality of Sons extends in most of the Funds only to age 21, when the allowances cease, and many withdraw at 14, or between that age and 21, or take a Bonus on resigning their claims to Pension. In the Bengal Military Orphan Society the observations which were furnished in a register to Mr. Griffith Davies in 1852, and in consequence of his severe illness, entrusted to Mr. Peter Hardy to investigate, are continued in a few cases to age 32, but as only 4 of the deaths out of 396 occurred after 21, I have thought it better to close the Lists at 20 as in the other Tables.

The records of the year of birth are evidently defective. Judging from the comparisons in this country and in graduating a Table therefrom some correction should be made, unless more careful returns should confirm the highly favourable rate for the first year of life in India. But at all other ages the returns appear trustworthy and the rates conform very much to the standard in Europe and show no indications of a tropical climate being injurious to children under the conditions which prevail of a partial residence in Europe, or in the more temperate hill climates of India.

General Hannyngton says, that the returns from the Bengal Army were obtained in reply to circulars addressed to every officer, particularly requesting that the names of none of the children born should be omitted whether subscribed for or not, and the answers are considered to be accurately and carefully filled in.

The observations from which the combined Totals are deduced are the same as those for Daughters, the total facts being as follows:—

	Entered.	Died.	Withdrew at 21 or before.	Annuitants.	Living at close of Observations.	Exposed to Risk till Age 20.
Madras Military Fund, 1808 to 1857	2,993	453	446	749	1,345	26770·5
„ Annuitants, „ „	749	45	455	..	249	5829·
Bengal Military Fund, 1808 to 1850	..	393	..	..	..	10887·
Madras Medical Fund, 1824 to 1855	480	66	143	..	271	5323·5
Bengal Civil Fund, 1850 to 1862 ..	1,014	104	231	..	679	7588·5
	..	1,061	..	..	..	56398·5

The following Abstract contains the numbers exposed to risk and deaths and the mortality per cent. amongst Male and Female Children in each of the first five years of life and in each quinquennial group of ages under 20 years of age:—

Abstract II.—*Showing the Number exposed to Risk and Number of Deaths and Mortality per Cent. in each Year of Age under 5 Years, and in Quinquennial Groups up to 20 Years of Age, amongst Male and Female Children born of European Parents in India.*

Ages.	C. Sons.		D. Daughters.		Force of Mortality per Cent.					
							Neison.		British Peerage.	
	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Sons.	Daughters	England and Wales.	Family Experience.	Male.	Female.
0–1	3552·	306	3404·	261	8·62	7·67	14·63	10·40	7·82	5·95
1–2	4781·	346	4526·	306	7·24	6·76	6·17	5·92	1·63	1·64
2–3	4292·	136	4071·	150	3·17	3·68	3·38	3·10	·87	·71
3–4	4011·	64	3740·5	70	1·60	1·87	2·39	2·12	·39	·56
4–5	3778·5	30	3495·	40	·79	1·14	1·77	1·53	·44	·56
0– 5	20414·5	882	19236·5	827	4·32	4·30	3·38	..	2·23	1·88
5–10	16024·5	82	14926·5	87	·51	·58	·94	..	·35	·44
10–15	12107·	39	11112·5	39	·32	·35	·66	..	·41	·66
15–20	7852·5	45	7435·5	41	·57	·55	·75	..	·66	·77
Total	56398·5	1,048	52711·	994	1·86	1·89				

As Mr. Neison took some pains to collect the statistics of the mortality of children, especially amongst families of the middling class in which accurate returns of 10,000 cases were sent to him in schedules prepared for the purpose, I have compared the present results with his.

I have also added a comparison with the mortality of children deduced by Messrs. Bailey and Day from the British Peerage, in which the mortality which prevails in the first five years of life is so low as to form quite a remarkable feature in the Tables.

In the first year of life the rate amongst both sexes in India is so low when compared with the second as to warrant the suspicion that many deaths must have escaped registration. The rate in the first year of life is generally, at least, double that of the second; but in the above returns it is amongst Males only 8·62 per cent. in the first year and 7·24 per cent. in the second, against 14·63 per cent. and 6·17 per cent. in England and Wales.

In the first two years the rate amongst Females is somewhat less than amongst Males; but in the three following years higher, although in the first quinquennial period they nearly agree on the average, being 4·32 and 4·30 per cent. against Neison's 3·38 per cent. The Registrar-General, however, makes the rate, Males 7·24 per cent. and Females 6·25 per cent. for the 30 years, 1838 to 1867.

In the three following quinquennial periods the rates are also very light amongst the children of European parents in India, not exceeding two-thirds of the rates in this country, and as there seems no reason to anticipate any defect in registration, except in the first year of life, we may conclude that either the climate or the treatment of the children is not unfavourable to their health or against their attaining an adult age. At these ages a considerable number of the children are sent to Europe, and though any deaths that take place there ought to be recorded on the books of the Fund, and the parents may still be in India, we cannot at these ages sufficiently distinguish the proportion exposed to the effects of a tropical climate. The inquiry, therefore, must be considered as limited by the conditions, that the children are of a class in which, if they survive the first few years of childhood, the parents generally have the means of removing them from the influence of a climate which may prove afterwards detrimental to health.

Mortality amongst the Natives of India.

We have now to examine a collection of facts relating to the mortality of the Natives of India by a very skilful and competent

authority, the late Dr. Haines of the Bombay Medical Service. The discussions connected with the Military, Medical and Civil Pension Funds, to which I have before alluded, brought forward at different times some of the Members, who, with sufficient mathematical knowledge and general ability, explained their views as to the financial condition of the Funds and commented on the valuations and their results. General Hannyngton, Colonel De Haviland, Colonel J. T. Smith, Colonel Oakes, and amongst other Medical Officers, Dr. Haines, contributed greatly towards obtaining original data and their proper application.

Extending his inquiries on the important question of native mortality in India, Dr. Haines gave great labour to the collection of original materials on the subject. The following Memorandum of Dr. Leith will explain the nature and object of his researches, and the Report of the Members of the Government and the letter addressed to Mrs. Haines by the Chief Secretary to the Government of Bombay will show the high appreciation which the Government entertained of his zealous and efficient services:—

MEMORANDUM.

Among the effects of the late Surgeon Robert Haines, M.B., there was a packet of papers comprising statistical compilations and tables, on which he had bestowed much labour and which he had nearly finished. During his last illness he advised that the work should be put into the hands of an Actuary for completion.

With regard to these papers it is ascertained that in the year 1859 the Government of Bombay had under consideration a scheme that had been proposed, to allow the commutation of *Enam* held for one or more lives into permanent alienations on purchase by the holders; but it was found that there were no means then available of knowing whether the financial effect of the scheme, if carried out, would be unfavourable or otherwise to the State.

In the year 1863, the matter was again under consideration, and still the Deputy-Auditor and Accountant-General, on being asked for a formula for calculating the values of annuities for one or more lives, had to reply, that he knew of no statistics showing the average duration of life of natives of India, and without information on this point, or an assumed average, it would be impracticable to frame the required table.

Mr. Haines, being considered a most competent actuary, was requested by Government to give valuations of the holdings, according to the best information at his command. As there were no Indian statistics on which to base the calculations, Mr. Haines, in conformity with the desire of Government, on the 9th January, 1864, furnished tables of the approximate valuations which he had founded on the Northampton table of mortality, which he assumed, by its higher ratios, to correspond better with the duration of life in this country than the more recent European tables.

Mr. Haines in submitting that Report on the valuation of successions,

stated that then he had amassed from the records of the Civil Paymaster, the Controller of Military Finance, and the several native regiments quartered in Bombay and its neighbourhood, a tolerably ample and trustworthy amount of information, on which to frame tables of the mortality at different ages, among the native community. Mr. Haines, however, continued the work of compilation, in order that the results might be the more reliable, and in furtherance of his object, in 1865 he visited, at his own expense, the military stations of Dhoolia, Maligaum, Sholapoor and Poona, that he might make extracts from the records of the regiments at those places.

Mr. Haines was well known to be scrupulously exact in all the work that he undertook, and from his established character in this respect, these tables which he has left may, with confidence, be considered as based on accurate data. The compilation of the register of pensioned native soldiers, including upwards of twenty-two thousand natives, and which forms part of the packet of papers, testifies to the care he took to ensure accuracy.

It cannot be doubted that native mortality and annuity tables would be of great use.

Cases in which it is desirable to know the value of a single or of joint lives are ever liable to occur, and as the Controller of Military Finance has stated, such tables would be very necessary in calculating the prospective burden to the State of the native pension establishment; other such questions are likely to require solution.

The tables would be most useful to the community generally, and with respect to this, the words of Mr. Haines may be quoted. They are, "Their applications are too numerous and varied to be summed up in a few words; but I may especially mention, that the extension of the system of Life Assurance to the native community, and the provident habits and relief from indigence which it promotes, is at present entirely arrested by want of such tables, for although applications have been frequently made by natives, I am told that the Insurance Offices feel themselves to be so completely in the dark as to the duration of life in this country, that they will not accept insurances on any terms."

Shortly before his death Mr. Haines completed the mortality tables, which formed by far the more laborious part of the task he had imposed on himself. The calculation of the annuity tables for which these furnish the basis, has yet to be made.

It is very desirable that the copyright of these tables should be secured for Government, and with a view to this, the sum of £500 should be offered for the papers to the trustees of the estate.

The documents in the packet consist of

(1.) A long register of Native Military Male Pensioners and a short register of European Pensioners.

(2.) A manuscript book, in which is shown the work of reducing to tables the information found in the registers of Native Regiments, of the Military and Civil Pension Establishment, and of Warden's Official Fund.

(3.) Another manuscript book which shows the work of reducing to tables the entries of Female Pensioners.

In this volume there are the completed mortality tables based on entries amounting to 53,564 natives and 1,213 Europeans.

The following is a list of the tables in this volume :—

Mortality among Natives of India.

Males.

1 Military Pensioners based on 22,463 entries.
12 Twelve separate Regiments or Battalions.
1 These twelve Battalions combined, the entries being 27,778.
1 Civil Pensioners on Warden's Fund.
1 Civil superannuated and invalided Pensioners.
1 The last two classes combined, amounting to 849.

Females.

1 Pensioned Widows and Daughters on Warden's Fund.
1 Pensioned Widows, Mothers and Daughters of Military Servants.
1 The above two classes combined, amounting to 2,494.

Mortality amongst Europeans.

1 Military Male Pensioners based on 648 entries.
1 Widows and Daughters of Soldiers, 565 entries.

Poona, 6th July, 1866. (Signed) A. H. Leith, M.D.

No. 66 of 1866. Political Department.

To Her Majesty's Principal Secretary of State for India, in Council, London.

My Lord,—In the accompanying collection we have the honour to forward a copy of our proceedings regarding a series of Mortality or Life Tables, applicable to the Native Army and other classes of the natives of India, which the late Surgeon R. Haines had completed shortly before his death. The collection also contains copy of Memorandum drawn up by Dr. Leith, explanatory of the laborious and useful nature of the task accomplished by Dr. Haines in collecting materials as the basis for these tables and in reducing those materials to order.

2. Dr. Leith has alluded to the advantages which may be expected to result from the preparation of Annuity Tables to correspond with the Life Tables prepared by Dr. Haines, and to the special value which attaches to the statistics and compilation now forwarded, as the work of an officer distinguished in an eminent degree, both for high scientific attainments and for habits of the most careful and accurate observation.

3. In order to preserve the materials collected by Dr. Haines, we were anxious that the copyright of the Tables and of the papers relating to them should, in the first instance, be secured to Government by purchase, and for that purpose we resolved that a sum of Rs. 5,000 should be offered for the papers to the Trustees of the Estate. It has been ascertained, however, that Dr. Haines' representatives in Bombay are not in a position to accept that offer.

We beg therefore that your Lordship will be so good as to cause a communication to be addressed to Mrs. Haines direct and the requisite arrangements to be made with her in England. Mrs. Haines' address is No. 4, Westcombe Park Road, Blackheath.

4. Should the copyright of the papers be thus secured, it is very desirable that they should at once be entrusted to the most competent actuary, whose services are available, in order that he may, without loss of time, complete the calculation of the Annuity Tables, and carefully revise and edit the entire work.

5. We beg to recommend that the completed Tables of both series should be published in an appropriate and handsome form, and that a sufficient number of copies should be furnished to each of the Indian Governments for distribution to the principal Public Offices.

We have, &c.,

H. B. E. Ferere.
R. Napier.
B. H. Ellis.
C. J. Erskine.

Bombay Castle, 28th September, 1866.

No. 405 of 1867. General Department.

Bombay Castle, *25th Feb.*, 1867.

To Mrs. R. Haines.

Madam,—I am desired to inform you that in recognition of the gratuitous services rendered to Government by the late Dr. Haines in the preparation of tables of mortality, applicable to the native army and other classes of the natives of India, His Excellency the Governor in Council has been pleased to direct that a sum of (5,000) five thousand rupees be paid on your account to T. B. Ferguson, Esquire, Barrister-at-Law in Bombay, on the understanding that the tables remain in the possession of Government, until their publication can be undertaken either by Dr. Haines' representatives or some competent and authorised person.

2nd. Though Government are unable to undertake the duty of publishing the results of Dr. Haines' labours in the form of Annuity Tables, the Right Honourable the Secretary of State has signified his willingness to subscribe for copies of such a work, if it be completed and published by the friends of your late husband.

3rd. I am desired to take this opportunity of assuring you how warmly the eminent services of Dr. Haines, in this, as in so many other ways, are appreciated by the Government, and how sincerely they lament the loss of so valuable an officer.

Yours obediently,

F. C. Chapman,
Chief Secretary to Government.

The summaries of these Tables, which I have given in Appendix E in quinquennial periods of age, are quite sufficient for the formation of graduated Tables therefrom. The headings of the Tables, 1 to 7, will suffice to show the classes of Lives observed, although it would be desirable to have a more precise account of the conditions under which they lived and the time during which they remained on the Registers.

The largest and most important Table, No. 4, contains no less than 27,778 entries, 188,072 exposed to risk and 3,128 deaths. They relate to 10 Regiments of Native Infantry and the Golundauze Artillery Battalion and the composition of each Regiment, with the totals "entered," "gone off," and "exposed to risk," are described in E.

Dr. Haines in all the Tables appears to have entered the numbers carefully at the age last birthday, carried on those remaining under observation from the previous year of age, the total under observation at each age, struck off or still living, and obtained the number "exposed to risk" by deducting from the total under observation at each age half of all who had entered or gone off from any cause, except death.

They are then grouped into quinquennial periods of age 0–5, 5–10, &c., and the rate of mortality computed on the number exposed to risk.

Table 4, just referred to, shows a steadily increasing rate of mortality from age 10 to 55, the greatest rise being from 15–20 to 20–25. Compared with that of European Military Officers in India, it shows about 1 per cent. less on the number exposed to risk from 15 to 60, and under 45 is just about 80 per cent. in excess of the rate at the same decennial ages of Farr's Healthy Life Table.

At the older ages, above 55, the observations nearly cease, and they must be continued by the Table of Pensioners, No. 3, which shows a great excess over European Officers who have served in India.

Dr. W. C. Coles of the Bombay Medical Service, who originally kindly brought these papers to my notice, and to whom I wrote for information as to the probable risks of Military Service to which these regiments might have been exposed, says, "The Tables were computed from the regimental records of Native Soldiers and Pensioners, and my impression is, that care was taken to select such Native Regiments as had not seen much, if any, service, in order to guard against including deaths occurring in action. I do not think, therefore, that the mortality included those killed in war. The only serious wars, in which Native Soldiers of the Bombay Army were engaged, were the Scinde Wars in 1844, those in the Southern Mahratta Country in 1846, the Persian War of 1857, the Mutinies of the same year, and the subsequent suppression of the Rebels."

Table No. 3 may be considered supplementary to the above, comprising Native Military Pensioners, chiefly Sepoys, with a few orphan sons of Sepoys, and a few clerks. In quinquennial groups the rates of mortality fluctuate much, but in decennial groups, from ages 15 to 45, they show a steady excess of about 55 in 10,000 living over those in active service. After that, up till age 75, the rate of mortality is greatly in excess of that of Retired Indian Military Officers.

Tables Nos. 1 and 2 combined also comprise Male Lives of Civil Pensioners, Natives. The mortality is high at ages 35–55 probably from many being then superannuated on medical certificate.

Tables Nos. 6 and 7 relate to the Lives of Female Pensioners, being mostly the Widows, Mothers and Daughters of Sepoys or of Civilians, Clerks in the Government Service. The rates of mortality in the decennial groups increase regularly from age 15 upwards, and present a strong contrast when compared with those amongst the Widows and Daughters of Soldiers constituting a class of Female European Pensioners in India. The number of Lives exposed to risk in Nos. 6 and 7 was 24,010, and the deaths 485, and in No. 5 only 3,620, with 78 deaths. In the latter, between ages 25 and 55, the rate of mortality was a constant of about 2·85 per cent., being almost the same as that of European Military Officers serving in India at the same ages. There is sufficient reason to conclude that Tables 6 and 7 afford the materials for a Mortality Table, fairly representing that of the Native Females of India.

These series of Tables were clearly looked upon by Dr. Haines as applicable both to Annuity and Assurance transactions amongst the Natives of India, and for the great pains, labour and skill which he gave to perfect them, they deserve the recognition of merit which the Government accorded to him for them.

As to the other Tables relating to the mortality of Europeans in India, I consider they are the most complete and authentic which have yet appeared, and well worthy of being combined in a form which will allow of a graduated Table being deduced for each class. For this purpose I have computed the rates of mortality in the decennial groups of ages, and I have added a series of Tables graduated by a very simple process, which, at every tenth age, 20, 30, 40, &c., will be found exactly to agree in the probability of living a year with the original observations.

I have been the more anxious to utilize the materials contained in the Reports of the Indian Pension Funds, as every year the development of the resources of India in productions and commerce, in railroads, canals, and other enterprises, calls an additional number of Europeans to seek their fortunes in our vast possessions there; and whilst the effects of the climate on health is an important subject in itself, it is to be hoped that these researches will lead to a great extension of Life Assurance business both amongst the classes for whose benefit the Pension Funds no longer exist, and amongst the general European, and even the native population of India.

APPENDIX A.—*Males.—Observations of Mortality, in Quinquennial groups of Ages, amongst European Military Officers and Civil Servants during Service in India and after Retirement and return to Europe.*

Ages.	A(1). Military Officers. Bengal, Madras and Bombay combined.		A(2). Military Officers, Unmarried. Madras and Bombay combined.		A(3). Military Officers, Married. Madras and Bombay combined.			A(4.) Civil Service. Bengal and Madras combined.		Force of Mortality per Cent. deduced from the preceding Observations, and compared.						Force of Marriage per Cent.	
	Exposed to Risk.	Died.	Exposed to Risk.	Died.	Married.	Exposed to Risk.	Died.	Exposed to Risk.	Died.	A(1).	A(2).	A(3).	A(4).	Peerage.	Farr's Healthy Life.	A(2). Indian Military Officers.	British Peerage.
15–20	18038·	271	6516·	133	14	25·	..	1063·	20	1·50	2·04	..	1·88	·66	·53	·21	·19·
20–	49883·5	1342	27718·	853	656	1374·5	26	8029·5	135	2·69	3·08	1·89	1·68	1·11	·74	2·37	4·21
25–	42612·	1181	20513·	680	1,051	5550·	105	7408·5	140	2·77	3·32	1·89	1·89	1·00	·80	5·12	7·70
30–	33303·	956	11700·	381	751	8271·	217	6031·	86	2·87	3·26	2·62	1·41	·86	·84	6·42	7·14
35–	24956·	706	6215·	212	366	8081·	204	4883·	103	2·83	3·41	2·52	2·11	·89	·89	5·89	5·47
40–	17488·5	504	3307·	124	155	6779·5	182	3966·5	81	2·88	3·75	2·68	2·04	1·10	·99	4·69	3·95
45–	11805·5	361	1835·5	57	79	5150·	156	3153·5	61	3·06	3·11	3·13	1·93	1·38	1·16	4·30	1·98
50–	8221·5	211	951·5	34	30	3796·5	93	2318·	56	2·57	3·57	2·45	2·42	1·63	1·44	3·15	1·07
55–	5180·5	151	459·5	22	13	2548·5	73	1767·	47	2·92	4·79	2·86	2·66	1·92	1·85	2·83	1·05
60–	2521·5	92	224·5	10	3	1552·5	58	1297·5	48	3·65	4·45	3·74	3·70	2·88	2·89	1·34	
65–	1180·	69	139·	3	..	859·	52	849·5	44	5·76	2·16	6·05	5·18	4·71	4·44		
70–	531·5	39	71·5	6	..	395·	29	506·	26	7·34	8·39	7·34	5·14	7·08	6·79		
75–	200·	27	21·5	3	..	147·	19	245·	30	13·50	13·95	12·92	12·24	11·03	10·37		
80–	64·	8	5·5	1	..	52·	6	76·5	19	12·50	18·18	11·54	24·84	17·06	15·66		
85–	11·5	3	..	..	..	9·5	3	16·5	2	26·09	..	31·58	12·12	33·44	23·03		
90-95	..	..	..	..	..	..	..	3·	2	..	..	..	66·67	52·17			
	215997·	5,920	79677·5	2,519	3,118	44591·	1,223	41664·	900	2·74	3·16	2·74	2·16	1·80	..	3·91	

Appendix B(1).—*Females.—Observations of Mortality amongst the Wives and Widows and of Remarriage amongst the Widows of European Military Officers and Civil Servants on Service in India and after Retirement.*

Ages.	B(1). Wives. Exposed to Risk.	Died.	Force of Mortality per Cent	B(2). Widows. Exposed to Risk.	Died.	Married.	Force per Cent. Mortality.	Marriage.
13–20	950·	18	1·90	66·5	1	8	1·50	12·03
20–	4647·5	83	1·79	652·5	4	48	·61	7·36
25–	5975·	72	1·21	1691·5	26	82	1·54	4·85
30–	5193·5	69	1·33	2523·	30	81	1·19	3·21
35–	3860·5	57	1·48	3056·	36	50	1·18	1·64
40–	2530·	26	1·03	3027·	35	25	1·16	·83
45–	1615·5	10	·62	2504·	36	21	1·44	·84
50–	917·	10	1·09	1953·	33	21	1·69	·61
55–	431·5	4	·93	1511·5	21	7	1·39	·46
60–	186·	5	2·69	1027·5	31	1	3·02	·10
65–	77·	..	..	572·5	20	..	3·49	
70–	22·5	..	..	268·	14	..	5·22	
75–	3·5	..	..	94·	5	..	5·32	
80–	..	..	..	42·	7	..	16·67	
85–90	..	..	..	2·	2	..	100·	
	26409·5	354	1·34	18991·	301	335	1·59	1·76

Observations of Mortality amongst the Children born of European Parents in India.

Age.	C. Sons. Exposed to Risk.	Died.	Force of Mortality per Cent.	D. Daughters. Exposed to Risk.	Died.	Force of Mortality per Cent.
0–	3552·	306	8·62	3404·	261	7·67
1–	4781·	346	7·24	4526.	306	6·76
2–	4292·	136	3·17	4071·	150	3·68
3–	4011·	64	1·60	3740·5	70	1·87
4–	3778·5	30	·79	3495·	40	1·14
0– 5	20414·5	882	4·32	19236·5	827	4·30
5–	16024·5	82	·51	14926·5	87	·58
10–	12107·	39	·32	11112·5	39	·35
15–20	7852·5	45	·57	7435·5	41	·55
	56398·5	1,048	1·86	52711·	994	1·89

Appendix A(1).—*Table of Mortality of European Military Officers on Service in India and after Retirement, deduced from the combined Experience of the Bengal, Madras and Bombay Military Funds.* (*Exposed to Risk*, 215,997 ; *Deaths*, 5,920.)

Age x	$\log q_x$ $=(1-p_x)$	Probability of Dying in a Year q_x	Probability of Living a Year p_x	$\log p_x$	$\log l_x$	l_x	d_x
15	$\bar{2}$·33584	·02167	·97833	$\bar{1}$·99049	4·00000	10000	217
16	·34296	·02203	·97797	·99033	3·99049	9783	215
17	·35008	·02239	·97761	·99017	·98082	9568	214
18	·35720	·02276	·97724	·99000	·97099	9354	213
19	·36432	·02314	·97686	·98983	·96099	9141	212
20	·37144	·02352	·97648	·98966	·95082	8929	210
21	·37856	·02391	·97609	·98949	·94048	8719	208
22	·38568	·02430	·97570	·98932	·92997	8511	207
23	·39280	·02471	·97529	·98913	·91929	8304	205
24	·39992	·02511	·97489	·98896	·90842	8099	203
25	·40704	·02553	·97447	·98877	·89738	7896	202
26	·41416	·02595	·97405	·98858	·88615	7694	200
27	·42128	·02638	·97362	·98839	·87473	7494	197
28	·42840	·02682	·97318	·98819	·86312	7297	196
29	·43552	·02726	·97274	·98800	·85131	7101	194
30	·44265	·02771	·97229	·98780	·83931	6907	191
31	·44325	·02775	·97225	·98778	·82711	6716	186
32	·44385	·02779	·97221	·98776	·81489	6530	182
33	·44446	·02783	·97217	·98774	·80265	6348	176
34	·44506	·02787	·97213	·98772	·79039	6172	173
35	·44567	·02790	·97210	·98771	·77811	5999	167
36	·44627	·02794	·97206	·98769	·76582	5832	163
37	·44687	·02798	·97202	·98768	·75351	5669	158
38	·44748	·02802	·97198	·98766	·74119	5511	155
39	·44808	·02806	·97194	·98764	·72885	5356	150
40	·44869	·02810	·97190	·98762	·71649	5206	146
41	·44884	·02811	·97189	·98762	·70411	5060	143
42	·44899	·02812	·97188	·98761	·69173	4917	138
43	·44914	·02813	·97187	·98761	·67934	4779	134
44	·44929	·02814	·97186	·98760	·66695	4645	131
45	·44944	·02815	·97185	·98760	·65455	4514	127
46	·44959	·02816	·97184	·98759	·64215	4387	124
47	·44974	·02817	·97183	·98759	·62974	4263	120
48	·44989	·02818	·97182	·98759	·61733	4143	117
49	·45004	·02819	·97181	·98758	·60492	4026	113
50	·45020	·02820	·97180	·98758	·59250	3913	110
51	·45433	·02847	·97153	·98746	·58008	3803	109
52	·45846	·02874	·97126	·98734	·56754	3694	106
53	·46259	·02901	·97099	·98721	·55488	3588	104
54	·46672	·02929	·97071	·98709	·54209	3484	102
55	·47086	·02957	·97043	·98696	·52918	3382	100
56	·47499	·02985	·97015	·98684	·51614	3282	98
57	·47912	·03014	·96986	·98671	·50298	3184	96
58	·48325	·03043	·96957	·98658	·48969	3088	94
59	·48738	·03072	·96928	·98645	·47627	2994	92
60	·49152	·03101	·96899	·98632	·46272	2902	90
61	·52062	·03316	·96684	·98535	·44904	2812	93
62	·54972	·03546	·96454	·98432	·43439	2719	96
63	·57882	·03792	·96208	·98321	·41871	2623	100
64	·60792	·04054	·95946	·98203	·40192	2523	102
65	·63702	·04335	·95665	·98075	·38395	2421	105
66	·66612	·04636	·95364	·97938	·36470	2316	108
67	·69522	·04957	·95043	·97792	·34408	2208	109
68	·72432	·05301	·94699	·97635	·32200	2099	111
69	·75342	·05668	·94332	·97466	·29835	1988	113

APPENDIX A(1)—(*continued*).

Age x	$\log q_x = (1-p_x)$	Probability of Dying in a Year q_x	Probability of Living a Year p_x	$\log p_x$	$\log l_x$	l_x	d_x
70	$\bar{2}$·78251	·06061	·93939	$\bar{1}$·97285	3·27301	1875	114
71	·81372	·06512	·93488	·97076	·24586	1761	114
72	·84494	·06998	·93002	·96849	·21662	1647	115
73	·87615	·07519	·92481	·96605	·18511	1532	116
74	·90737	·08079	·91921	·96341	·15116	1416	114
75	·93858	·08681	·91319	·96046	·11457	1302	113
76	·96980	·09328	·90672	·95747	·07513	1189	111
77	$\bar{1}$·00101	·10023	·89976	·95413	·03260	1078	108
78	·03223	·10770	·89230	·95051	2·98673	970	105
79	·06344	·11573	·88427	·94658	·93724	865	100
80	·09466	·12435	·87565	·94233	·88382	765	95
81	·12151	·13228	·86772	·93838	·82615	670	89
82	·14837	·14072	·85928	·93413	·76453	581	81
83	·17522	·14970	·85030	·92957	·69866	500	75
84	·20208	·15925	·84075	·91467	·62823	425	68
85	·22894	·16941	·83059	·91939	·55290	357	60
86	·25579	·18021	·81979	·91370	·47229	297	54
87	·28265	·19171	·80829	·90757	·38599	243	46
88	·30950	·20394	·79606	·90095	·29356	197	40
89	·33636	.21695	·78305	·89379	·19451	157	34
90	·36322	·23079	·76921	·88604	·08830	123	29
91	·49057	·30944	·69056	·83920	1·97434	94	29
92	·61793	·41489	·58511	·76724	·81354	65	27
93	·74528	·55626	·44374	·64713	·58078	38	21
94	·87264	·74583	·25417	·40512	·22791	17	13
95	1·00000	..	..	..	0·63303	4	4

APPENDIX A(4).—*Table of Mortality of European Civil Servants on Service in India and after Retirement, deduced from the combined Experience of the Bengal and Madras Civil Service Funds.* (*Exposed to Risk*, 41,664; *Deaths*, 900.)

Age x	$\log q_x$	Probability of Dying in a Year q_x	Probability of Living a Year p_x	$\log p_x$	$\log l_x$	l_x	d_x
15	$\bar{2}$·22932	·01696	·98304	$\bar{1}$·99257	4·00000	10000	170
16	·22881	·01694	·98306	·99158	3·99257	9830	166
17	·22830	·01692	·98308	·99259	·98515	9664	164
18	·22779	·01690	·98310	·99260	·97774	9500	160
19	·22728	·01688	·98312	·99261	·97034	9340	158
20	·22677	·01686	·98314	·99262	·96295	9182	154
21	·22626	·01684	·98316	·99262	·95557	9028	153
22	·22575	·01682	·98318	·99263	·94819	8875	149
23	·22524	·01680	·98320	·99264	·94082	8726	146
24	·22473	·01678	·98322	·99265	·93346	8580	144
25	·22422	·01676	·98324	·99266	·92611	8436	142
26	·22371	·01674	·98326	·99267	·91877	8294	139
27	·22320	·01672	·98328	·99258	·91144	8155	136
28	·22269	·01670	·98330	·99269	·90412	8019	134
29	·22218	·01668	·98332	·99269	·89681	7885	131
30	·22167	·01666	·98334	·99270	·88950	7754	130
31	·23086	·01702	·98298	·99254	·88220	7624	129
32	·24005	·01738	·98262	·99239	·87474	7495	131
33	·24924	·01775	·98225	·99222	·86713	7364	130
34	·25843	·01813	·98187	·99205	·85935	7234	132
35	·26762	·01852	·98148	·99188	·85140	7102	131
36	·27681	·01892	·98108	·99170	·84328	6971	132

Appendix A(4)—(*continued*).

Age x	log q_x	Probability of Dying in a Year q_x	Probability of Living a Year p_x	log p_x	log l_x	l_x	d_x
37	$\bar{2}$·28600	·01932	·98068	$\bar{1}$·99153	3·83498	6839	132
38	·29519	·01973	·98027	·99135	·82651	6707	132
39	·30438	·02016	·97984	·99116	·81786	6575	133
40	·31356	·02059	·97941	·99096	·80902	6442	133
41	·31478	·02064	·97936	·99094	·79998	6309	130
42	·31600	·02070	·97930	·99092	·79092	6179	128
43	·31723	·02076	·97924	·99089	·78184	6051	125
44	·31845	·02082	·97918	·99086	·77273	5926	124
45	·31967	·02088	·97912	·99084	·76359	5802	121
46	·32089	·02094	·97906	·99081	·75443	5681	119
47	·32212	·02100	·97900	·99078	·74524	5562	117
48	·32334	·02105	·97895	·99076	·73602	5445	114
49	·32456	·02111	·97889	·99073	·72678	5331	113
50	·32579	·02117	·97883	·99071	·71751	5218	110
51	·34168	·02196	·97804	·99036	·70822	5108	112
52	·35757	·02278	·97722	·98999	·69858	4996	114
53	·37346	·02363	·97637	·98961	·68857	4882	116
54	·38935	·02451	·97549	·98922	·67818	4766	117
55	·40524	·02542	·97458	·98882	·66740	4649	118
56	·42113	·02637	·97363	·98839	·65622	4531	119
57	·43702	·02735	·97265	·98796	·64461	4412	121
58	·45291	·02837	·97163	·98750	·63257	4291	122
59	·46880	·02943	·97057	·98703	·62007	4169	122
60	·48468	·03053	·96947	·98653	·60710	4047	124
61	·50637	·03209	·96791	·98584	·59363	3923	126
62	·52806	·03373	·96627	·98510	·57947	3797	128
63	·54975	·03546	·96454	·98432	·56457	3669	130
64	·57144	·03728	·96272	·98350	·54889	3539	132
65	·59313	·03919	·96081	·98264	·53239	3407	133
66	·61482	·04119	·95881	·98173	·51503	3274	135
67	·63651	·04330	·95670	·98078	·49676	3139	136
68	·65820	·04552	·95448	·97977	·47754	3003	137
69	·67989	·04785	·95215	·97871	·45731	2866	137
70	·70158	·05030	·94970	·97759	·43602	2729	137
71	·74653	·05579	·94421	·97507	·41361	2592	145
72	·79148	·06187	·93813	·97226	·38868	2447	151
73	·83643	·06862	·93138	·96913	·36094	2296	158
74	·88138	·07610	·92390	·96563	·33007	2138	162
75	·92633	·08440	·91560	·96171	·39570	1976	167
76	·97129	·09360	·90640	·95732	·25741	1809	169
77	$\bar{1}$·01624	·10381	·89619	·95240	·21473	1640	171
78	·06119	·11513	·88487	·94688	·16713	1469	169
79	·10614	·12769	·87231	·94067	·11401	1300	166
80	·15109	·14161	·85839	·93368	·05468	1134	161
81	·16294	·14553	·85447	·93170	2·98836	973	143
82	·17479	·14955	·85045	·92965	·91906	830	124
83	·18663	·15368	·84632	·92753	·84871	706	109
84	·19848	·15794	·84206	·92534	·77624	597	94
85	·21033	·16230	·83770	·92309	·70158	503	82
86	·22218	·16679	·83321	·92075	·62467	421	70
87	·23403	·17141	·82859	·91834	·54542	351	60
88	·24587	·17614	·82386	·91585	·46376	291	51
89	·25772	·18102	·81898	·91327	·37961	240	44
90	·26957	·18603	·81397	·91061	·29288	196	36
91	·41565	·26041	·73959	·86899	·20349	160	42
92	·56174	·36454	·63546	·80309	·07248	118	43
93	·70782	·51029	·48971	·68994	1·87557	75	38
94	·85391	·71435	·28565	·45583	·56551	37	26
95	0·00000	1·00000	..	..	·02134	11	11

APPENDIX B.—*Table of Mortality of the Wives and Widows of European Military Officers and Civilians on Service in India and after Retirement, deduced from the combined Experience in the Three Presidencies.* (*Exposed to Risk*, 45400·5; *Deaths*, 655.)

Age x	$\log q_x$	Probability of Dying in a Year q_x	Probability of Living a Year p_x	$\log p_x$	$\log l_x$	l_x	d_x
15	$\bar{2}$·28028	·01907	·98093	$\bar{1}$·99164	4·00000	10000·	191·
16	·26856	·01856	·98144	·99186	3·99164	9809·	182·
17	·25684	·01807	·98193	·99208	·98350	9627·	174·
18	·24511	·01758	·98242	·99230	·97558	9453·	166·
19	·23339	·01712	·98288	·99250	·96788	9287·	159·
20	·22167	·01666	·98334	·99270	·96038	9128·	152·
21	·20994	·01622	·98378	·99290	·95308	8976·	146·
22	·19822	·01578	·98422	·99309	·94598	8830·	139·
23	·18650	·01536	·98464	·99328	·93907	8691·	133·
24	·17478	·01496	·98504	·99345	·93235	8558·	128·
25	·16305	·01456	·98544	·99363	·92580	8430·	123·
26	·15133	·01417	·98583	·99380	·91943	8307·	118·
27	·13961	·01379	·98621	·99397	·91323	8189·	113·
28	·12788	·01342	·88658	·99413	·90720	8076·	108·
29	·11616	·01307	·98693	·99429	·90133	7968·	104·
30	·10444	·01272	·98728	·99444	·89562	7864·	100·
31	·10272	·01267	·98733	·99446	·89006	7764·	99·
32	·10100	·01262	·98738	·99448	·88452	7665·	97·
33	·09928	·01257	·98743	·99451	·87900	7568·	95·
34	·09756	·01252	·98748	·99453	·87351	7473·	93·
35	·09584	·01247	·98753	·99455	·86804	7380·	92·
36	·09412	·01242	·98758	·99457	·86259	7288·	91·
37	·09240	·01237	·98763	·99459	·85716	7197·	89·
38	·09068	·01232	·98768	·99462	·85175	7108·	87·
39	·08896	·01227	·98773	·99464	·84637	7021·	87·
40	·08725	·01223	·98777	·99466	·84101	6934·	84·
41	·08863	·01226	·98774	·99464	·83567	6850·	84·
42	·09001	·01230	·98770	·99463	·83031	6766·	83·
43	·09139	·01234	·98766	·99461	·82494	6683·	83·
44	·09277	·01238	·98762	·99459	·81955	6600·	82·
45	·09415	·01242	·98758	·99457	·81414	6518·	81·
46	·09553	·01246	·58754	·99455	·80871	6437·	80·
47	·09691	·01250	·98750	·99454	·80326	6357·	79·
48	·09829	·01254	·98746	·99452	·79780	6278·	79·
49	·09967	·01258	·98742	·99450	·79232	6199·	78·
50	·10105	·01262	·98738	·99448	·78682	6121·	77·
51	·11908	·01316	·98684	·99425	·78130	6044·	80·
52	·13711	·01371	·98629	·99400	·77555	5964·	82·
53	·15515	·01429	·98571	·99375	·76955	5882·	84·
54	·17318	·01490	·98510	·99348	·76330	5798·	86·
55	·19122	·01553	·98447	·99320	·75678	5712·	89·
56	·20925	·01619	·98381	·99291	·74998	5623·	91·
57	·22729	·01688	·98312	·99260	·74289	5532·	93·
58	·24532	·01759	·98241	·99229	·73549	5439·	96·
59	·26335	·01834	·98166	·99196	·72778	5343·	98·
60	·28139	·01912	·98088	·99162	·71974	5245·	100·
61	·30834	·02034	·97966	·99108	·71136	5145·	105·
62	·33529	·02164	·97836	·99050	·70244	5040·	109·
63	·36225	·02303	·97697	·98988	·69294	4931·	113·
64	·38920	·02450	·97550	·98923	·68282	4818·	118·
65	·41615	·02607	·97393	·98853	·67205	4700·	123·
66	·44310	·02774	·97226	·98778	·66058	4577·	127·
67	·47006	·02952	·97048	·98699	·64836	4450·	131·
68	·49701	·03141	·96859	·98614	·63535	4319·	136·
69	·52396	·03342	·96658	·98524	·62149	4183·	140·

Appendix B—(*continued*).

Age x	$\log q_x$	Probability of Dying in a Year q_x	Probability of Living a Year p_x	$\log p_x$	$\log l_x$	l_x	d_x
70	$\bar{2}$·55092	·03556	·96444	$\bar{1}$·98428	3·60673	4043·	143·
71	·58745	·03868	·96132	·98287	·59101	3900·	151·
72	·62398	·04207	·95793	·98133	·57388	3749·	158·
73	·66051	·04576	·95424	·97976	·55521	3591·	164·
74	·69704	·04978	·95022	·97782	·53487	3427·	171·
75	·73357	·05415	·94585	·97582	·51269	3256·	176·
76	·77010	·05890	·94110	·97364	·48851	3080·	182·
77	·80663	·06407	·93593	·97124	·46215	2898·	185·
78	·84316	·06969	·93031	·96863	·43339	2713·	189·
79	·87969	·07580	·92420	·96577	·40202	2524·	192·
80	·91622	·08245	·91755	·96263	·36779	2332·	192·
81	$\bar{1}$·00699	·10162	·89838	·95346	·33042	2140·	217·
82	·09776	·12524	·87476	·94189	·28388	1923·	241·
83	·18853	·15436	·84564	·92719	·22577	1682·	260·
84	·27930	·19024	·80976	·90836	·15296	1422·	270·
85	·37007	·23446	·76554	·88397	·06132	1152·	270·
86	·46084	·28896	·71104	·85189	2·94529	882·	255·
87	·55161	·35613	·64387	·80880	·79718	627·	223·
88	·64238	·43892	·56109	·74903	·60598	404·	178·
89	·73315	·54094	·45906	·66187	·35501	226·	122·
90	·82391	·66667	·33333	·52287	·01688	104·	69·
91	·85912	·72297	·27703	·44253	1·53975	35·	25·
92	·89434	·78404	·21596	·33437	0·98228	10·	8·
93	·92956	·85028	·14972	·17528	0·31665	2·	1·7
94	·96478	·92211	·07789	$\bar{2}$·89148	$\bar{1}$·49193	·3	·28
95	1·00000	..	..	..	$\bar{2}$·38341	·02	·02

Table of Mortality of Children born of European Parents, Military Officers and Civilians on Service in India and after Retirement.

C. Sons. (*Exposed to Risk*, 56398·5; *Deaths*, 1,048.)

Age x	$\log q_x$	Probability of Dying in a Year q_x	Probability of Living a Year p_x	$\log p_x$	$\log l_x$	l_x	d_x
0	$\bar{2}$·91719	·08264	·91736	$\bar{1}$·96254	4·00000	10000	826
1	·84430	·06987	·93013	·96854	3·96254	9174	641
2	·49423	·03121	·96879	·98623	·93108	8533	267
3	·20066	·01587	·98413	·99305	·91731	8266	131
4	$\bar{3}$·89592	·00787	·99213	·99657	·91036	8135	64
5	·83277	·00680	·99320	·99704	·90693	8071	55
6	·76962	·00588	·99412	·99743	·90397	8016	47
7	·70647	·00509	·99491	·99778	·90140	7969	41
8	·66646	·00464	·99536	·99798	·89918	7928	36
9	·62596	·00423	·99577	·99816	·89716	7892	34
10	·58546	·00385	·99615	·99832	·89532	7858	30
11	·54496	·00351	·99649	·99847	·89364	7828	28
12	·50446	·00320	·99680	·99861	·89211	7800	25
13	·55449	·00359	·99641	·99844	·89072	7775	27
14	·60453	·00402	·99598	·99825	·88916	7748	32
15	·65456	·00451	·99549	·99804	·88741	7716	34
16	·70460	·00507	·99493	·99779	·88545	7682	39
17	·75464	·00568	·99432	·99753	·88324	7643	44
18	·80468	·00638	·99362	·99722	·88077	7599	48
19	·85472	·00716	·99284	·99688	·87799	7551	54
20	·90476	·00803	·99197	·99650	·87487	7497	

Table of Mortality of Children, &c.—(continued).

D. Daughters. (*Exposed to Risk*, 52711; *Deaths*, 994.)

Age x	log q_x	Probability of Dying in a Year q_x	Probability of Living a Year p_x	log p_x	log l_x	l_x	d_x
0	$\bar{2}$·86845	·07387	·92613	$\bar{1}$·96667	4·00000	10000	739
1	·81551	·06539	·93461	·97063	3·96667	9261	605
2	·55793	·03614	·96386	·98401	·93730	8656	313
3	·26780	·01853	·98147	·99188	·92131	8343	155
4	·05443	·01134	·98866	·99505	·91319	8188	93
5	$\bar{3}$·95701	·00906	·99094	·99605	·90824	8095	73
6	·85959	·00724	·99276	·99684	·90429	8022	58
7	·76217	·00578	·99422	·99748	·90113	7964	46
8	·71839	·00523	·99477	·99772	·89861	7918	42
9	·67462	·00473	·99527	·99794	·89633	7876	37
10	·63085	·00427	·99573	·99814	·89427	7839	33
11	·58708	·00386	·99614	·99832	·89241	7806	30
12	·54331	·00349	·99651	·99848	·89073	7776	28
13	·58248	·00382	·99618	·99834	·88921	7748	29
14	·62165	·00418	·99582	·99818	·88755	7719	32
15	·66082	·00458	·99542	·99801	·88573	7687	36
16	·69999	·00501	·99499	·99782	·88374	7651	38
17	·73917	·00548	·99452	·99761	·88156	7613	42
18	·77834	·00600	·99400	·99739	·87917	7571	45
19	·81751	·00657	·99343	·99714	·87656	7526	49
20	·85669	·00719	·99281	·99687	·87370	7477	

Appendix E.—*Summary, in Quinquennial and Decennial Periods of Age, of Tables of Mortality amongst the Natives of India, in the Bombay Presidency, collected by the late Dr. Robert Haines.*

Table.	Description of Table.	Entered.	Total under Observation.	Struck off or still Living.	Half of Entered or gone off.	Died.	At Risk.	Mortality per Cent.
1	Civil Pensioners, chiefly Clerks and their Orphan Sons under age 16—Wardens Official Fund, Males	527	6,932	387	457·	140	6475	2·162
2	Civil Pensioners, chiefly Clerks superannuated on Medical Certificate—Males	322	3,118	158	240·	164	2878	5·698
3	Native Military Pensioners, Sepoys chiefly, a few Clerks, &c., and a few Orphan Sons of Sepoys	22,463	148,346	16,108	19285·5	6,355	129060·5	4·924
4*	Total of 10 Regiments of Golundauze Artillery Battalion	27,778	214,286	24,650	26214·	3,128	188072·	1·663
5	Females, European Pensioners, Widows and Daughters of Soldiers..................	565	4,146	487	526·	78	3620·	2·155
6	Females, Native Pensioners, Widows, Mothers, and Daughters of Sepoys	1,890	17,731	1,554	1722·	336	16009	2·099
7	Females, Native Pensioners, Civilians, Widows and Daughters of Clerks	604	8,531	455	529·5	149	8001·5	1·862

* See "Summary of each Regiment contained in Table 4," p. 214.

Appendix E—(*continued*).

Summary of each Regiment contained in Table 4.

Sepoys.	Entered.	Under Observation.	Struck off or living.	Half Entered or gone off.	Died.	At Risk.	Mortality per Cent.
(1). 3rd Regt. N.L.I., chiefly Hindoostanees and Purwarrees, about equal proportions, 1841 to 1865 (24 years)	4,201	27,866	3,791	3996·	430	23870·	1·801
(2). 4th Regt. N.I. (Rifles), majority Purwarrees, a few Hindoostanees, and others, 1848 to 1865 (17 years)	1,859	18,109	1,579	1719·	280	16390·	1·708
(3). 7th Regt. N.I., Castes not noted, 1838 to 1859 (21 years). great mortality in 1845	4,111	..	3,835	3973·	276	20575·	1·341
(4). 9th Regt. N.I., all Castes, 1846 to 1865 (19 years)	2,671	20,240	2,357	2514·	313	17726·	1·766
(5). 10th Regt. N.I., Castes not noted, 1849 to 1861 (12 years), does not include Native Officers..	1,470	10,801	1,346	1408·	124	9393·	1·320
(6). 14th Regt. N.I., Hindoos of all Castes, a few Mussulmans and Jews, 1834 to 1864 (30 years)	3,134	28,598	2,737	2935·5	397	25662·5	1·547
(7). 18th Regt. N.I., Caste not noted, 1848 to 1863 (15 years) ..	1,568	15,005	1,389	1478·5	179	13526·5	1·323
(8). 21st Regt. N.I., Marine Battalion, mostly Purwarrees, some Mussulmans and a few Jews—30 casualties lost in the "Cleopatra," foundered at sea, are included amongst "gone off"—1842 to 1864 (22 years)	2,741	26,938	2,162	2451·5	579	24486·5	2·365
(9). 25th Regt. N.I., Hindoos of all Castes, a few Mussulmans and Jews, 1848 to 1864 (16 years)	2,315	15,187	2,133	2224·	182	12963·	1·404
(10). 29th Regt. N.I. (disbanded 1860), Hindoos of all Castes, a very few Mussulmans—great mortality in Oct. 1858, probably cholera—1850 to 1860 (10 years)	2,107	10,570	1,951	2029·	156	8541·	1·826
(11). 3rd & 4th Battalions, Artillery (Native), almost all high Caste Hindoos, Brahmins, and Rajpoots, age at enlistment not entered in the Register, assumed to be 18 or 19, 1847 to 1863 (16 years)........	1,571	16,345	1,359	1465·	212	14880·	1·428
Total of 10 Regiments and Golundauze Battalion ...	27,778	214,286	24,650	26214·	3,128	188072·	1·663

APPENDIX E—(*continued*).

	1. M. Civil Pensioners, chiefly Clerks and their Orphan Sons under 16. Wardens Official Fund.			2. M. Civil Pensioners, chiefly Clerks Superannuated on Medical Certificate.			3. Native Military Pensioners, Sepoys chiefly, a few Clerks and a few Orphan Sons of Sepoys.			4. Total 10 Regiments N.I. and Artillery Battalion.			5. Females. European Pensioners, Widows and Daughters of Soldiers.			6. Females. Native Pensioners, Widows, Mothers, and Daughters of Sepoys.			7. Females. Native Pensioners and Widows and Daughters of Clerks.		
	At Risk.	Died.	Rate of Mortality per Cent.	At Risk.	Died.	Rate of Mortality per Cent.	At Risk.	Died.	Rate of Mortality per Cent.	At Risk.	Died.	Rate of Mortality per Cent.	At Risk.	Died.	Rate of Mortality per Cent.	At Risk.	Died.	Rate of Mortality per Cent.	At Risk.	Died.	Rate of Mortality per Cent.
0–	8·	..	..	..	..	..	·5	..	..	..	..	..	33·	..	..	..	..	..	6·		
1–	23·	..	..	·5	..	..	4·5	..	..	..	..	..	68·5	2	..	2·5	..	..	17·5		
2–	34·	..	..	1·5	..	..	11·5	1	..	..	..	..	72·	1	..	5·	..	..	30·		
3–	45·	..	..	2·	..	..	17·5	..	..	1·	..	..	76·5	..	..	6·5	..	..	44·	1	
4–	58·	..	..	2·	..	..	22·	..	..	1·5	..	..	82·5	..	..	10·	..	..	61·		
0– 5	168·	0	..	6·	..	..	56·	1	1·786	2·5	..	..	332·5	3	·902	24·	..	..	158·5	1	
5–	531·5	3	·564	17·5	1	5·714	139·5	..	..	38·	..	..	491·5	..	..	72·5	..	..	600·5	5	
10–	973·5	2	·205	17·5	..	..	82·	1	1·220	247·5	2	·808	541·5	..	..	202·5	..	.	837·	4	
15–	336·5	..	..	17·	..	..	43·	2	4·651	10014·	110	1·098	58·5	..	..	486·	2	1·029	265·	..	
20–	..	..	..	20·	..	..	204·5	3	1·467	50119·	784	1·564	86·5	4	4·624	998·	5	·501	183·	..	
25–	6·	..	..	22·5	..	..	305·5	7	2·291	47240·	792	1·677	137·	4	2·920	1500·5	19	1·267	354·	3	
30–	10·	..	..	26·	..	..	1646·5	37	2·247	38992·	673	1·726	243·	7	2·881	2021·5	31	1·534	576·5	4	·694
35–	15·	1	·667	25·	1	4·	12812·5	246	1·920	25269·5	455	1·801	291·	10	3·436	2225·	28	1·258	767·	7	·913
40–	100·5	..	..	71·	5	7·042	17280·	477	2·760	10677·5	205	1·920	344·	8	2·326	2047·	35	1·710	905·	9	·994
45–	285·	4	1·405	127·	13	10·236	16182·5	609	3·764	4180·	81	1·938	287·	7	2·439	1596·5	30	1·879	878·5	22	2·504
50–	603·5	16	2·651	270·5	10	3·697	24960·	1,078	4·319	1103·	23	2·085	237·5	8	3·368	1431·5	25	1·746	769·5	17	2·209
55–	820·	21	2·561	435·	18	4·138	22096·	1,149	5·200	173·5	3	1·729	212·5	6	2·824	1254·	38	3·030	577·5	20	3·463
60–	834·	21	2·518	506·	26	5·138	14724·	965	6·554	15·5	..	..	166·5	8	4·805	964·	34	3·527	412·	13	3·155
65–	698·	37	5·301	456·	32	2·794	9232·5	768	8·318	..	..	..	96·5	7	7·254	613·	42	6·852	339·	13	3·835
70–	460·5	19	4·126	343·5	28	8·151	5453·5	536	9·831	..	..	..	49·	3	6·122	348·5	20	5·739	211·5	17	8·038
75–	313·	11	3·514	223·5	10	4·474	2476·5	291	11·750	..	..	..	13·	1	7·692	173·	14	8·092	89·5	8	8·939
80–	179·	3	1·676	141·	14	9·929	922·5	114	12·324	..	..	..	7·	..	..	45·5	11	22·135	50·	4	8·
85–	86·5	1	1·156	93·	3	3·226	331·	47	14·199	..	..	..	13·	1	7·692	3·	2	66·667	21·5	2	9·302
90–	41·5	1	2·410	38·	3	7·895	88·	18	20·455	..	..	..	8·5	..	..	3·			6·		
95–	8·5	..	..	12·5	..	..	18·5	4	21·622	..	..	..	1·5	..	..						
100–	4·5	..	..	1·	..	..	6·	2	33·333	..	..		2·5	1	40·						
105–109	..	..	..	8·5	..	..															
	6475·	140	2·162	2878·	164	5·698	129060·5	6,355	4·924	188072·	3,128	1·663	3620·	78	2·155	16009·	336	2·099	8001·5	149	1·862

Summary of the above Tables in Decennial Periods of Age.

	Nos. 1 and 2.		Rate of Mortality per Cent.	No. 3.		Rate of Mortality per Cent.	No. 4.		Rate of Mortality per Cent.	No. 5.		Rate of Mortality per Cent.	Nos. 6 and 7.		Rate of Mortality per Cent.
	At Risk.	Died.		At Risk.	Died.		At Risk.	Died.		At Risk.	Died.		At Risk.	Died.	
0– 5	174·	..	..	56·	1	1·786	2·5	..	..	332·5	3	·902	182·5	1	..
5– 15	1540·	6	·390	221·5	1	451	285·5	2	701	033·	..	..	1712·5	9	·526
15–	373·5	..	..	247·5	5	2·020	60133·	894	1·487	1145·	4	2·759	1932·	7	·362
25–	64·5	..	..	1952·	44	2·254	86232·	1,465	1·699	380·	11	2·895	4452·5	57	1·208
35–	211·5	7	3·310	30092·5	723	2·403	35947·	660	1·836	635·	18	2·835	5944·	79	1·329
45–	1286·	43	3·344	41142·5	1,687	4·104	5283·	104	1·969	524·5	15	2·860	4676·	94	2·010
55–	2595·	86	3·314	36820·	2,114	5·741	189·	3	1·587	379·	14	3·694	3207·5	105	3·274
65–	1958·	116	5·925	14686·	1,304	8·879	..	..	..	145·5	10	6·873	1512·	92	6·085
75–	856·5	38	4·437	3399·	405	11·915	..	..	..	20·	1	5·	358·	37	10·334
85–	259·	8	3·089	419·	65	15·513	.	..	..	21·5	1	4·65	33·5	4	11·940
95–110	35·	..	..	24·5	6	24·490				4·	1	25·			
..	9353·	304	3·250	129060·5	6,355	4·924	188072·	3,128	1·663	3620·	78	2·155	24010·5	485	2·020

On the Risk attaching to the grant of Life Assurances. By DR. C. BREMIKER, *of Berlin. Translated by* T. B. SPRAGUE, M.A., *Vice-President of the Institute of Actuaries.*

WHILE life insurance premiums, irrespective of the additions for the cost of management, are so calculated, that assuming a given mortality table and a given rate of interest, the receipts of the Insurance Office improved at compound interest will exactly suffice to pay the claims that occur according to the table of mortality; yet the Office suffers a loss in those cases where death occurs soon, and on the contrary makes a profit in the case of the persons who live long. In the grant of life annuities, the contrary is the case. It can be calculated exactly how long an insured person must live in order that the Office may make neither profit nor loss on his insurance; and in the same way it can be calculated what gain or loss the Office experiences when the life assured dies after a given number of years, n. If all these gains and losses are added together, the losses being considered as negative gains, the sum must be $=0$. This, however, is far from being the case in practice. The datums of the mortality table, altho' they may be deduced from observations extending over a long series of years, are to be considered as mean values, which are strictly applicable only when there is a very great number of deaths; and we ought to

expect that with a small number of insurances, and still more in single instances, considerable deviations from the mean will take place.

If on the one side it is the object of the assured to protect himself against the consequences of premature death, or of too long a life, so on the other side the Insurance Office must be in a position to undertake the risk attaching to those contingencies, that is to say, it must, in addition to the reserve based on the mean rate of mortality, and determined so as to provide for the payment of all claims that will arise, possess a fund by means of which all deviations from the mean mortality shall be covered. It is the object of the present paper to determine this fund, and to fix its relation to the number and amount of the insurances. The Guarantee Fund of the Office must in no case be less than this fund. Whether it should be made equal to the latter, or for greater security should be two or three times as great, is unimportant. In any case we must first determine the scale by which it is to be measured, and the want of such a scale explains the extreme divergence of opinion as to the Guarantee Fund.

I.

The risk, or the danger, to which the Office and the assured are respectively exposed, depends partly on the magnitude of the sum assured, and partly on the unequal distribution of the gains and losses which the one assured has over the other through the earlier or later happening of death. There is not to be found among the authors who have considered the subject a precise conception of the manner in which the risk is to be calculated. This conception must rather be taken from the calculus of probabilities. Tetens* appears to have felt this, and therefore prefaces his chapter on risk with a long mathematical discussion of the coefficients of an expanded binomial, enquiries which were much in vogue among mathematicians in the second half of the last century, and which, so far as they relate to probabilities, led, about the beginning of this century, to the method of least squares. Tetens, however, did not arrive at that. Indeed, he makes no use of the propositions he demonstrates, but confines himself exclusively to the greatest possible gain and loss. His ideas as to the nature of risk vacillate backwards and forwards, and result in no clear conception. It was reserved for the method of least squares, that most fruitful application of the calculus of probabili-

* Einleitung zur Berechnung der Leibrenten und Anwartschaften. Leipzig, 1785.

ties, to throw the necessary light on this subject. That method affords us the means of estimating the risk of committing an error to which we are exposed in observations of any kind. By its means definite conceptions were for the first time formed as to mean and probable errors, and as to the risk of error in single events of a given kind subject to chance. In the case of various observations of one and the same event, this method leads to the arithmetic mean as the result. But just as the arithmetic mean had already been regarded as the most probable result before the method of least squares was known, so in the same way has the value of an insurance been calculated according to the arithmetic mean without regard to its error, inasmuch as that value was assumed to be the average of all the payments to be expected according to the mortality table. This cannot, strictly speaking, be called an application of the calculus of probabilities; and if nevertheless we find both in old and new textbooks on life assurance a chapter on the calculus of probabilities, this might just as well have been left out, since in reality no application is made of it. The only application that could have been made of probabilities would have been to calculate the trustworthiness of the results obtained from the mortality table, or the probability of their agreeing with observation within given limits. In this direction a wide and fruitful field would have presented itself for the calculus of probabilities; and its cultivation would have led to most important results, among others to the conception of risk. In the present state of science this path would offer no difficulties worth speaking of; but it has not been trodden since the time of Laplace,* either because people have disregarded probability altogether, or because they have not known how to manage it. People have been rather contented with calculating the average values of life annuities and insurances according to some table of mortality, without even bestowing any special care on the choice of the table. That in this no use is made of the calculus of probabilities may now, although it is not necessary, be proved by some instances.

Let l_x, l_{x+1}, l_{x+2}, ... l_{x+n} ... denote the numbers of persons living according to the mortality table, at the ages x, $x+1$, $x+2$, ... $x+n$, ... and $d_{x+\frac{1}{2}}$, $d_{x+1+\frac{1}{2}}$, ... $d_{x+n+\frac{1}{2}}$ ... the numbers dying in the years of age $x+1$, $x+2$, ... $x+n+1$, ... so that

* Théorie des probabilités. Paris, 1812.

$$
\begin{aligned}
d_{x+\frac{1}{2}} &= l_x - l_{x+1} \\
d_{x+\frac{3}{2}} &= l_{x+1} - l_{x+2} \\
\&c. &= \&c. \\
d_{x+n+\frac{1}{2}} &= l_{x+n} - l_{x+n+1}
\end{aligned}
$$

Furthermore, let l'_x, l'_{x+1}, ... denote the discounted numbers of the living, and $d'_{x+\frac{1}{2}}$, $d'_{x+\frac{3}{2}}$, ... the discounted numbers of the dying, these numbers being given by the equations

$$
\begin{aligned}
l'_x &= l_x v^x & \qquad d'_{x+\frac{1}{2}} &= d_{x+\frac{1}{2}} v^{x+\frac{1}{2}} \\
l'_{x+1} &= l_{x+1} v^{x+1} & \qquad d'_{x+1+\frac{1}{2}} &= d_{x+1+\frac{1}{2}} v^{x+1+\frac{1}{2}} \\
&\&c. & &\&c.
\end{aligned}
$$

where v is the value of 1 due in a year.

In order to calculate the present value of an annuity of £1 for persons of the age x, we proceed as follows.

If each of the l_x persons living at the age x, according to the table of mortality, receives immediately £1, this makes £l_x. Of these persons there are still living at the beginning of the second year l_{x+1}, each of whom receives £1; and the present value of this £1 being only £v, the present value of all the payments is $l_{x+1}v$. Just in the same way the l_{x+n} persons living at the beginning of the $\overline{n+1}|$th year receive £1 each, the present value of the payments being $l_{x+n}v^n$; and so on until all the persons are dead. The sum of all these payments is

$$
\begin{aligned}
l_x + l_{x+1}v + l_{x+2}v^2 + \ldots\ldots &= v^{-x}(l'_x + l'_{x+1} + \ldots\ldots) \\
&= v^{-x}\Sigma l'_{x+n} \quad \begin{matrix} \text{from } n=0 \\ \text{to } \quad n=\infty \end{matrix}
\end{aligned}
$$

where the symbol of summation refers to the values of l' in which n is put equal to 0, 1, 2, 3, ... up to the extreme limit of life. But this extreme limit may be put equal to ∞, because all the terms following a fixed one vanish. If now each of the l_x persons pays down a sum a as the purchase money of an immediate annuity payable in advance, the Office receives $l_x\mathsf{a}$, which ought to be equal to the present value of all the annuities to be paid. We have therefore the equation

$$
\mathsf{a} = \frac{v^{-x}\Sigma l'_{x+n}}{l_x} = \frac{\Sigma l'_{x+n}}{l'_x} \quad \left.\begin{matrix} \text{from } n=0 \\ \text{to } \quad n=\infty \end{matrix}\right\} \quad . \quad . \quad (1)
$$

and this quantity a is called the value of an annuity payable in advance for the life of a person aged x.

Again, if the value of an assurance of £1 payable on the death of a person x years of age is to be calculated, the course of

proceeding is exactly the same. Out of l_x persons alive at the age x, there die according to the mortality table, $d_{x+n+\frac{1}{2}}$ in the $\overline{n+1}|$th year, where n may be any number from to 0 to ∞. These receive each £1; and since it can be assumed that the deaths are uniformly distributed over the year, and that therefore the average time that each has lived since the age x is $n+\frac{1}{2}$ years, it follows that each £1 has the present value $v^{n+\frac{1}{2}}$. The present value of all the insurances to be paid is therefore

$$\Sigma d_{x+n+\frac{1}{2}}v^{n+\frac{1}{2}}=v^{-x}\Sigma d'_{x+n+\frac{1}{2}};$$

and if this value is to be paid by the l_x persons in equal shares, each will have to pay

$$\mathrm{A}=\frac{v^{-x}\Sigma d'_{x+n+\frac{1}{2}}}{l_x}=\frac{\Sigma d'_{x+n+\frac{1}{2}}}{l'_x},\ \left.\begin{array}{l}\text{from } n=0\\ \text{to} \quad n=\infty\end{array}\right\}. \quad . \quad (2)$$

This then is the value of an assurance of £1.

A man must be very much biassed to see in these calculations an application of the theory of probabilities. They are only calculations of averages, such as occur in the application of the arithmetic mean, it being assumed that whatever the number of persons of the same age alive at the outset, the number living at any subsequent age will be proportional to that in the table of mortality; or in symbols, if out of l_x persons alive at the age x, l_{x+n} are still alive after n years, then out of N persons of the same age x, there will be $\mathrm{N}\cdot\frac{l_{x+n}}{l_x}$ alive after n years. This is just the same thing as saying that out of different bodies of persons all of the same age, the numbers living after n years will always be proportional to the original numbers. It would be as correct to say that the theory of probabilities must be applied to decide as to the price of a ticket in a lottery, where the number of tickets is 1,000 and the sum to be divided among the winners is £10,000, in which case everyone will instantly fix the price at £10, without waiting for any calculation of probabilities.

In the same way as we have shown in these instances, we can calculate the values of all kinds of insurances, and the premiums for temporary, whole life, and reversionary annuities, whether the premiums are deferred, temporary, increasing or decreasing, without the least application of the theory of probabilities; and we have the further advantage of obtaining a much clearer view, inasmuch as all that is foreign to the calculation is avoided. For instance, if we wished to calculate the average duration of marriage or of

widowhood among N married couples in which the age of the husbands is x and that of the wives y, we should proceed as follows, making use of a mortality table for women which we will suppose gives the numbers living at the ages y, $y+1$, ... as λ_y, λ_{y+1}, ... After n years there are $N \cdot \frac{l_{x+n}}{l_x}$ of the husbands still alive, and there would be that number of married couples still existing, and $\frac{N}{l_x} \cdot (l_x - l_{x+n})$ widows, if none of the wives had also died in the same time. But since of λ_y women of the age y there are only λ_{y+n} alive after n years, therefore out of the $N \cdot \frac{l_{x+n}}{l_x}$ wives there are after n years $N \cdot \frac{l_{x+n}}{l_x} \cdot \frac{\lambda_{y+n}}{\lambda_y}$ alive. This is also the number of the marriages that are still subsisting; and the number of the surviving widows is reduced in the same proportion to $\frac{N}{l_x \lambda_y} \cdot (l_x - l_{x+n})\lambda_{y+n}$. Just in the same way we get the number of the widowers surviving after n years equal to $\frac{N}{l_x \lambda_y} \cdot (\lambda_y - \lambda_{y+n}) l_{x+n}$, and the number of cases in which both husband and wife are dead $\frac{N}{l_x \lambda_y} \cdot (l_x - l_{x+n})(\lambda_y - \lambda_{y+n})$. From this can be immediately deduced the calculation of the average duration of widowhood, and of joint life and reversionary annuities.

In consequence of the theory of insurances having been based on probabilities, there have arisen several ideas which belong to the latter, such as the probable durations of life, of marriage, and of widowhood, of which however, so far as I know, no practical application has ever been made. All that has been done is to calculate and tabulate their numerical values.

(*To be continued.*)

NOTICES OF NEW BOOKS.

The Life of John Heysham, M.D., and his Correspondence with Mr. Joshua Milne relative to the Carlisle Bills of Mortality. Edited by HENRY LONSDALE, M.D., o.ε.μ. London: Longmans, Green & Co., 1870.

Notwithstanding the interest of the biographical and other details relative to the distinguished author of the Carlisle Table of Mortality, we purpose, in our present notice, to confine our attention chiefly to the more technical portion of the book, as best suited to the pages of this *Journal.*

The main interest of the work centres in the origin and history of the Carlisle Table—a table probably still more in use by the Life Assurance Companies of England and America than any other single table whatever. "Before Dr. Farr constructed the English Life Tables, Professor De Morgan "pronounced the 'Carlisle Table' to be the best existing table of healthy "life in England. The Life Assurance Offices of the United States of "America seem to have endorsed this opinion and made the same table "applicable to their interests. And the magnitude of these interests at "home and across the Atlantic resting entirely on Dr. Heysham's opera-"tions is far beyond ordinary computation." We shall see in due course how nearly this old table agrees with those of more modern construction.

Dr. Heysham was born in 1753. After receiving a liberal education, he was apprenticed to a surgeon at Burton, and entered a school of medicine in Edinburgh, studying physic under Munro, Cullen, Black and others for three years, and obtaining his doctor's degree in 1777. In the following year he settled as a physician in Carlisle, where he resided until his death in 1834.

He is described as having devoted himself at a very early age to the study of Natural History, and especially of Ornithology. Anyone interested in this branch of Natural History will read with pleasure the highly amusing account of the birds of the district in which Dr. Heysham lived for many years, a full record of which will be found in Hutchinson's History of Cumberland. We may record here that Dr. Heysham aided the celebrated Dr. Paley in the anatomical and Natural History inquiries upon which the Natural Theology of the latter was founded.

Dr. Heysham is stated by his biographer to have been a handsome, strong, energetic and hard-working man. He lived well, indeed almost freely, as was the frequent custom of his day. He attained however the age of 80, transacting his magisterial business up to the day before his death.

At the present time, when smallpox is so prevalent in this country, it is interesting to note that Dr. Heysham was one of the first advocates of vaccination. In 1785 he was a firm believer in the virtues of inoculation—which was first performed in England in 1721—being influenced by his personal observation of the generally mild form of disease following that practice. But he "very early recognised the great import of Dr. Jenner's "notable observations, that persons who had become affected by cowpox "inoculation on the dairy farms in the south, either escaped smallpox "altogether, or had the disease in a greatly modified form. He proved "his trust in this, one of the greatest discoveries ever vouchsafed to the "followers of physic, by vaccinating his youngest daughter, Isabella, on "the 23rd October, 1800, when she was scarcely a month old. For a "good and obvious purpose he made this fact publicly known in Cumber-"land, and people listened to his commendations the more eagerly, that the "physician had experimented upon his own child, and with entire success. "He was the first to introduce vaccination to Carlisle, if not the north-"western district of England—at a time, too, when the practice stood in a "more or less doubtful position, both in London and the provinces. As he "joined heartily in the homage paid to Dr. Jenner, so he continued through "life to urge the use of vaccination as an almost infallible preventive of the "worst forms of smallpox. Lady Mary Montagu had done much by the

" introduction of inoculation or natural pox to lessen the virulence of the " disease. Jenner came to sweep it altogether away. And had the " opinions expressed in 1869, or thereabouts, by the late Sir J. Y. Simpson, " the greatest physician of our day, been acted upon, there is no doubt the " disease would have been stamped out from the British Isles, and for ever. " In proof of this, look to Ireland, where compulsory vaccination exists, " only one death from smallpox is reported as occurring in the whole of " that country during the quarter ending June, 1870."

Vaccination is now compulsory. The Act requires, under a penalty of 20s., the vaccination of every child born in England and Wales before it is three months old, unless a medical man shall certify its unfitness for the operation. Revaccination, however, which probably after a certain interval becomes equally necessary, could scarcely be made compulsory by law. The desirability of this practice is now fast becoming recognised, one revaccination after arriving at adult age being considered by the best medical authorities indispensable, and public opinion will, it may be hoped, bring about what the law would be powerless to effect.

The mortality at the present time among smallpox patients in this country is stated in a document published by the Royal College of Physicians to be about 35 per cent. among those that have not been vaccinated, and less than 1 per cent. in the class that have been properly vaccinated.

Dr. Edward Jenner, when residing at Sudbury, in or about the year 1796, found that a popular notion existed that milkers, who had been infected with a peculiar eruption, communicated from the udder of the cow, were proof against smallpox. He ascertained, after much labour, that the cow was subject to a variety of eruptions, only one of which, the true cowpox, had this protective power, and that this disease could be effectually communicated to the milkers at one particular period of its course only. The first case of vaccination was that of a boy aged eight years, who was on the 14th May, 1796, vaccinated with matter taken from the hands of a milkmaid—he took the cowpox, and was, on the 1st of July following, inoculated for smallpox, without the least effect.

Dr. Heysham appears to have been greatly devoted to the promotion of sanitary matters in Carlisle. He was the founder of a public Dispensary, and assisted in the establishment of a house of recovery or Fever-house. In 1782 he published an account of Jail-fever, or *Typhus Carcerum*, as it appeared in Carlisle in 1781, which he treated successfully by the free use of port wine and bark in large doses in the earlier stages of the disease, then a novelty in medical practice. Though probably not the first to advocate the use of stimulants in the treatment of fever, Dr. Heysham prescribed much larger doses, both of wine and bark, than most other physicians of his day, and evidently with great success. He ascertained that the " predisposing causes were poor diet, uncleanliness, intemperance, " mental depression; and these he endeavoured to counteract." His mode of treatment of typhus and other forms of fever, does not appear strange in the present day, when "alchoholism" is probably overdone; but he must have been a bold practitioner to have originated a system so opposed to the bleeding and depletive system, then so much in vogue.

Next to his strictly professional pursuits Dr. Heysham's chief interest seems to have been the subject of vital statistics, and to this accidental

circumstance we owe the construction by the late Mr. Joshua Milne, the Actuary of the 'Sun' Life Assurance Company, of the celebrated Carlisle Table of Mortality, which is not only still in use by many of the existing Life Assurance Companies, but which seems to agree to a remarkable extent with other more modern tables. "The accuracy which guided "Heysham's observations on the rate of mortality in Carlisle proved of the "highest significance to the general community of Britain, and, it may be "added, to the world at large. Wherever the question of life assurance "has been discussed throughout the civilised globe, Heysham's labours "have been recognised and extolled for their meritorious application and "usefulness. For, it need hardly be said that the Life or Mortality Table "is the basis upon which the whole science of life assurance rests; it is "essential to the vital statistician as the barometer is to the meteorologist, "the balance to the physicist, and the test tube to the chemist. In explanation of this agreement Carlisle is stated to have offered a sanitary "condition reflective of England at large. As a city of middle size—the "population of its two parishes being five-sixths urban and one-sixth "rural—its trading interests demanding a large proportion of operatives; "its crowded lanes, like larger towns, favouring the spread of epidemics, "small-pox, fever; and lastly, but probably first in significance, its large "infantile population. These circumstances conspired to make Carlisle a "fair epitome of the borough and landed interest, *quoad* the general health "of the kingdom. . . . . . The value of Dr. Heysham's work in "Carlisle can be best instanced by the serious errors that sprang from "incorrect observation and records made elsewhere. When Dr. Price was "constructing the Northampton Tables, a great number of Baptists lived "in the town who did not sanction infant baptism, and thereby reducing "the ratio of the christenings to the births, led Dr. Price to believe that "the population of Northampton was stationary. The average lifetime in "Northampton was in reality about 30 years, but Dr. Price, overlooking "the Baptists, assumed it to be only 24. It is *now* 37½, or 13 years "(*one-third*) more than he took it to be! 'and as a curious confirmation "'of the error, the mortality of the Equitable Society (which first used "'the Northampton Table) was *one-third less than that Table predicted.* "'But the most serious part of the business remains yet to be told. The "'Government adopted these tables as the basis of its annuity schemes. "'The same error which gave the Equitable and other societies using the "'Table *one-third too much premium* induced the Government to grant "'annuities by *one-third too large* for the price charged, and before the "'error was rectified, about two millions of money were lost to the country "'by these annuity transactions.'" It is needless to point out that this passage was not written by an actuary. Mr. Milne was of opinion "that "although the Carlisle Table had been constructed from the mortality of "two parishes only, the results it exhibited would probably vary very "little 'from the general' law that obtains throughout the kingdom, taking "'town and country together, if we except the children under five years "'of age, or at most under ten.'" An opinion which has been completely corroborated.

Dr. Heysham, shortly after taking up his residence in Carlisle, determined to make a census of the inhabitants and to compile Bills of Mortality of the city. "He prefaced his observations on the advantages to be

" derived from accurate registers of mortality by the following passages " from Percival's *Essays*, vol. ii.:—' The establishment of a judicious and " ' accurate register of the births and burials in every town and parish " ' would be attended with the most important advantages, medical, " ' political, and moral. By such an institution the increase or decrease of " ' certain diseases, the comparative healthiness of different situations, " ' climates, and seasons; the influence of particular trades and manufac- " ' tures on longevity; with many other curious circumstances, not more " ' interesting to physicians than beneficial to mankind, would be ascer- " ' tained with tolerable precision. In a political point of view, exact " ' registers of human mortality are of still greater consequence, as the " ' number of people and progress of population in the kingdom, may, in " ' the most easy and unexceptionable manner, be deduced from them. " ' They are the foundation, likewise, of all calculations concerning the " ' value of assurances on lives, reversionary payments, and of every " ' scheme for providing annuities for widows and persons in old age. In " ' a moral light such *Tables* are of evident utility, as the increase of vice " ' or virtue may be determined by observing the proportion which the " ' diseases arising from luxury, intemperance, and other similar causes, " ' bear to the rest; and in what particular places distempers of this class " ' are found to be most fatal.' . . . . Early in 1779 Dr. Heysham " commenced his statistical observations, and set earnestly to work, from " week to week and month to month, to record the births, marriages, the " diseases, and deaths of the inhabitants of Carlisle." The city contained two parishes, St. Mary's and St. Cuthbert's; those parishes including the city, its suburbs, and certain outlying districts. " In January, 1780, a " very careful and accurate survey was made by Mr. Stanger and Mr. " Howard, under Dr. Heysham's own inspection, when there were found in " the district before surveyed, 891 houses, 1,605 families, and 6,299 " inhabitants." The total number of inhabitants of both sexes under observation was 7,677. He next summarised the single and married persons as follows:—

Total of persons who are or have been married . .	3,073
Total of persons who were never married . . .	4,604
Total	7,677

" The astonishing increase of 2,141 inhabitants, above half of the " original number in the city and suburbs, in the small space of 17 years " (1763–80), Heysham attributed to the establishment of calico-printing " manufactories. There were four companies, who daily employed about " 800 industrious poor in this kind of work. ' This increase of popula- " ' tion,' Dr. Heysham remarked, ' had taken place during that very period " ' in which Dr. Price asserts the depopulation of Great Britain to have " ' been rapid and progressive. And what makes this increase more " ' remarkable, Carlisle has, during the whole period alluded to, been con- " ' stantly supplying the army, the navy, the metropolis, and even the " ' distant regions of India, with her hardy, active and enterprising sons.' "

He then showed the ages at death of 258 persons, and the causes of their deaths—classified under Cullen's *Genera Morborum*, in the year

1779. Another table shows the total of deaths in each season of the year, viz:—

In Winter	33
,, Spring	34
,, Summer	75
,, Autumn	116
Total	258

From these tables he was enabled to compare the mortality of Carlisle with that of other towns of England and the continent, and he showed that " 'in Vienna, about 1 in $19\frac{1}{2}$ of the inhabitants die annually; in London, " '1 in $20\frac{3}{4}$; in Edinburgh, 1 in $20\frac{4}{5}$; in Leeds, 1 in $21\frac{3}{5}$; in Dublin, " '1 in 22; in Rome, 1 in 23; in Amsterdam, 1 in 24; in Breslau, 1 " 'in 25; in Berlin, 1 in $26\frac{1}{2}$; in Northampton and Shrewsbury, 1 in " '$26\frac{1}{2}$; in Liverpool, 1 in $27\frac{1}{2}$; in Manchester, 1 in 28; in Chester, 1 " 'in 40; but in the year 1774, when the small-pox was very general " 'and fatal, 1 in 27; whilst in Carlisle the mortality was in 1779, 1 in " '$30\frac{3}{5}$.' " The extreme salubrity therefore of Carlisle was made apparent in what happened to be a very unhealthy year, no less than 129 persons having died from two epidemic diseases, viz., small-pox and scarlet fever. These tables further showed " that women live longer than men. During " 1779, twelve persons died between 80 and 90 years old, eight of these " were females, and the two persons who lived between 90 and 100 were " both females. There were four widows to one widower, an astonishing " disproportion, which he endeavoured to account for on the following " grounds:—1st. Men are in general more intemperate than women; " 2nd. They are exposed to greater hardships and dangers; 3rd. " Widowers perhaps in general have greater opportunities of getting wives " than widows have of getting husbands; 4th. Widows have a greater " propensity to live in towns than widowers; 5th. Women in general, " and more especially among the middle ranks of life, marry earlier than " men; 6th. That as men have 'firmer and more robust' constitutions " than women, their 'muscular and nervous fibres' sooner become stiff and " rigid, and less able to meet the 'functions necessary to health and life.'" These five tables were compiled by Dr. Heysham to illustrate the bills of mortality for 1779. He adopted the same forms in subsequent years up to the end of December, 1787.

Dr. Heysham found from a summation of the nine years that 1 in $39\frac{1}{4}$ of the inhabitants died annually. It is important to observe that " every " infectious and epidemic disease, to which the human body is subject (the " plague excepted) prevailed during this period in Carlisle."

The highly interesting correspondence in the Appendix between Dr. Heysham and Mr. Milne originated with the latter in 1812. Mr. Milne was at that time engaged in inquiries relative to human mortality, and had constructed a table based upon Dr. Heysham's observations, which showed that the inhabitants of Carlisle surpassed in longevity those of any other place for which a similar table had then been constructed. In order to make his table as correct as possible, Mr. Milne entered into a long correspondence with Dr. Heysham, discussing with him many minute points, and evidencing the extreme care exercised by both in order to reconcile apparent

discrepancies, and to ensure the greatest accuracy in the smallest details. We may mention particularly the following, viz.:—

The non-entry among the births of Quakers, Roman Catholics, and Dissenters;

The effect of immigration;

The influence of vaccination;

The value of nosological tables;

The condition of the people as regards prices, and wages, and sanitary matters, in relation to mortality; and lastly,

The particular circumstances connected with the town of Carlisle at the period during which the observations were made, which rendered it, as previously stated, peculiarly suitable for the purpose in question.

To show the remarkable agreement of the Carlisle Tables, based on observations commenced in 1779 upon a very limited number of lives, with other tables of more recent construction, founded on records of a later date and of far more extensive character, we extract the following figures from a complete table of expectation of life contained in the last edition of the *Insurance Register*, by Mr. William White—a useful record of life assurance statistics.

Expectation of Life.

Completed Age.	Northampton Experience.	Carlisle Experience.	Equitable Society's Experience.	Actuaries' Combined Experience.	Actuaries' New Combined Experience	English (or National) Experience No. 3 (Males).	American Experience.	Completed Age.
	1780.	1815.	1834.	1843.	1869.	1864.	1867.	
	Years.	Years.	Years.	Years.	Years.	Years.	Years.	
5	40·84	51·25			53·83	49·71		5
10	39·78	48·82	48·83	48·36	49·89	47·05	48·72	10
15	36 51	45·00	44·81	44 96	45·90	43·18	45·50	15
20	33·43	41·46	41·06	41·49	41·98	39·48	42·20	20
25	30·85	37·86	37·44	37·98	38·44	36·12	38·81	25
30	28·27	34·34	33·98	34·43	34·75	32·76	35·33	30
35	25·68	31·00	30·66	30·87	31·15	29·40	31·78	35
40	23·08	27·61	27·40	27·28	27·57	26·06	28·18	40
45	20 52	24·46	24·10	23 69	23·98	22·76	24·54	45
50	17·99	21·11	20·83	20·18	20·51	19·54	20·91	50
55	15·58	17·58	17·85	16·86	17·14	16·45	17·40	55
60	13·21	14·34	15·06	13·77	13·99	13·53	14·09	60
65	10·88	11·79	12·35	10·97	11·17	10·82	11·10	65
70	8·60	9·18	9·84	8·54	8·68	8·45	8·48	70
75	6·54	7·01	7·52	6·48	6·56	6·49	6·28	75
80	4·75	5·51	5·38	4·78	4·93	4·93	4·38	80
85	3·37	4·12	3·73	3·36	3·58	3·73	2·77	85
90	2·41	3·28	2·65	2·11	2·68	2·84	1·42	90
95	·75	3·53	1·05	1·12	1·90	2·17	·50	95

The extreme accuracy of Mr. Milne's Tables has been strikingly shown by the calculations of Mr. Benjamin Gompertz, F.R.S., the results of which were published in the *Philosophical Transactions* of the year 1825.

The correspondence concludes with a "case" submitted by Dr. Heysham to Mr. Milne for his opinion as an actuary. Dr. Heysham held the lease of an estate dependent upon three lives, aged in 1814, 22, 21, and 17 respectively, renewable on the failure of each life for ever on pay-

ment of a fine of 20s. Mr. Milne, according to the Carlisle Table, taking interest at 5 per cent., estimated the present value of the fines at 14s. 1d., or, according to the Northampton Table, with the same rate of interest, at £1. 1s. 10½d. The same case was submitted to Mr. William Morgan, the then actuary of the "Equitable," who valued the fines at £2. 4s. These results varying so widely, we have tested the computations by the formula given by Mr. Milne in his treatise, and found them quite correct. The Northampton Table, with interest at 3 per cent., produces Mr. Morgan's result exactly.

As. Mr. Milne points out, "This is one of the cases wherein the "difference in the law of mortality makes a greater difference in the value "sought than perhaps any other." We refer particularly to this to show the difficulties against which, in the earlier days of life assurance, actuaries, all working independently of one another, had to contend. Here are two actuaries, both of the highest eminence, who, in a practical question of not unfrequent occurrence differ, even supposing they were agreed upon the mortality table to be used—which they were not—exactly 100 per cent. in their results; while between the Carlisle 5 per cent. computation of the one, and the Northampton 3 per cent. of the other, the difference is more than 214 per cent. The actuarial profession owes a deep debt of gratitude to the Institute of Actuaries, for, through the freer intercourse that now prevails among actuaries, in consequence of its establishment, and perhaps still more through the publication of this journal, some sort of agreement has necessarily been arrived at, which renders it now impossible, or at least highly improbable, that such discordant opinions as those referred to should go forth to the public, to the manifest discredit of actuarial science.

Mr. Milne's treatise *On the Valuation of Annuities and Assurances, and on the Construction of Tables of Mortality*, is even now one of the best works on these subjects, and is, we believe, as highly thought of by the present generation of actuaries as it was at the period of its issue in 1815 by Dr. Heysham and other eminent men of that day. Mr. Milne, in one of his letters to Dr. Heysham, dated 2nd May, 1815, says, in reference to the work in question, "If it meet with the reception of the "public, which these and the opinions of some other distinguished judges "would lead me to hope for, it must be obvious to every reader that it "will be owing in a great measure to your own valuable observations." The effect of what Carlyle has termed "the attrition of intellect" in the case of these two men, Dr. Heysham and Mr. Milne, each on the other, is apparent in a remarkable degree throughout this interesting correspondence.

We cannot conclude this notice without paying a just tribute to the editor, Dr. Henry Lonsdale, of Carlisle, who has succeeded in producing a very interesting and instructive volume. The mystic letters *ο. ε. μ.* following his name in the title page show that he was a member of the Brotherhood established in Edinburgh in 1835 for mutual assistance and encouragement, of which order Professor Edward Forbes was one of the chief promoters. It was termed the Oineromathic Society. The words ΟΙΝΟΣ, ΕΡΩΣ, ΜΑΘΗΣΙΣ (wine, love, learning) were the motto of the brotherhood, which consisted of artists, scholars, physicians, naturalists, poets, priests and mathematicians. Interesting accounts of this society will be found in Dr. George Wilson's "Memoir" of Professor Forbes, F.R.S., and in the "Anatomical Memoirs" of Professor Goodsir, F.R.S., by the author of the present biography.

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On the Liquidation and Reconstruction of an Insolvent Life Insurance Company. By T. B. SPRAGUE, M.A., *Vice-President of the Institute of Actuaries.*

THE first question that must be considered in connection with this subject is,—When is a Life Insurance Company insolvent? This question has recently acquired greater practical importance in consequence of the passing of the "Life Assurance Companies Act, 1870," by which it is for the first time in effect enacted that an insolvent Life Office may be wound up, altho it has not committed any act of bankruptcy. Under the old law, even if such a Company were notoriously insolvent, it may be said that practically there was no means of putting a stop to its operations until it failed to pay an accrued claim. This has now been altered, and a Company that is proved to be insolvent can be wound up. The 21st section of the above Act provides that the Court of Chancery "may order "the winding up of any Company in accordance with the Companies "Act, 1862, on the application of one or more policyholders or "shareholders, upon its being proved to the satisfaction of the "Court that the Company is insolvent, and in determining whether "or not the Company is insolvent the Court shall take into account "its contingent or prospective liability under policies and annuity "and other existing contracts." It will be noticed, however, that the Act has laid down no standard by which the solvency or insol-

vency of the Company is to be decided. How, then, is the Court to decide on this important point, so as to carry into effect the above provision? Suppose a petition presented for winding up a Company on the ground that it is insolvent; and that the petitioners have produced their actuaries, who state that in their opinion the Company is insolvent and unable to meet its liabilities. It may be taken for granted that the Company will also produce actuaries, who will state with equal positiveness, that in their opinion the Company is perfectly solvent, and will duly discharge its liabilities as they arise; and the circumstances of the Company may be such, that is to say, it may be so nearly bordering on insolvency, that the actuaries produced on the two sides may be equal in standing and authority, and the arguments adduced for and against the solvency of the Company may be so nearly balanced, that a very slight allowance on one side or the other will turn the scale. It is not easy to see a way out of this difficulty except by further legislation, which shall fix the legal standard of solvency.

This is what has been done in America. It is to be noticed, however, that since life insurance legislation in the United States is held to be within the province of the several State Governments, and not of the Central Government, each of the States that chooses to place Life Offices under restrictions must fix its own standard of solvency. The standard fixed by the State of Massachusetts is that the Office must possess the reserve shown by a net-premium valuation by the Experience Table of Mortality at 4 percent interest; and of this standard it may be said with safety that altho every Company which comes up to it is certainly solvent, still a Company may fall very far short of it without becoming insolvent. The reserve required at the present time in the State of New York is that brought out by a valuation by the American Table of Mortality with 4½ percent interest. The reserve required in Massachusetts is thus about 10 percent greater than that required in New York; so that it may very easily happen that a particular Company shall possess the amount of assets required by the standard of New York, but fall short of the standard of Massachusetts. According to the present state of things, therefore, a Company might be declared solvent by the Superintendent of the State of New York, and at the same time be declared insolvent by the Commissioner of Massachusetts; and unless some change is made, we shall no doubt in the course of a few years have instances of this kind actually occurring, so that one or more Life Insurance Companies will be declared insolvent in Massachusetts and compelled to

discontinue business in that State, while no objection is made to their continuing to do business in the State of New York.

It must not be forgotten that if a legal standard of solvency were established, there would be some tendency on the part of the managers of weak Companies to think that so long as they come up to that standard they may be satisfied; but this would be partly or wholly counteracted, if it were clearly understood that a Company falling below the standard, would be forthwith wound up, unless the deficiency were supplied by the shareholders.

If now we assume that it is considered on the whole desirable to fix a legal standard of solvency, such for example as that to be presently explained, I think there would be no possible objection to the appointment of a public officer who should be charged with the duty of making valuations of all the Life Offices by the legal datums; and who should publish an annual report showing how much each Office exceeds or falls below the legal standard. His certificate would of course be received by the Court of Chancery as evidence of insolvency, in a petition for winding up an Office; but would no doubt be liable to be set aside if the Company could prove error on his part. Such a public officer should of course have full right to verify the returns made by each Company by comparison with their books, and should be authorized to examine the officials of the Company on oath. He should, however, have none of the penal powers conferred on the American Commissioners, his function being restricted to informing the public.

The New York and Massachusetts standards above mentioned have the defect in common, that they take no account whatever of the magnitude of the premiums charged by the Companies, being both based upon what is technically called the net-premium method of valuation. For a full consideration of the objections to that method of valuation, I beg leave to refer to my paper, read before the Institute of Actuaries in their last session. (See *Journal,* vol. xv p. 411.) On the present occasion, I will only observe, that when the question is merely as to the solvency of a Life Insurance Company, no method can be expected to give satisfactory results which does not take into consideration the actual premiums receivable by it. In fact, it is easy to understand that a Company charging high premiums may be perfectly solvent and able to meet its liabilities, while another Company with the same sum assured, and the same amount of invested assets, but charging low premiums, may be quite insolvent.

When the question is one simply of solvency, there can be

no doubt, I think, that a Company must be held solvent, provided it has sufficient funds to pay the bare sums assured as they become payable by the death of the lives assured; so that altho the Company may have charged high premiums in consideration of promises made or expectations held out to the assured of large bonuses, it must be considered, in making an estimate as to its solvency, that it has received those premiums for the bare sums assured. On the other hand, any commission allowed by the Company to its agents, or to solicitors and other persons, ought to be deducted, so that we give the Company credit only for the smaller amount of premiums it virtually receives. At the same time, a proper provision must be made for the future expenses of conducting the business. Bearing then in mind these requirements, I think the most suitable method of valuing policies will be a modification of that method which I have termed the "hypothetical." It will be necessary to fix, first, on a table of premiums, which should be so calculated as to allow of the payment of no commission to agents, and only a very moderate expense in conducting the business; and secondly, on a rate of interest on which the calculations are to be based. Then a very simple calculation shows how much of the sum assured is provided for by the future premiums on the policy, reduced by any commission payable; and the value of the liability of the Office under the policy is such a sum as, being applied by way of single premium, will insure the remaining part of the sum assured and bonus.

Thus, to take a very simple instance, suppose that a person paying a premium of £30 a year, which may be either the original premium or one permanently reduced by bonus, has a policy for £1,000, to which a bonus of £100 has been added; and that according to the scale of premiums adopted, the premium at his present age would be £4 per £100; then we see that the £30 a year which he pays will insure £750, and deducting this from £1,100, we have a sum of £350 to be otherwise provided for. The value of the policy will then be the single premium that would be required for insuring £350 at his present age. The rate of single premium can be deduced from the annual premium in the following way, it being understood that it would not be equitable to use a net (or unloaded) single premium for the purpose of calculating the value of the policy.

Let S be the sum assured together with any bonus that has been added, and P_x the premium on the policy, after deduction of the commission, as stated above. Also suppose that the present age of the life assured is $x+n$, x being the age at which the policy was

effected, and n the number of years which have since elapsed; and that ϖ_{x+n} is the premium to insure 1 at the present age of the life assured. Then the sum which can be insured for the premium payable is $\frac{P_x}{\varpi_{x+n}}$; and the portion of the sum assured which remains to be provided for, is $S-\frac{P_x}{\varpi_{x+n}}=\frac{S\varpi_{x+n}-P_x}{\varpi_{x+n}}$; and we have to find the single premium for which this can be insured. In order to do this, we consider what sum can be insured for a single premium of 1. Now i being the yearly interest on 1, and v the present value of 1 payable in a year, a single premium of 1 will produce a return of i at the end of each year; hence a single premium of $1+i$ will produce a return of i payable yearly in advance; and a single premium of 1 will produce an annual return of $\frac{i}{1+i}$, or $1-v$, payable yearly in advance, and the amount which can be insured on a life of $x+n$ for that annual sum of $1-v$, is $\frac{1-v}{\varpi_{x+n}}$. Hence the sum which can be insured for 1 paid as a single premium is $1+\frac{1-v}{\varpi_{x+n}}$ or $\frac{\varpi_{x+n}+1-v}{\varpi_{x+n}}$; from which it follows that the single premium to insure 1 is $\frac{\varpi_{x+n}}{\varpi_{x+n}+1-v}$. Combining this with our previous conclusion, we find that the value of the policy, or the single premium required, is

$$\frac{S\varpi_{x+n}-P_x}{\varpi_{x+n}}\cdot\frac{\varpi_{x+n}}{\varpi_{x+n}+1-v}=\frac{S\varpi_{x+n}-P_x}{\varpi_{x+n}+1-v}.$$

This value may also be obtained more directly. Thus, let V be the value of the policy required; then V, together with its interest, and together with the premium P_x, ought to insure the sum S; or the interest on V, together with P_x, ought to insure $S-V$. But V will produce the sum of $(1-v)V$ payable annually in advance, so that we have the annual premium in advance of $(1-v)V+P_x$, and this is to be equal to the annual premium on $S-V$; whence

$$(1-v)V+P_x=\varpi_{x+n}(S-V)$$

and
$$V=\frac{\varpi_{x+n}S-P_x}{\varpi_{x+n}+1-v}$$

—the same result as before.

This is the same result as we should obtain if by an inverse process we deduced a table of annuities from the premiums charged,

and valued the actual sum assured and the actual premium (less commission) by those hypothetical annuity values. In fact, ϖ_{x+n} being the annual premium, the value of the hypothetical annuity-due deduced from it is $\dfrac{1}{\varpi_{x+n}+1-v}$; and the value of the hypothetical single premium is $\dfrac{\varpi_{x+n}}{\varpi_{x+n}+1-v}$.

Now writing the above value of V in the form

$$\frac{\varpi_{x+n}}{\varpi_{x+n}+1-v}\cdot S-\frac{1}{\varpi_{x+n}+1-v}\cdot P_x$$

we see that it is equal to

hypoth. single premium $\times$ S $-$ hypoth. annuity-due $\times\ P_x$.

Again, putting the value of V into the form

$$\frac{1}{\varpi_{x+n}+1-v}\times(S\varpi_{x+n}-P_x)$$

and observing that $S\varpi_{x+n}$ is the annual premium required to insure S according to the table of premiums used in the valuation, we see that the value of the policy is the value by the hypothetical table of annuities, of the difference between the premium paid (less commission) and that required to insure the same total sum according to the premiums used in the valuation.

To fix on a scale of premiums in the way here suggested appears very much preferable to prescribing a table of mortality; inasmuch as when the table of mortality has been chosen, there still remains the important question of how the premiums ought to be loaded, as to which great difference of opinion exists.*

The method of valuation here suggested entirely meets the difficulty that has been felt in America and on the continent, as to the proper mode of making allowance in the value of a policy when

* As stated in my recent paper "On the proper method of estimating the liability of a Life Insurance Company under its policies," I have been led by subsequent reflection to modify considerably the views which I formerly held (see *Journal*, vol. xi) as to the respective merits of the hypothetical and net-premium methods of valuation. In particular, my remarks on the net-premium method contained on p. 102 are to be considered as superseded by those contained in my paper above referred to. On the other hand, the objections to the hypothetical method of valuation on the top of p. 103 do not seem to me to be entitled to so much weight as I formerly attributed to them. It is undeniable that in some cases, such as in the transfer of policies from one Company to another, the premium actually payable is an important element in the determination of the value of the policy, and the net-premium method of valuation is quite inapplicable.

I may here add that the arguments advanced by Mr. Tucker in his well-known paper (see this *Journal*, vol. x p. 312) in recommendation of the hypothetical, or as he calls it, the reinsurance method of valuation, are not in my opinion the most convincing that can be found. The above explanation of its operation, which was suggested to me by a remark made by Mr. R. P. Hardy, the Actuary and Secretary of the London and Provincial Law Office, appears to place it in a new and more favourable light.

the commission payable thereon has been either wholly or partly commuted by a single payment. If the premium is freed from commission, the full amount receivable will be valued, and the value of the policy will be consequently less.

Of course, in employing the above method in any case to value the policies of a Company, the greatest vigilance must be used to see that all policies in which its application would bring out a negative value, *i.e.*, in which $P_x > S\varpi_{x+n}$, are omitted and considered as of no value.

I have thus shown how the "amount of the estimated liabilities" mentioned in the 21st section of the Act may properly be ascertained. It may now be useful to consider how the 22nd section of the Act may best be carried into effect, which provides that the Court may reduce the amount of the contracts in place of making a winding-up order. The object aimed at by this section appears to be the very proper one, that the insolvency of a Company shall not necessitate the termination of the contracts. Under the present process of winding up, all contracts under policies are considered as broken by the insolvency of the Company which has entered into them, and each policyholder is left without any other remedy than his claim against the Company for damages for breach of contract. This course in practice gives rise to very great, in fact, insuperable difficulties. If the life assured is still in good health (in a word, an insurable life) it is easy for him to ascertain upon what terms another Company will take up the broken contract; and the same is true in the not uncommon case where the health of the life assured has become impaired, so that an extra premium has to be paid in effecting a new insurance. But among a large number of persons whose lives have been insured for many years, there will always be some who have become uninsurable, in consequence of the existence of some chronic disease which is morally certain to shorten life, but is not attended with immediate risk; and there is absolutely no means whatever for estimating in a correct manner the loss to the policyholder in that case. If, however, the contracts are continued in force, subject only to such a reduction of the sums assured as may be necessary, this difficulty does not arise.

It may further be said that the contracts of the assured were for a certain amount of assurance; and if the Company is unable to perform its contracts in full, it is more in accordance with reason and justice that the composition to be paid by the Company to its creditors, the policyholders, should be a reduced amount of assurance, than that they should receive a cash payment which was not contemplated in the contract.

The 22nd section of the Act runs as follows:—"The Court, in "the case of a Company which has been proved to be insolvent, "may, if it thinks fit, reduce the amount of the contracts of the "Company upon such terms and subject to such conditions as the "Court thinks just, in place of making a winding-up order." Now it will be seen on consideration that this section does not provide for the cases that may arise in practice. It appears to have been thought by the framers of the Act, that the winding up of a Life Insurance Company must of necessity involve the termination of all the insurance contracts, by putting an end to the payment of all premiums on the one side and of all claims on the other side. But this is clearly not the case; for altho the Company may be wound up as far as concerns the shareholders, the policyholders may continue to pay their premiums into a common fund. For this purpose, no aid is required from liquidaters or the Court of Chancery. In the worst case, where there should be no existing or recoverable assets available to meet the claims of the policyholders, the premiums they are paying would suffice, as we have seen above, to insure a certain portion of their several sums assured; and their policies might be reduced, if necessary, to those portions. The winding up of the Company thus only necessitates the finding of a new machinery for the receipt of premiums, investment of funds, and payment of claims; and the two operations, of winding up the Company, and of reducing the amounts of the contracts, which are opposed to each other in the Act, are by no means inconsistent with each other, but may coexist.

On the other hand, if a proprietary Life Insurance Company is proved to the satisfaction of the judge to be insolvent: that is to say, if the realized assets, inclusive of the shareholders' paid-up capital, falls short of the estimated liability of the Company, it would be quite contrary to justice that the amounts of the contracts should be reduced before the shareholders had paid up the full sums for which they are liable. In the case supposed, if the shareholders, being liable and presumably able to make good the deficiency, do not voluntarily take any steps for that purpose, it appears to me that a winding-up order is essential in order to compel them to contribute rateably towards making good the deficiency. It is true there is a course short of this, namely, that the Court should order a call to be made on the shareholders, of sufficient magnitude to make good the deficiency; but whether the Court has the power to do this, or, having the power, would consider it a preferable course, I am unable to say. I assume

then that in the case supposed a winding-up order is made. There are then two distinct cases which may occur. Either the shareholders may under pressure make good the deficiency, or they may be unable to do so. In both cases the winding up will terminate the contract between the shareholders and the policyholders, giving the former a final discharge from their liabilities. But in neither case does it appear to be a necessary result of winding-up that the funds realized shall be divided among the policyholders in proportion to their several interests; and in both cases, for the reasons above stated, such a course appears contrary to the justice of the case. If under the winding up the deficiency in the assets is made good, there will be a fund which is the property of the policyholders collectively, and which, being invested, will, with the future premiums, suffice to pay all the claims under the policies. In this case, instead of dividing the fund, as would now be done under a winding-up, it seems clearly to meet the justice of the case better, either to enter into a contract with some other Company to adopt the policies in consideration of receiving the realized assets and the future premiums, or to constitute the policyholders into a Mutual Insurance Society, with the terms and conditions of all their policies unaltered; and in the latter case they might either appoint their own officers to receive premiums, pay claims, etc., or they might arrange with some other Office to administer the business. If, on the other hand, the winding-up should not produce a sufficient sum to make good the deficiency, so that the assets are insufficient to pay the claims in full, in this case, too, it will be inequitable to divide the fund, and the proper course will be to reduce the contracts to such an amount as the assets will provide for.

I now proceed to show how this may be done with due regard to the rights of all the policyholders. In the first place, we must throw off from the premiums actually payable the additional sums paid by the participating policyholders for the purpose of entitling them to a share in the profits. These sums, being the difference between the participating and non-participating premiums for the sums assured under their policies, may be called *the overpayments* of the participating policyholders, and must be considered as sums staked by them on the chance of getting a bonus; and the speculation having thus far turned out unfavourably, we must consider the stakes as forfeited (or, more strictly, impounded), and give each participating policyholder credit for paying only such a premium as he would have paid for a non-participating policy of the same amount. We must next fix on a table of premiums to

be used in the valuation, as explained in the earlier part of this paper. These should be based on a table of mortality and a rate of interest such as appear most likely to agree with the future experience of the Society; and should be loaded only to the extent necessary to provide for the expenses of conducting the business. We have then to calculate for each policy the sum which the premium actually payable (reduced in the case of the participating policyholders, as already explained) will insure at the present age of the life assured under the new assumed table of premiums.

Nothing is to be deducted from the premiums for commission. Of course, as the original Company has been wound up and its business put an end to, all claims for payment of commission will be extinguished. In fact, the commission must be considered as a trade expense of the old Company, which the new Society is as little liable for as for any other of the expenses. Altho, therefore, in estimating the solvency of the original Company, we deduct from the premiums receivable the commission allowed on them, there must be no such deduction made in the case of the new Society; and it might thus possibly happen, that tho the original Company was proved to be insolvent, yet the new Society receiving its assets, might be in a position to pay all claims in full as they arise.

We have thus, then, found that according to the scale of premiums adopted, the actual (or modified) premium payable by each policyholder will insure a certain part of his sum assured, viz. $\frac{P_x}{\varpi_{x+n}}$, using the notation of the earlier part of this paper. (It must be borne in mind, however, that P_x now has a rather different meaning, as it represents in the case of a non-participating policyholder, the full premium without deduction for commission; and in the case of a participating policyholder, the full premium he would have paid according to the non-participating scale of premiums.) This part of the sum assured there can be no question about. The future payments of the policyholder must be considered as fully providing for it, whatever may be his state of health; and the average deterioration in the health of the lives assured, must be taken into account in fixing on the scale of premiums to form the basis of the new Society's operations. Any reduction must fall exclusively upon the remaining part of the sum assured, that part, namely, which his past payments have paid for. If then, by the method described above, we calculate the liability in respect of these already-paid-for portions of the sums assured, or the sum

which the new Society should have in hand, in order to be able to pay the claims in full as they arise, and find that it exceeds the amount of the available assets, including those raised under the winding-up, justice requires that these portions of the sums assured shall be all rateably reduced, so that the liability under the reduced sums shall be exactly equal to the amount of the available assets. This reduced sum being added to the sum that can be insured at the present age, as already found, we get the amount of the new policy in each case.

Thus, by the formula investigated above, the value of the policies, or the sum which the Office should have in hand, is

$$\Sigma \frac{\mathrm{S}\varpi_{x+n}-\mathrm{P}_x}{\varpi_{x+n}+1-v},$$

which, for brevity, we will denote by Σ simply; and if the amount of the assets $=k\Sigma$, where $k<1$, then the general formula for the reduced sum assured will be

$$\frac{\mathrm{P}_x}{\varpi_{x+n}}+k\frac{\mathrm{S}\varpi_{x+n}-\mathrm{P}_x}{\varpi_{x+n}},$$

or

$$k\mathrm{S}+(1-k)\frac{\mathrm{P}_x}{\varpi_{x+n}}.$$

The case of the policyholders who have been insured only a few years requires special consideration. In this case, the premium payable will be more than sufficient to insure the whole of the sum assured according to the scale of premiums adopted, and it follows at once that these policies should not be reduced at all. But ought they not to be increased? As regards the non-participating policyholder, we may say that if he is paid in full, he gets all he ever expected, and he cannot properly advance any claim for an increase of his sum assured. As regards the participating policyholders, again, it is clear that their contract was that they might expect to receive an increase of their benefits, but only contingently on the Company realizing a profit; so that they can claim no immediate increase of their sums assured, but must wait until the subsequent operations of the Society show a realized surplus. In the case, therefore, of recent policies where the payments of the assured are more than would be required according to the scale of premiums adopted, the policyholders, whether participating or non-participating, will according to this view rank for the full sum assured, neither more, nor less. But considering the circumstances of the case more closely, and bearing in mind particularly that it may be said the premiums charged by the Office are reduced under the new arrange-

ment, it seems politic, even if not required by justice, to allow the sums assured under the recent policies to be increased to such amounts as could, according to the new scale of premiums, be insured at the present ages for the premiums payable. But if this more liberal view be taken, it is clear that strict proof of present good health may properly be required in order to entitle the policyholder to such increase of the sum assured.

The new Society being thus started, it will be necessary to make provision for the equitable distribution of any profit or loss that may arise in its future working. A new valuation must be made on precisely the same principles, after the expiration of an agreed time; and if the result of it is to show a fresh deficiency, there will be no alternative but to further reduce the sums assured according to principles to be explained presently; and so on, from time to time, as often as there may be a deficiency. If, on the contrary, the valuation shows a surplus beyond what is necessary to provide for the payment of the reduced sums assured, in what way should this be distributed? Is the whole of it to be appropriated to lessen the deductions made from the sums assured? Or, on the other hand, should the whole of it be allotted to the participating policyholders? Or should some intermediate course be pursued? In order to answer these questions, we must, as in all other cases where we have to do with the equitable division of surplus, consider the sources from which the surplus has arisen. We must thus remember that the participating policyholders have contributed in a special way towards the surplus, by paying a higher rate of premium than the non-participating policyholders, for the same sum assured. They are therefore in justice entitled to have an account taken of the "overpayments" they have made since the starting of the new Society; and if it should appear that the existing surplus is less than the amount of those overpayments, then the whole of the surplus should be divided among the participating policyholders in proportion to their several overpayments. If the surplus should exceed the amount of their overpayments, this latter amount should first be divided among the participating policyholders, and the remaining portion of the surplus, which may be called the "general surplus," should then be applied to reduce rateably all the deductions made from the sums assured. The only class of policies about which any doubt can arise will be those in which no deduction at all was made from the sums assured. It seems just that the participating policyholders in this position should derive some benefit from the surplus to which they have undoubtedly contributed; but any method that could be suggested

of increasing their sums assured, would introduce inconvenient contrasts between the policies which were reduced very slightly and those not reduced at all. It seems therefore better that an arrangement should be made that in consideration of the policyholders in question renouncing all right to the "general surplus," their policies shall in no event be reduced. This arrangement is also clearly the best that can be made as regards the non-participating policyholders whose policies were either not reduced or were increased.

We are now in a position to explain how the second and subsequent reductions of the sums assured should be made, if such should be found necessary. The method that will best suit all cases appears to be that in the event of a further deficiency, deductions should be made from the still existing policies proportional to those originally made. Thus we consider that an estimated deficiency in the assets necessitates a certain reduction of each policy, this being calculated by the best datums that are available; but if the subsequent experience of the Society shows that, in the events which have occurred, the deficiency was either greater or less than originally estimated, then the deductions made from the still existing policies will be increased or diminished. Of course, claims arising between any two valuations would be paid according to the scale fixed on at the last valuation quite independently of the results shown by the succeeding valuation.

It would much simplify matters if the distinction above drawn between participating and non-participating policies were neglected, so that for every policy there should be calculated the sum which the premium payable will insure according to the present age and the new scale of premiums. In this case there would be no "overpayments" of participating policyholders, and the whole of any surplus would be applied to reduce the deductions made from the policies. This course, however, tho simpler, and perhaps more practically convenient, would not so well carry out the spirit of the original contracts.

It will be noticed that, according to the method here suggested, the participating policyholders will receive a smaller insurance for the same premium than the non-participating policyholders, but this is no objection to the method. The remarks of Lord Justice James in his judgment (when Vice-Chancellor) *in re* Albert Life Assurance Company (2 May, 1870), may here be quoted in illustration of this point.

"With regard to the participating policies, it appears to me that I cannot make any difference. No doubt the participating policyholder will

be worse off, because I cannot give him a share of profits in the new office I have imagined.* His bargain was that he should share in the profits of the *Albert.* He was himself, more or less, in a certain degree a partner. He took upon himself the chance of that office making a profit. As things have turned out now no profit can possibly be made. At the time of the winding-up his bargain was this: for £100 a year, we will say, which he was to pay during his life, he was to receive £3,000 at his death; in addition to £3,000 he was to receive a certain thing, x, which was to represent profits. Now, in estimating any damage, I am to take into consideration that he was to be paid £3,000 and x. I am satisfied as a matter of fact, that x is to be converted into zero; and, therefore, for £100 a year, in the events which have happened, he was to have £3,000 plus zero. That is the basis of calculation in that case. It is true this man might say, 'But there is another man whose age and life were exactly the same as mine. I have for years paid £100, he has only paid £95; and yet you provide him with a new policy for the same amount as mine, I paying the £100 and he paying the £95 for the rest of our respective lives.' That may be. He did not choose to enter into a speculation, and you did choose to enter into a speculation. You are not, it is true, to contribute *quâ* partner to the losses, but, having taken the chance of making profit you cannot complain of the result of the speculative policy that you entered into; you cannot complain that the result of the winding-up has put you upon less favourable terms than the person who did not enter into the speculation with a chance of getting profit. So far as the speculation has resulted in an ascertained profit, so far as it has resulted in a bonus being declared, so far as it has resulted in cash paid, so far as it has resulted in a diminution of premium —all that is settled and bound by the contract between the parties. I must act accordingly, and take each contract exactly as it stands, considering that the chance of profit is now reduced to *nil.*"

The result of the principles laid down in this judgment is that the participating policyholder will not only receive a policy in the new Office of the same amount as the non-participating policyholder, but that he will have a smaller claim against the Albert—the Company which has failed.

This distinction between the participating and non-participating policyholders is not necessary in estimating the solvency of a Company, since, as I have pointed out, the Company must be held solvent provided that it is found to be in a position to pay the bare sums assured; but when a new Mutual Society has to be founded out of the wreck of the old Company the case is entirely altered. The spirit of the original contracts made by the two classes of policyholders is better carried out by treating the overpayments of participating policyholders as forming a sort of guarantee fund of the new Society. They were originally payments made to provide for future bonuses and contingencies; and by the method above

* This refers to the Office that is contemplated in the judgment as taking over the policies of the Albert. If the policies are not taken over, but formed into a Mutual Society, there is, as we have seen, no difficulty in allowing profits to the participating policyholders.

sketched out, they are retained for the same purpose in the reconstructed Society.

It may be considered by some to be an objection to the process here described that the annuitants of an Office will be more heavily mulcted than the policyholders; but, setting aside any legal questions as to right to priority of payment, there is no doubt that the result in question is equitable, for the annuitants can have no possible right to claim payment of their annuities out of the premiums to be paid in future by the policyholders.

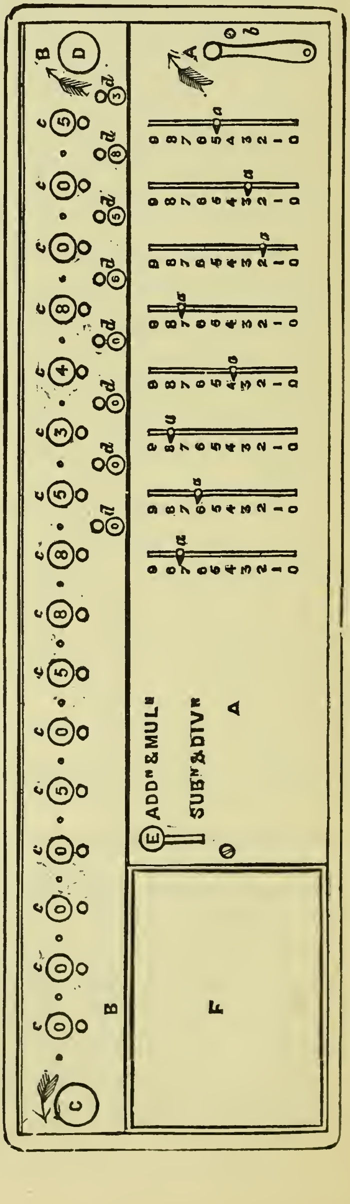
A few practical questions still remain to be considered. Assuming a Company to have been proved insolvent and a reconstruction of the kind here indicated to be resolved upon, the sums assured under the policies being reduced to the extent rendered necessary by the deficiency in the assets, the amount of such deficiency will of course depend upon the sum which can be realized by making a call upon the shareholders. It will not, however, be necessary to suspend operations until this amount is ascertained; but the reduced amounts of the policies can be first calculated on the supposition that nothing will be obtained from the shareholders, and any amount that is subsequently obtained from them will then of course have to be applied towards lessening the deductions so made.

Again, how are policyholders wishing to withdraw from the Society to be dealt with? In the case of the failure of a Life Insurance Company, the shock to public confidence will of course be so great that even if the Company be reconstructed on a sound footing, many persons will decline to pay anything more, considering that the first loss is the least. The fairest manner of dealing with these persons will probably be to issue to them paid-up policies, representing their share in the assets of the Company. Thus, $\frac{P_x}{\varpi_{x+n}}$ being the portion of the sum assured which is provided by their future premiums, and $k \cdot \frac{S\varpi_{x+n} - P_x}{\varpi_{x+n}}$ being the paid-up portion, which represents their share in the available assets, the utmost that can be claimed by a policyholder wishing to be relieved of all future payments, is a paid-up policy of this latter amount, and something should be deducted from it on account of the option exercised by the policyholder. It would be unwise to offer to pay surrender values in cash, and this is a point which had better be left for consideration, after experience has been had of the working of the new Society.

On the Use of M. Thomas de Colmar's Arithmometer in Actuarial and other Computations. *By* Major-General Hannyngton, *Associate Member of the Institute of Actuaries.*

[Read before the Institute, 27th February, 1871.]

The accompanying illustration gives a top view of the instrument. The following description is taken from the *Engineer*, 20th May, 1870.



It is constructed chiefly of a brass plate A A furnished with eight slots as shown; directly under these slots are mounted eight drums, each having nine elongated cog-teeth of successively decreasing length; over each drum, and between it and the slot, is mounted a square shaft on which slides a pinion wheel, so as to catch any number of teeth on the drum. Each of these pinion wheels is moved by a button *a*, of which there is one in each slot, the figures at the sides of the slots showing the proper position of each button *a* for any work to be performed by the instrument, so that not the least trouble is encountered in arriving at the result. The cogged drums gear by bevel wheels with a long horizontal shaft which is also in gear with the vertical shaft moved by the handle *b*, by which the instrument is worked. B B is a moveable brass plate, which can turn and slide on a round bar hinge at the back; in this plate there are sixteen holes, *c*, under each of which is a moveable disk numbered from 0 to 9, and arranged so that any one figure of each disk may be brought under its corresponding hole *c*. These disks have bevel wheels, which gear with bevel wheels on the before-mentioned square shafts. The moveable plate B B is also furnished with the holes *d*, having disks numbered from 0 to 9 underneath, and are for showing the number of turns of the handle, giving by this means the quotient in division and showing the multiplier in multiplication. The knobs C and D are for bringing the figures under the holes *c* and *d* respectively to zero before commencing an operation, and the knob E is for setting the instrument to work addition and multiplication, or subtraction and division. F is a small plate for memorandums. If the knob

E be placed at addition, each turn of the handle will carry the figures marked by the buttons a under the indicator holes c, while if the knob be placed at subtraction it will subtract from the figures under the holes c the number marked by the buttons a.

The instructions given with the machine are sufficient for the performance of the elementary operations, but it will nevertheless be convenient in this paper to explain these operations anew. A reference to the diagram will render what follows intelligible.

The fixed brass plate A A may be called the Face Register or *Face,* on which numbers expressed by eight or fewer figures may be indicated by the moveable *markers a.* When we desire that any given number should be so indicated, the direction will be to *set on* that number. The moveable plate BB we shall call the Sliding Register or *Slide;* and when we desire that a number be transferred from the Face to the Slide, the direction will be to *put up* that number. In the diagram the number 76847235 is "set on" and the number 505885348005 is "put up," while the number 6583 appears in the quotient holes. The figures in the upper holes show the product of the first and last of these numbers. Sometimes it will be convenient to put up numbers by hand, which may be done by turning the *Studs* which are under the *figure holes c* (sixteen in number in the diagram). These studs are placed on the axes of the *figure disks,* which disks turn in either direction, and have the numerals 0 to 9 so ranged on them as to appear, one at a time, in the figure holes. A smaller set of holes, *d,* studs, and disks (in the diagram eight in number) are placed below the others on the right-hand half of the slide; these may be called the "*quotient*" *holes, studs,* and *disks,* being, as will appear, serviceable chiefly in the operation of division. The small holes between the figure holes, *c,* serve to hold an ivory pin to mark, when requisite, the decimal point. By turning the milled head C towards the left hand, and the head D towards the right hand, the figures in the holes *c* and *d* respectively will become zero; these heads may therefore be called *Effacers.* Before commencing any operation, it is necessary to see that all significant figures have been effaced. The button E, which slides along the slit shown in the diagram, may be called the *Regulator,* because, as marked on the Face, its position determines the action of the instrument. The *Winch, b,* turned by hand, always in one direction, to the right, gives motion to the mechanism. One turn "puts up" the number that has been "set on" the face; and if this number be not changed, another turn will put it up again, and so continually. Under the

winch lies a pin, not shown in the diagram. This may be called the *Stopping pin*, at which the winch should always be stopped. When the winch is not at the stopping pin, the regulator cannot be moved. Attached to the markers, under the face, is a spring, to keep them, when in use, in their proper position. F is a small plate of ground glass to serve as a writing tablet.

We may now commence numerical operations. They will be adapted to an eight-figure machine.

Addition.

Place the regulator at addition; set on the several addends in succession, giving one turn of the winch for each.

Example.—Find the sum of 94765, 34287, and 52931.

Set on, and put up		94765,	result	94765
„	„	34287	„	129052
„	„	52931	„	181983

In this way any series of numbers, each consisting of not more than eight figures may be summed, so long as the sum does not exceed 9999999999. A further example will be given, in showing how to extend the power of the machine.

The operation of addition is rather tedious; but when the sum of many numbers is required, the result is safe.

Subtraction.

Place the regulator at addition; set on the minuend, and one turn of the winch will put it up; or the minuend may be put up on the slide by hand, which is often more convenient. Place the regulator at subtraction; set on the subtrahend and give one turn of the winch.

Example.—From 181983 take 94765.

Put up the minuend 181983; set on the subtrahend 94765. Placing the regulator at subtraction, give one turn of the winch, and the result, 87218, will appear on the slide.

A series of addends and subtrahends, arranged in any order of succession, may be summed by attending to the signs, and placing the regulator accordingly. In such case, the occurrence of an intermediate negative value will not embarrass the action of the machine, or interfere with the exhibition on the slide of a final positive result. But if the sum of the subtractive quantities exceed that of the additive quantities, the result will show one or more superfluous figures, nines, on the left of the slide. These nines must be rejected, and the complements of the other figures be

taken. Thus, if the preceding example be inverted, the result will show 9999912782, which represents −87218.

Multiplication.

Place the regulator at addition and set on the multiplicand. Give as many turns of the winch as are indicated by the digit in the units, or in the lowest place. This digit will now appear in the quotient hole on the right, and a corresponding product in the larger figure holes. Now draw out the slide towards the right hand one or more places, as indicated by the place of the next significant figure of the multiplier reckoned from the extreme right, and give the number of turns indicated by that figure, which will then appear in its proper quotient hole. The product of the multiplicand and the two figures of the multiplier will be seen in the large figure holes. Proceed similarly, digit by digit, until all the digits of the multiplier appear in their order in the quotient holes. The complete product of the two factors will then be exhibited in the figure holes of the slide. It will be convenient to designate the factor that is set on the machine as the *in* factor, and the other, which is ultimately seen in the quotient holes, as the *out* factor.

As an example, let us multiply 42967 by 3752.

Place the regulator at multiplication; set on 42967.

The units figure being	2, give two turns, the result is	85934
	Draw out the slide one place;	
The next figure being	5, give five turns, the result is	2234284
	Draw out the slide one place;	
,,	7, give seven turns, the result is	32311184
	Draw out the slide one place;	
,,	3, give three turns, the result is	161212184

The operation is now complete, and the careful worker will notice that the out factor 3752 appears in the quotient holes, and shows that the right number of turns has been made.

Division.

Draw out the slide to its full extent, *i.e.*, so that eight holes only are left on the face. Put up the dividend. Be careful before commencing the division to have the quotient disks effaced. Set on the divisor, place the regulator at division, and turn until the figures on the left of the slide become less than those under them on the face. Bring in the slide one or more places to the left, as may be requisite to make the figures remaining thereon exceed in arithmetical value those under them on the face, and turn as

before. Repeat the process until the slide be wholly brought home, or until the dividend be exhausted. The quotient will now appear in the quotient disks.

As an example, we may find $\frac{161212184}{42967}=3752$, and the process will exhibit in reverse order the same figures as the example of multiplication above given.

The four arithmetical rules having been thus exemplified within the ordinary limits of the machine, it is proper to show how these limits may be extended.

The addition or subtraction of large numbers can be performed in parts, and to this method there are practically no limits. A single example will suffice.

3579	62176543
5962	75311739
7346	16753761
2517	93665716

Taking eight places from the units inclusive, the sum 247907759 appears on the slide; then drawing out the slide eight places, the four places on the left hand of the vertical line will be summed in their proper position, and the total, 1940647907759, will be found.

As to multiplication, some detail is necessary.

The machine to which these illustrations refer, is an eight-figure machine, giving products up to sixteen places of figures, which is the full extent of the slide. But with intermediate record by the operator in the manner shown below, the power of the machine can be virtually doubled, and a product containing thirty-two places of figures be readily obtained. For if each factor be severed into parts of eight figures each, and these parts be denoted by the letters *a*, *b*, *c*, *d*, we have

$$(a+b)(c+d)=ac+ad+bc+bd$$

whereby not only the partial products but also their local positions are indicated, so that they may be written in due order.

For example,

$$\underbrace{12345678}_{a}\ \underbrace{87654321}_{b}\times\underbrace{86427531}_{c}\ \underbrace{13572468}_{d}$$

$$\begin{array}{rl}
ac= & 1067006468061018 \\
ad= & 0167561319593304 \\
bc= & 7575746545511451 \\
bd= & 1189685466834228 \\
\hline
 & 10670065454940967700160966834228
\end{array}$$

Rarely indeed can such a product as this be required, but often there is need for partial products to a considerable number of places, and that these should be obtained without intermediate record. The extent to which this is possible is $n+\frac{1}{2}n$ places, true in the last place, n being the number of places on the face of the machine. Hence, an eight-figure machine will give partial products of twelve figures, true in the last place. The nature of the process is shown above, and the practical rule is as follows, for twelve figures by twelve figures:—

Divide each of the factors into parts, of eight figures and four figures, reckoned from the highest place, and call $(a+b)$ the *in* factor; set on the first eight places of the *in* factor, multiply by first eight places of the *out* factor; set on the first four places of the *in* factor, multiply by the last four places of the *out* factor; set on the last four places of the *in* factor, multiply by the first four places of the *out* factor.

The work is now complete, and is true in the twelfth figure, reckoned from the highest place.

We repeat the former example, cut down to twelve figures,

$$\underbrace{12345678}_{a}\ \underbrace{8765}_{b}\times\underbrace{86427531}_{c}\ \underbrace{1357}_{d}$$

The result is

$$1067006545491451$$

of which twelve places are true.

Division is less tractable than multiplication, but so long as the divisor contains no more than eight figures the dividend and the quotient may be unlimited, for the remainders can be transferred to the left or highest place on the slide, the partial quotient be recorded, then effaced, and the operation be carried on to any extent. But a divisor of more than eight figures offers difficulties that can perhaps best be surmounted by decomposing it into two or more factors, and then working by successive divisions. This branch of the subject may be left to inquiring students.

Among the powers of the machine may to some extent be included that of the difference engine; for a second difference can often be supplied by the operator. For instance, a table of square numbers having the second difference constant, requires merely that the operator should continually add 2 to the difference on the face of the machine. Thus in fact any quadratic form could easily be tabulated. Hence, also, two or more operators working together on separate machines, might compute tables requiring differences of

the third or higher orders. Such an application of these machines might have important uses.

To the actuary these machines may be very helpful, and I shall therefore offer a few examples based on life contingencies. The summation of Columns N_x, S_x and the like, where each successive value has to be exhibited, may be very conveniently effected.

Detached operations, such as $D_x = l_x v^x$, do not require explanation. To obtain the values directly, and without having to record the intermediate steps, is an obvious advantage.

When $D_{x.y} = l_x v^x . l_y$ for all combinations of x and y is required, the machine affords peculiar aid.

Since each value of D_x has to be combined with each value of l_y, any given value of D_x being set on as an *in* factor may be multiplied by successive values of l_y, which operation can be made continuous by either adding or subtracting $D_x d_y$. It is immaterial whether we begin with the youngest or oldest age, for the machine performs addition or subtraction with equal facility.

The type of the calculation is either

$$D_x l_y + D_x d_{y+1} + D_x d_{y+2} + \ldots .$$

or

$$D_x l_y - D_x d_{y-1} - D_x d_{y-2} - \ldots .$$

and the successive values are to be entered in their proper columns as they arise. As a check, $D_{x.y}$ for each tenth year of difference of ages should be directly computed, and the agreement of these with the continuous process will prove the work.

The data for Carlisle Table, 3 per cent., are as follows, to a sufficient extent for our example:—

x	l_x	d_x	v^x	D_x
104	1	1	·04623050	0·04623050
103	3	2	·04761742	0·14285226
102	5	2	·04904594	0·24522970
101	7	2	·05051732	0·35362124
100	9	2	·05203284	0·46829556
99	11	2	·05359382	0·59853202
98	14	3	·05520164	0·77282296
97	18	4	·05685769	1·02343842
96	23	5	·05856342	1·34695866
95	30	7	·06032032	1·80960960
94	40	10	·06212993	2·48519720
93	54	14	·06399383	3·45566682
92	75	21	·06591364	4·94352300
91	105	30	·06789105	7·12856025
90	142	37	·06992778	9·92974476

In this l_x and l_y are taken from the same life table, but they may, and often should be, taken from different life tables, as for instance when the lives represent husband and wife, or father and daughter.

The operation will now be,—Set on $D_{104}=\cdot 04623050$ (this remains on the face). Multiply by $l_{104}=1$; record the result, ·04628050, in Column 0, and let it remain on the slide. Efface the out factor. Multiply by $d_{103}=2$; record the result, 0·13869150, in Column 1, and let it remain on the slide. Efface the out factor. Multiply by $d_{102}=2$; record the result, 0·23115250, in Column 2, and let it remain on the slide. Efface the out factor. Multiply by $d_{101}=2$, and proceed as already shown.

The speed and certainty of this method will be appreciated by those who have had experience in the construction of commutation tables.

The effacing of the out factors is not a part of the numerical details, but it is a precaution against error that will not be neglected by careful workers.

The following values of $D_{x.y}$ were computed on the machine (at full power) by the foregoing process. They will serve as a guide to those who may desire to travel in this path. The decimals are here recorded to an extent not required in practice:—

$x-y=$	0	1	2	3	4	5
$x=104$	0·0462305	0·1386915	0·2311525	0·3236135	0·4160745	0·5085355
„ 103	0·4285568	0·7142613	0·9999658	1·2856703	1·5713749	1·9999316
„ 102	1·2261485	1·7166079	2·2070673	2·6975267	3·4332158	4·4141346
„ 101	2·4753487	3·1825912	3·8898336	4·9506974	6·3651823	8·1332885
„ 100	4·2146600	5·1512512	6·5561378	8·4293201	10·7707979	14·0488668
„ 99	6·4848522	8·2534483	10·6115764	13·5592365	17·6859606	23·5812808
„ 98	10·8195214	13·9108133	17·7749281	23·1846888	30·9129184	41·7324398
„ 97	18·4218912	23·5390832	30·7031520	40·9375360	55·2656736	76·7578800
„ 96	30·9800501	40·4087610	53·8783480	72·7357698	101·0219025	141·4306635
„ 95	54·2882880	72·3843840	97·7189184	135·7207200	190·0090080	256·9645632

Note.—The first entries in Columns 0, 1, and 2 are found as shown in the foregoing explanation, and in like manner all the values are computed in lateral order from the initial term in Column 0. The vertical columns are $D_{x.y}$ from the oldest ages downwards, for differences of age noted in the headings. They might, with equal facility, be formed from the younger ages, as is usual in the construction of Column D.

In a similar way survivorship assurances may be computed, but an example is unnecessary. What is here shown brings out one of the most useful powers of the machine, namely that of giving the *sum* of a series of *products* without exhibiting the several quantities.

A simple and striking example of continuous calculation is afforded in the construction of temporary annuities. If we set on the reciprocal of D_x at any age x, and then multiply this by the successive values of D for the higher ages, never altering what appears on the slide, our end will be attained. The form is expressed thus,

$$\frac{1}{D_x}(D_{x+1}+D_{x+2}+D_{x+2}\ldots\ldots+D_{x+n})={}_{n)}a_{x+n}$$

The successive values as they arise on the slide must be recorded.

The formation of tables having a constant difference requires no more than the setting on of an initial value, and thenceforth each turn of the winch gives a new term. Thus, by placing 1 in a proper position on the slide, and $\frac{1-v}{100}$ on the face, and setting the regulator to subtraction, we might grind out tables for the conversion of annuities into assurances more quickly than the values could be transcribed.

To pursue the subject further would be tedious. I will therefore conclude with two examples in a different branch of science. They are, I think, remarkable and important.

1. Given a, b, c, three sides of a spherical triangle to find an angle—suppose the angle A.

2. Given two sides b, c, and the included angle A of a spherical triangle to find the side a.

The formulas that we shall use are

$$\sin^2\tfrac{1}{2}A=\frac{\sin^2\frac{1}{2}a-\sin^2\frac{1}{2}(b-c)}{\sin^2\frac{1}{2}(b+c)-\sin^2\frac{1}{2}(b-c)} \quad . \quad . \quad . \quad (1)$$

$$\sin^2\tfrac{1}{2}a=\{\sin^2\tfrac{1}{2}(b+c)-\sin^2\tfrac{1}{2}(b-c)\}\sin^2\tfrac{1}{2}A+\sin^2\tfrac{1}{2}(b-c) \quad (2)$$

It appears from these formulas that the data are all of one name—that is, the squares of the sines of half arcs.

Let the sides and angle be as follows:—

	θ °	′	″	$\mathrm{Sin}^2\frac{1}{2}\theta$
a =	79	45	39	0·4111212
b =	73	45	51	·3602051
c =	39	53	20	·1163552
A =	93	25	25	·5298593
$(b+c)$=	113	39	11	·7005986
$(b-c)$=	33	52	31	·0848735

Now, to find $\sin^2\frac{1}{2}A$, we have

$$\frac{\cdot4111212-\cdot0848735}{\cdot7005986-\cdot0848735}=\frac{\cdot3262477}{\cdot6157251}=\cdot5298593=\sin^2\tfrac{1}{2}A\,;$$

and to find $\sin^2\frac{1}{2}a$,

$$\begin{array}{r} \cdot7005986 \\ -\cdot0848735 \\ \hline \cdot6157251 \end{array} \times \cdot5298593 = \begin{array}{r} \cdot3262477 \\ +\cdot0848735 \\ \hline \cdot4111212 \end{array} = \sin^2\tfrac{1}{2}a$$

The simplicity of these operations is evident; and as these two problems comprise the whole practice of nautical spherics, they are in this form worthy of especial notice.

A table of $\sin^2\frac{1}{2}\theta$ for every tenth second of space, with suitable formulas, was published in 1805 by James Andrew, M.A. This is one of the most valuable aids ever offered to navigators, and yet, so far as I know, it has been utterly neglected by them. Conjointly with the arithmometer it would smooth all the difficulties of lunars, and double altitudes, and of great circle sailing; and if this mention of it be somewhat out of place, it is nevertheless well deserved. Extensive tables of natural versed sines exist, and they afford similar but not in all respects the same advantage.

I have noticed that tables quadratic in form may be computed on the machine. A conspicuous instance may be mentioned. Sir John Macneill's Earthwork Tables are purely quadratic, and they might thus, with comparatively small labour, be recomputed and extended. There are many other tables having second differences that change but slowly; these also are within the range of the machine.

In conclusion, I may say that, having had considerable experience in actuarial computations, I have never found the machine fail to afford help; though it may happen that the right process is not always that indicated by the seemingly most appropriate formula. The machine asks for peculiar methods, and such as are not easily to be described. Herein there is room for skill and intelligence, so that hand and head may work together with mutual advantage. I will only add, that I stand here, not as a teacher but as a student and fellow worker with the members of this Institute, whom I have the honour to address.

On Mechanical Aids to Calculation. A Lecture to the Actuarial Society of Edinburgh. By Edward Sang, F.R.S.E., *Fellow of the Faculty of Actuaries in Scotland.*

In every branch of business, even in the very rudest stage of barter, men have to count. A skin is exchanged for so many

cowries, a rifle for so many skins. Do what we will, go where we will, the necessity for counting meets us. Yet, though numbers be essential to all our operations, we do not easily form an idea of large numbers. Thus in no language do the separate names for numbers reach to twenty. In one, I believe, the numeration goes as far as fifteen; in the northern languages of Europe it reaches to twelve, but in most languages it stops short at ten.

More advanced numbers are held as composed of so many tens and so many units; they are named accordingly, as in English thirty-seven, meaning three tens and seven. In the Greek, Latin, and modern European languages, the names of the multiples of ten are derived from the names of the units; in some Asiatic languages the first five of them have independent names.

When we arrive at ten tens a new name, as *hundred* in English, is introduced, and we reckon in hundreds up to ten hundreds, which also receives a special name, with us *thousand;* and here, with only one exception that I know of, the special nomenclature ceases; all larger numbers being named by compounding the names of inferior numbers.

In all this we see the use of helps to counting. Were numbers to receive independent names, the stock of words would be exhausted, the memory would fail to keep them in order, mistakes would be frequent. But the contrivance of articulate numeration introduces ease, clearness, and certainty.

[The above remarks appear to us extremely suggestive and worthy of further development. As regards the separate names of the numbers, however, it appears to us that the English names, *twenty*, *million*, *trillion*, *quadrillion*, must be considered as separate independent names. For it cannot be said that either *twenty* or *million*, or either of the others, is formed by compounding the (English) names of inferior numbers.

The help to counting furnished by a simple and systematic method of naming numbers, seems to have been carried further in English than in most other modern languages. Thus, as compared with French, the old-fashioned numeration so familiar to us in the English Bible, of *threescore*, *threescore and ten*, *fourscore*, &c., has entirely given way to *sixty*, *seventy*, *eighty*, &c., while in French *septante*, *octante*, *nonante*, are to this day provincialisms, and the words in common use are *soixante-dix*, *quatre-vingts*, *quatre-vingts-dix*. The method of numeration in Danish is still more clumsy. For *sixty* and *eighty* the names are *tresindstyve* and *firsindstyve*, which correspond to *threescore* and *fourscore*; but for *fifty*, *seventy*, and *ninety*, the names are *halvtresindstyve*, *halvfirsindstyve*, and *halvfemsindstyve*, which will correspond to *half* (-way towards) *threescore fourscore*, *fivescore*.

In comparing English with German, the idioms of the two languages closely coincide as regards the above points; but when we come to join units with tens, as in reading out the number 4321, the Englishman reads

four thousand three hundred and *twenty-one*, while the German says *vier tausend drei hundert ein* und *zwanzig*, an inversion which must slightly increase the difficulty of counting, and which, we are informed, does in practice frequently cause mistakes in setting down the results of a multiplication.—Ed. *J. I. A.*]

As in the naming, so in the marking of numbers recourse was had to various contrivances for expediting the operations. The simple but prolix plan of putting down a counter for each article in barter, was soon improved by using a second kind of counter for each *ten*, or *dozen*, or *score* as the case might be. Thus in the very rudest kind of numeral notation which yet subsists, that used by the Romans, a stroke was made for each unit, and, to break the monotony, a cross stroke was used at the fifth, thus gradually giving rise to the use of the mark V for five and X for two fives or ten. The Arabs, however, adopted a more ingenious plan by using the first ten letters of their alphabet for the ten units; the second ten for the tens, and the third set of ten for the hundreds. A contrivance adopted by the Greeks, who, to supplement their defective alphabet, introduced some arbitrary marks. In India, however, the great improvement was made of using nine arbitrary marks for the nine units, and of indicating the tens, hundreds, and thousands by a change of position. This Hindi Rakkam was adopted by the Arabian arithmeticians, and by them introduced into Southern Europe; for which reason they are commonly but erroneously called by us Arabic numerals.

This Hindi contrivance, in daily use by all of us, is in reality a geometrical aid to counting: it takes a mechanical form in the shape of the swan-pan, or of the wire frame with strung beads.

When we mark down the successive numbers, the continual recurrence of the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, introduces the idea of circulation; and so leads us to divide the circumference of a wheel into ten parts and to write the numerals in them. In this way, by turning the wheel one step at a time and always in one direction we have the successive units. And here we have the first element in machine-counting.

By arranging a second wheel for the tens, and so placing it as that at each turn of the unit's wheel it shall be moved one step, we are enabled to count to one hundred; and by continuing this arrangement we can carry the numeration to any desired extent. Such is the construction of ordinary counting-machinery. In some of these machines the movements are continuously connected by means of toothed wheels, as in the cases of clock-work and of

the ordinary gas-meter index. In others the motions are by jerks; the ten's wheel remains at rest until the units are passing from 9 to 0, at which time the tens are advanced one step; and the indication changes from say 69 to 70. Some of these latter machines are so contrived as that the wheels are locked until the carrying take place. Such are the machines used for registering the number of operations performed by the bank-note printing press. From their very construction continuously connected machines are always locked.

Such counting machines are of great use in many situations, as at loading and landing wharves, in large warehouses, at turnpike gates and in general wherever extensive tale has to be made, or wherever a check is required upon operations. They receive various forms according to the purpose for which they are intended; and by modifications and extensions they become the calculating machines to which our attention is principally to be directed.

As soon as we attempt to use a counting machine we find that there must be some provision for setting it to zero, or to whatever may be the number from which we wish to begin. This provision is made in two ways. In one the index is fitted stiffly, not permanently, to its axis, so that it may be turned round whenever a sufficient pressure is applied to it. We have a familiar example of this in our clocks and watches: without it we should not be able to correct the error of the time-keeper. In the other way the circumference of the index wheel is notched and a knuckled holder enters into the notch, being kept there by a spring. By this arrangement the wheel cannot be turned either backwards or forwards without raising the holder, so that it is kept steadily in its place until a pressure be applied sufficient to overcome the elasticity of the spring. Moreover, as soon as the tooth of the wheel is brought past the angle of the knuckle, the index is thrown forward or backward to the next number. In such a case the index wheels are independent of each other; but complete independence would not subserve our purpose; wherefore a provision is made to cause an advance of unit on any one index when the index below passes from 9 to 0.

An instrument arranged in this way may be set to any initial number and may be used at once for counting. But it is now capable of much more extensive use.

If I turn the unit's index forward *seven* steps I shall add *seven* to the indication, for should the units overpass the 9, as say from 8 to 5, the ten's index would be brought forward one step by

means of the carrier. If I now turn the ten's index *four* steps forward, I shall add *forty* to the indication, that is to say, I shall have added *forty-seven* to the previously recorded number. In this way the instrument has become an *addition machine;* its action contains all that is essential to this operation. It is also a subtraction-instrument; the very movements which perform addition may be made to perform subtraction, and even at the same time. All that is needed is to inscribe on the indices numbers in the inverse order.

Various contrivances have been made for the purpose of facilitating the addition; in all of these the object aimed at is to have the addend set upon the machine, and the summation performed by the motion of the instrument, generally by one turn of the handle. Much ingenuity has been expended on these contrivances, and considerable apparent diversity of arrangement has been the result; yet the essential characters of the action are the same in all.

We desire, for example, to bring the ten's index forward four steps by one action. For this purpose we may arrange that a hook or propeller shall have a stroke of four divisions; or we may cause four teeth of some wheel to act upon the index: both of these methods are in use; the latter is seen in Thomas' machine; the former in the striking part of our common house clocks, to which I shall have occasion afterwards to revert.

In Thomas' machine there is placed a cylinder to make a complete turn at each operation; on its surface there are fixed nine parallel bars, or as we may call them elongated teeth, which are to work the teeth of the counting wheel: the lengths of these bars are equi-different, so that at one place of the cylinder the whole *nine* are ready to act, at the next place *eight,* and so on. The counting wheel is made to slip longitudinally upon a squared axis, so that we can bring it opposite to any desired part of the cylinder.

If we bring the counting wheel so as to be acted on by four bars, one turn of the cylinder will cause an addition of *four,* and each succeeding turn will cause a new addition of *four.*

Now for each of the indices, that is for units, tens, hundreds, and so on, we have a special cylinder, and therefore if we set the counting wheels to specified digits, and turn each of the cylinders once round (for the present we shall suppose once in succession) we shall have added the specified number to the previous indication.

If we do this for one cylinder after another some considerable time will be required. The contrivance for economising that time is exceedingly ingenious in Thomas' machine. He causes all the

cylinders to be actuated at once by the working handle, so that all the additions go on nearly at once. This, however, would cause confusion in the carrying. He therefore makes the index wheels entirely independent of each other, and arranges the carrying in another way. On each index wheel there is fixed a stud to come in contact with a lever whenever the indication passes from 9 to 0, and so to push this lever aside; and a detent is provided to keep this lever back after the stud has passed onwards. This lever brings into action the carrier fixed on the axis, and this carrier only acts after the addition by the bars has been completed. Thus by arranging the cylinders one tooth in arrear at each step, the carrying from one rank to another is completely effected, and the whole addition completed by one turn of the handle.

By help of this machine, then, we can perform addition, and by turning the handle repeatedly, we can perform successive additions, and so form an equi-different progression.

On the same instrument there is an arrangement, by means of bevelled wheels, for changing the direction of the motion of the indices, and thereby converting addition into subtraction.

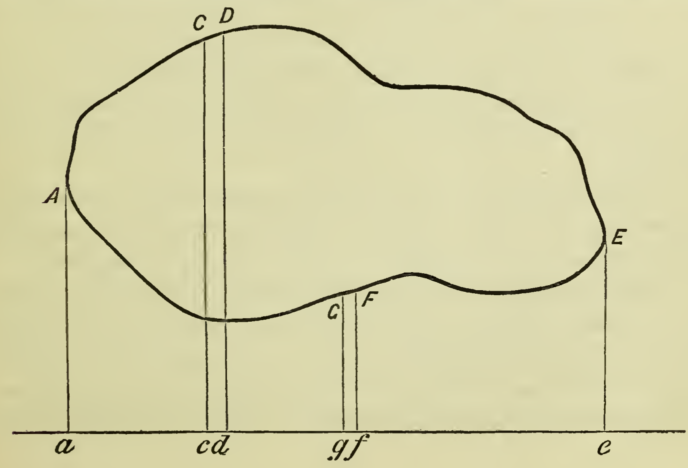
M. Thomas has provided the means for shifting the addend or subtrahend to a different position on the numeration scale. This he accomplishes by sliding the whole frame carrying the indices sideways, at the same time preserving the recorded number. By this means, when we have set the addend to say 3734, we are able, by shifting the index frame two steps to the right, to convert the addend into 373400. He has also arranged a set of indicators to shew how many times the addition or subtraction has been performed in the various positions, and so to shew the multiplier.

This very ingeniously-constructed instrument exhibits the various characters and requirements of a calculating machine, in a tolerably complete state of development. After all, however, it is only an addition-machine, the only multiplication which it performs being that by the powers of ten, done by the transposition of the frame. A multiplying-machine proper would be such, that when the two factors are set upon the instrument, their product should be exhibited automatically. As yet, we have scarcely any approach to such an instrument.

The first attempt at an apparatus for multiplying was made by John Nepair, the inventor of logarithms; and the study of this attempt may serve to throw considerable light upon the general subject of mechanical aids to calculation. The contrivance is known under the name of Nepair's Bones, and is described in a

work called *Rhabdologia*. It is simply a moveable multiplication table. The multiplicand is written on the top of a rod or other moveable slip, its multiples in lines below, the units being separated from the tens by diagonal lines; and these slips are sufficiently numerous to allow of the formation of any number by their initial figures. If the multiplicand be, for example, 43429448, we arrange slips showing that number at the top, and then in any of the horizontal lines we have its multiple by carrying the tens on one slip to the units of that on its right. In this way the computer obtains the successive lines of his product: these he has to write down and add together in the usual way. I am not acquainted with any other contrivance for showing the product of two numerical factors. There exist, however, several instruments for showing the product of factors represented geometrically, and for the summation of such products. It may be useful to examine these, in order to understand the kind of instrument that is desireable for purely arithmetical operations.

In order to obtain the surface of an irregular figure drawn upon paper, we divide it into a number of parts by parallel lines, and get the area of each of these by multiplying the length by the breadth; and finally we add all these products together. The instruments to which I allude perform all of these operations, and show the result at once. In order to understand how this is accomplished, we shall draw a straight line outside of the figure, and bring the parallels to cut it, as in the example. Here we at once perceive that the surface of the figure is the sum of all the rhomboids, such as *c* C D *d*, drawn to the farther side of the contour, less the sum of all such as *f* F G *g* drawn to its inner side.



In order to represent mechanically the areas of such rectangles we must have a motion whose extent is in the compound ratio of the breadth *c d*, and of the length *c* C. In what is, I believe, the first machine of this kind ever contrived, the Platometer of Mr. John Sang, this compounding of two ratios is accomplished by help of a cone, mounted on two wheels. Let us suppose that the axis of the cone is laid in the direction *c* C, and that, in rolling, its apex describes the line *a c*, then while it moves from *c* to *d*, the quantity of turning is proportional to *c d*. But the surface of the cone opposite C D must move through a distance also proportional to *c* C, wherefore, the actual motion there must be proportional to the surface of the rectangle *c* C D *d*. This motion is measured and recorded by a wheel rolling on the cone, and capable of being slid upon it lengthways. Thus when the tracer of the instrument is led along the contour A C D E, the wheel measures and adds up all the minute rectangles from *a* A to E *e*. Similarly when the tracer is led along the other part of the boundary from E to A, the same wheel measures all the rectangles *f* F G *g*, but now in the opposite or subtractive direction; so that when the tracer is brought back to A, the index shows the number of square inches in the surface of the figure.

The Platometer contrived by Mr. Beverley, of Dunedin, New Zealand, exhibits the same principle brought out in a different way. If a cylinder be dragged along a surface endwise it does not turn; if it be moved in a direction at right angles to its axis it rolls; in other positions it partly slides and partly rolls, and in general the quantity of turning is proportional to the sine of the angle between the direction of the axis and the direction of the motion. Taking advantage of this law Mr. Beverley arranges his instrument so that the quantity of angular motion is proportional to the distance of the tracer from a certain straight line; it is, at the same time, proportional to the extent of the actual displacement, and hence the indication is proportional to the area of the rectangle.

These instruments are called Platometers because of the use to which they have been applied; they are in reality integrating machines, and may be applied to a great variety of purposes.

Thus, let us take Mr. Sang's Platometer, and instead of making it to roll like a carriage on wheels, let us fix the frame, leaving the wheels and cone free to turn; and seek to apply it to the solution of such a problem as this, "to find the value of an assurance." We shall divide along the periphery of one of the wheels parts to

represent £1; and in the direction of the slope of the cone, other parts to represent the decreasing values of £1 payable $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$ years hence. The product of £1 payable for each person who dies in the year, into the value of £1 for that year is obtained by bringing the index wheel to the proper distance from the apex of the cone, and then turning the cone round by as many divisions as there are deaths in the mortality table. And if we perform this operation regularly for each year during the whole of the table, the index wheel will at once record the sum total of all these products. Beverley's machine may easily be arranged to do the same work.

Both of these instruments, however, and indeed all machines of this kind, depend on a combination of sliding and rolling, and are thus liable to considerable inexactitude in their indications. They fall far short of the certainty which attends the use of toothed wheels, and would be unfit for such calculations as come under the notice of the actuary.

Having thus given a sketch of the general construction and mode of action of calculating machines, I proceed to consider how these instruments are to be applied and what is the amount of help to be expected from them.

Beginning with the earliest contrivance, that by Nepair, let us place a set of his rods before us, and proceed to make use of them in calculation. We wish, for example, to multiply 397364 by 7. Having collected and arranged the sticks headed 3, 9, 7, 3, 6, 4, we pass down to the seventh row and there find

$$\left|\ {}^{2}/{}_{1}\ \right|\ {}^{6}/{}_{3}\ \left|\ {}^{4}/{}_{9}\ \right|\ {}^{2}/{}_{1}\ \left|\ {}^{4}/{}_{2}\ \right|\ {}^{2}/{}_{8}\ \right|$$

giving the result 2781548. Now the very least attention is enough to show that the trouble of seeking out the sticks, of arranging them, and of writing down the product, must be many times greater than that of writing down the product without the sticks. It must, not, however, be thought that Nepair's rods were of no use: in his day men learned to count after they were grown old, the Indian numerals were novelties, and the multiplication table was by no means at the finger-ends of computers; thus the rods were then of real use even for a solitary operation such as this.

When we have to find the product of two numbers, each of several digits, the utility of the contrivance is better seen. Thus,

if the product of the above number by 782935 were wanted, one arrangement of the rod serves for all the six multipliers, and the six products may be easily written down for addition. There can be no doubt but that this was a considerable aid to those who were unaccustomed to the use of numbers, and on whom the necessity for heavy work had come like an oppression. Astronomy was beginning to assume an exact form, trigonometrical tables had been extended to seven decimal places; the dimensions of the planetary orbits had been ascertained, and even their eccentricities with considerable precision. Hence just as the need for trigonometrical calculations grew greater, the number of decimal places also increased, and the burden of the multiplications and divisions became so intolerable that even Kepler exclaimed, "The duration of human life is too short to meet the requirements of the astronomer." Under such circumstances, the assistance to be obtained from Nepair's rods was by no means to be despised.

Now-a-days, however, an ordinarily good computer, and particularly one who has acquired the easy art of working from the left hand, finds that the time occupied by the mental operation is but an insignificant fraction of that taken up by the manual operation of writing; and to him a set of Nepair's rods would only prove a troublesome incumbrance.

Thus we can only estimate the value of any such mechanical aid, by taking into account the attainments of the operator. To those who can hardly tell whether seven times nine make sixty-five or sixty-three, a set of Nepair's rods may safely be recommended; yet even to such a one the evil may overbalance the good, since he will be prevented from acquiring expertness through practice.

Passing from the elementary contrivance of Nepair to the elaborate and ingeniously contrived machine of Thomas, we have to arrange the indices so as to show the multiplicand, setting at the same time the result indices to zero. On now turning the handle one, two, three times we obtain the product of the multiplicand by the successive digits one, two, three. Having thus obtained the product by the units, we shift the apparatus one step along, turn the handle once for every figure in the tens, so on for the hundreds, until the multiplication be completed: the result has now to be inspected and written down upon our paper. In order to prevent mistake, the multiplier actually used is recorded by the machine, this record has also to be inspected and compared with our data.

The question whether or not this machine manipulation be a help to the computer has to be settled by two considerations; one,

the ease of managing the machine, the other, the attainments of the computer himself; it is therefore not susceptible of a general answer.

The transference of the factors from the paper to the machine, the comparisons needed to prevent mistake, and the transference of the result from the machine are serious inconveniences, which appear the greater when we consider that a computer is far more liable to make a mistake in copying than to commit an error in calculation. When we take into account the many short cuts and the abbreviations occurring in general practice, and consider that these are quite lost in the machine work, we begin to think that the balance may not be quite so much in favour of the machine as appears at the first sight of it; and may even be inclined to enquire whether, to an expert computer, the instrument would not prove to be a serious hindrance. And then there is one obvious drawback, that the use of such an aid will not tend to make us good computers.

The application of this machine to division, consists merely in successive subtractions. You by one turn of the handle subtract the divisor, shifted to the proper step on the numeration scale, once from the dividend, and you continue this until the remainder be less than the subtrahend. Having thus discovered the highest digit of the quotient, you shift the dividend one step lower—repeat the subtractions and so continue till the last figure of the quotient appears.

This process contrasts much less favourably than that for multiplying with the ordinary paper work: indeed, to a moderately rapid computer it is irksome, and for this reason that attention has to be given to each successive remainder, lest we should turn the handle once too often; we have, in fact, much more than the mental fatigue of a common division.

The advantages of calculating machines in multiplying or dividing may with some be a matter of doubt; but there can only be one opinion when we apply them to desultory additions or subtractions. We have to add one number to another, or we have to sum up a column of numbers. Having brought the result indices to zero, we set the first addend upon the machine and turn the handle; then set the second addend and give another turn; and so on to the end of the chapter. Why, the time spent in setting the indices would greatly exceed that needed for running up the column and inscribing the result.

The great benefit to be conferred upon us by calculating

machines has always been looked for in the compilation of tables; and that benefit is expected in two ways, one in rapidity, and one in certainty of operation.

The simplest kind of table is that of equi-different quantities, such as multiples of the modulus of denary logarithms, the lengths of circular arcs; and for this work the machine seems to be admirably contrived. We have only to put the indices to the common difference, and then each turn of the handle will produce a new term in the series; with a rapidity with which the hand cannot compute, and also with, if the machine be properly made, absolute certainty. True; but where is this result? It is shown on the moveable indices, and, to be of any use to us, must be copied upon paper. Now, the time expended in this copying will be considerably greater than that needed for summing and writing the result in the ordinary way: and although the indication be correct, the copy of it is liable to error; while, if the addition be made continuously on paper, and tested in the usual way, we are almost absolutely freed from error, because an error would necessarily be continued, and so could not escape notice.

In order to obtain the advantage of the certainty of machine action, the machine itself must make its marks upon the paper. Such is the action of numbering machines for railway tickets, for pages of books, and the like. A proper calculating machine for this class of tabular work, punching the results upon a plate, or operating by help of a type-composing machine, would undoubtedly produce trustworthy results.

Nor is the table of equi-different quantities the only one to which machinery may be applied. In our house clocks we have an arrangement whereby changing differences are obtained; these differences increasing by unit each hour. So, by causing the indices of the addend to change at each step, that is, by introducing second differences, we may construct tables of a higher order, such as tables of squares. And again, by employing a third addition table to change the second difference, we may construct tables of cubes. And thus, in general, calculating machines may be made to compute and tabulate the values of all algebraic functions having integer indices. But even such machines are only advantageous when they entirely dispense with pen work.

The great mass of tables, costing labour and thought to the computer, such as trigonometrical canons, tables of logarithms, have their orders of differences interminate, and so it is impossible to compute them by machinery. For then the differences of the

last available order must be computed intellectually, and then, indeed, we might set up the type by machinery, provided each successive difference were set by hand. The instrument would therefore fall to be classed among composing machines.

Thus, on the whole, arithmeticians have not much to expect from the aid of calculating machines. A few tables, otherwise very easily made, comprise the whole extent of our expected benefits; and we must fall back upon the wholesome truth that we cannot delegate our intellectual functions, and say to a machine, to a formula, to a rule, or to a dogma, I am too lazy to think, do please think for me.

[The author overlooks the advantage of the arithmometer when used in a very long series of calculations, namely, that the work is almost entirely mechanical, and in consequence much less fatiguing after a very moderate degree of use, than direct calculation, which requires a greater mental strain. This was remarked to us some years ago, by Dr. Zillmer, President of the German Life Insurance Institute, who has used the machine extensively, and who has described some of its applications in a paper of which a translation is printed in the number of this *Journal* for April, 1869, p. 25.—Ed. *J. I. A.*]

Mr. W. J. Hancock, Actuary and Secretary of the Patriotic Assurance Company, who has had much experience in the use of the Arithmometer, has favoured us with the following observations, in further elucidation of the points discussed in the preceding papers of Major-General Hannyngton and Mr. Sang:—

In the foregoing papers by Major-General Hannyngton and Mr. Sang we have a very full description of M. Thomas' arithmometer, but there appears to be a difference of opinion as to its practical utility.

Major-General Hannyngton's recommendation is based on a lengthened practical experience of its working.

Mr. Sang, while stating that on the whole arithmeticians have not much to expect from the aid of calculating machines, does not appear to have used it to any extent.

It is admitted that the machine does the first four rules of arithmetic correctly, and the question is—does it save time, risk of error and mental labour? Take the case of multiplication; we have three methods open to us.

1st. By direct multiplication of the multiplicand by the multiplier, figure by figure.

2nd. By the manipulation of certain other numbers, which bear known relations to the numbers we wish to multiply together, such as logarithms and quarter squares.

3rd. By mechanical aid, such as the slide rule and the arithmometer.

If we take 8 figures to be multiplied by 8 figures—

By the 1st method we have 64 separate multiplications; we have to consider 64 to 72 times what is to be carried, to write down from 64 to 72 figures; then to make 14 additions, some of 8 figures each, and to consider 14 or 15 times what is to be carried.

By the 2nd method the logarithms of the multiplicand must be sought out, and the proportional parts for those figures not given directly in the table used taken out, and the result written down. The same must be done for the multiplier. The numbers so obtained must then be added, and the resulting logarithms converted into the answer required.

By the 3rd method, using the arithmometer, the multiplicand is put on the lower plate or face of the machine; the attention of the operator is then only required to turn the handle the proper number of times, and shift the slide one step for each figure in the multiplier. If on inspection it be found that the figures on the lower plate and on the multiplier disks be correct, the product shown on the upper disks must be correct, unless the machine be broken in some part.

In using the machine we may consider placing the buttons on lower plate as being equal to writing the multiplicand on a piece of paper, and the turning of the handle and shifting the slide to produce the multiplier as a little more than writing the multiplier under the multiplicand. Then we save the mental labour and loss of time incident to long multiplication. The risk of error in the machine is reduced to the risk that some one or more of the figures in the multiplicand and multiplier are wrong. In long multiplication there is, in addition to this risk (a figure may be written on paper wrong as well as be put on the machine wrong), the risk that some of the numerous mental operations to which I have referred may be wrong.

I think it is therefore manifest that the machine is far superior to the long multiplication by hand, when 8 figures by 8 figures are involved, and that it saves mental labour, time and risk of error. Of course the value of the machine diminishes with the number of figures required to be dealt with, and is not perhaps marked when the figures in multiplicand or multiplier do not exceed two.

If a man has not half a mile to travel, there is not much difference between walking and going in a railway train; but when the distance is one or two hundred miles, the advantage of going by train instead of walking becomes evident.

In comparing the machine with the use of logarithms there are two things to be considered:—First, the mental labour and risk of error in finding out the logarithms of multiplicand and multiplier, in their addition, and then in finding the number corresponding to the resulting logarithms. The mental labour here is proportionately greater for the total number of figures used in the operation than in the case of long multiplication. Second, there are many persons who are neither actuaries nor computers who do not know the use of logarithms, and yet have to multiply and divide large figures.

With regard to division the assistance afforded by the machine is, other things equal, only a little less than in the case of multiplication. I cannot agree with Mr. Sang that the mental fatigue of watching the quotient, when working the machine, is greater than in long division. Should the handle be turned once too often, it is at once detected by seeing one or two 9's on the left of the dividend on the slide, and this can be

rectified in an instant by putting the machine to addition, when one turn of the handle corrects the error.

The difference between multiplication and division, when performed by logarithms, being only the difference between adding and subtracting the logarithms, the advantage of the machine for division, so far as time and mental labour are concerned, is not quite so great as for multiplication; but, with regard to risk of error, the advantage for both multiplication and division is the same.

Mr. Sang's objections appear to amount to about three.

First. That a computer is far more liable to make a mistake in copying than commit an error in calculation.

Assuming that he is correct in this statement—which I do not admit—I think it is evident that when more than a few figures are dealt with, the computer has far more opportunities of committing an error in calculation than of making a mistake in copying.

Second. That short cuts and abbreviations occurring in general practice are lost in machine work. It is no doubt true that in many classes of cases the quantities to be dealt with bear such a relation to each other that short cuts are of great use. A butcher's boy will sometimes be able to tell how much so many pounds of beef, at so much a pound, will amount to, in a time that would astonish very able computers. Major-General Hannyngton has pointed out in his paper that the machine has short cuts and methods of its own, and that from his experience the machine requires special formulas; and Dr. Zillmer has pointed out the same thing.

The following example of a decreasing annuity calculation will illustrate the use of the machine.

Carlisle $3\frac{1}{2}$ percent (Chisholm):—

(24.) $\dfrac{(a+b)\mathrm{N}_{x+1|n}-b(\mathrm{S}_{x+1|n}-n\mathrm{N}_{x+1+n})}{\mathrm{D}_x}$ =the present value of an annuity for n years, commencing at £a and decreasing by £b annually until the end of the term.

$$x=35,\ x+1=36,\ n=23$$

$$x+1+n=59$$

$$a=\begin{cases}16{\cdot}525 & \text{Instalment}\\ 15{\cdot}203 & \text{Interest on £380·075 at 4 percent}\end{cases}$$

$$\begin{array}{lrl} & 31{\cdot}728 & \\ b & {\cdot}661 & \text{Interest on instalment at 4 percent} \\ \hline a+b & 32{\cdot}389 & \text{At 4 percent.} \end{array}$$

$$\begin{array}{l}\mathrm{N}_{36}=27615{\cdot}6\\ \mathrm{N}_{59}=\ \ 5608{\cdot}3\\ \hline \end{array}$$

$$22007{\cdot}3=\lambda 4{\cdot}3425667$$

$$32{\cdot}389\ =\lambda 1{\cdot}5103975$$

$$\overline{5{\cdot}8529642}=712794$$

$S_{36} = 397165\cdot$
$S_{59} = 50314\cdot$

$\phantom{23N_{59}}\ 346851\cdot$
$23N_{59}\ \ 128991$

$217860 = \lambda 5\cdot 3381775$
$\cdot 661\ \ \ \lambda \bar{1}\cdot 8202415$

$5\cdot 1583790 = 144005$

$N_{59}\ \ 5608\cdot 3$
$\phantom{N_{59}\ \ 560}23 \qquad\qquad 568789 = \lambda 5\cdot 7549511$

$16824\cdot 9 \qquad\qquad D_{35} = 1608\cdot 48 = \lambda 3\cdot 2064159$
112166

$\qquad\qquad\qquad\qquad 2\cdot 5485352 = 353\cdot 6185.$

$128990\cdot 9$

By machine:—

$a+b \quad 32\cdot 389$
$\times \qquad \ldots\ldots = 712794$
$N_{36} - N_{59} = 22007\cdot 3$
$b \ \ldots\ldots\ldots = \cdot 661 \qquad - \qquad = 568789$
$S_{36} - S_{59} = 346851\cdot \quad \times \quad = 144005\cdot$
$23 \qquad - \qquad = 217860 \qquad\qquad 353\cdot 6189$
$\times \quad = 128991\cdot$
$N_{59} \qquad\qquad\qquad \div$
$D_{35} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1608\cdot 48$

The calculation is done as follows:—N_{59} is put on the face of the machine, which is set for multiplication, the handle is turned three times, the slide shifted one step to the right, the handle is then turned twice and 128991 is read from the upper holes and recorded, and the slide effaced. S_{36} is put on the face, the handle turned once and it appears in the upper holes. S_{59} is then put on the face, the machine set for subtraction, and the handle turned once, and 346851 appears in the upper holes. 128991 is now put on the face, the handle turned once, 217860 appears in the upper holes. This number is then put on the face, the slide effaced, the machine set for multiplication, the handle turned once, the slide shifted one step, the handle turned six times, the slide shifted one step, the handle turned six times, and 144005· appears in the upper holes and is recorded. The slide being again effaced, N_{36} is put on the face, the machine set for addition, the handle turned once, and it appears in the upper holes. N_{59} is then put on the face, the machine set for subtraction, the handle turned once, 22007·3 appears in the upper holes. This number is then put on the face, the slide effaced, the machine set for multiplication, the handle turned nine times, the slide shifted one step, the handle turned eight times, and so on, until 32·389 appears on the multiplier disks, then 712794 will appear in the upper holes. 144005 is now put on the face, the machine set for subtraction, the handle turned once, and 568789 appears in the upper holes; the slide can be so placed in the multiplication of $(a+b)N_{x+1|n}$ that these figures will be in the left-hand holes of the slide—otherwise they must be

set up in those holes. 1608·48 is then set on the face, the machine set for division, the quotient disks effaced, the handle turned three times, the slide shifted one step, the handle turned five times, and so on until 353·6189 appears in the quotient holes. If the reciprocal of 1608·48 be taken, then 568789 is put on the face, and multiplication is performed until that reciprocal appears in the multiplier holes; then 353·6189 will appear in the upper holes. This calculation is made on a six-figure machine, so that the last figure taken down often has the usual increase of 1 when the next figure is 5 or upwards. The time taken to turn the handle and shift the slide is almost instantaneous. The decimal points are regulated by the ivory pins.

Thirdly. That the use of the machine will not make us good computers.

This objection is exactly of the same nature as the objection often made to the use of machinery in place of hand labour.

In an interesting lecture delivered by the Rev. Professor Haughton, F.T.C.D., M.D., at the Royal Institution of Great Britain, on 27th May, 1871, he points out that the human muscle, like the planet moving in its orbit, or the bee making its cell, performs its work on the principle of least action. I do not see why the brain should not do its work on the same principle; so that if the arithmometer can do those calculations for which it is suited, in less time, or in the same time, but with less mental labour than the hand alone can do them, it should be used. We do not expect the calculating machine to think for us, but to save the brain in doing mechanical work.

During the very interesting discussion which followed the reading of General Hannyngton's paper at the Institute of Actuaries, every gentleman who spoke of having used the machine for any length of time referred to the saving of time and mental labour it effected. Mr. Sprague pointed out that logarithms would be more convenient where three, four, or more factors were to be multiplied together; or where several factors were to be divided by two or three others.

No doubt there are many calculations in which the machine would not give assistance; but that does not make it less useful where it can be applied. A sewing machine is not condemned because it will not sew on buttons. On the whole, I think the balance of argument is in favour of the arithmometer; and I have no doubt that with more extensive use improvements will be made in construction and workmanship.

On the Equitable Apportionment of a Fund between the Life Tenant and the Reversioner. *By* A. Baden, *Fellow of the Institute of Actuaries.*

[Read before the Institute, 27th March, 1871.]

If any excuse be wanting for once more bringing this question forward for your consideration, it is not because the question itself is unimportant. The frequency of the cases involving it upon which the opinion of actuaries is sought, and the magnitude of the

interests concerned, will be allowed by all to furnish ample grounds for the utmost possible care and pains in determining the principles by which the advice given should be regulated. Upwards of 15 years have elapsed since Mr. Jellicoe's well-known paper on this subject was read before the Institute. How far the practice of actuaries has been influenced by the conclusions laid down in that paper I do not know; but from the fact that (so far as I am aware, at least,) the subject has not been reopened since, it would seem that these conclusions have been accepted by the profession, and that in the opinion of those most competent to form a judgment there is nothing more to be said about them. If this be so, I am afraid that I shall incur the suspicion of rashness when I say that the excuse I have to urge for reopening the question is, that the conclusions laid down by Mr. Jellicoe do not commend themselves to my mind as perfectly satisfactory. I propose very briefly to state my reasons for differing from him; and should I fail, as I well may, in convincing others of the correctness of my views, I shall be consoled by the reflection, that however firmly established, and apparently beyond discussion, any truth may be, it is a profitable thing for most, perhaps for all of us, if a rash sceptic now and then compels us to review the grounds of our confidence, and to restate to ourselves, if not to others, the reasons for the faith that is in us.

It is my intention to deal only with the condition in which there is presumed to be an equal desire on the part of the life tenant and the reversioner to bring the fund under immediate control, and in which, consequently, no consideration but that of strict equity is to be admitted in determining the respective share of the fund to be allotted to each of the parties.

It cannot be denied that a great improvement in the practice of our profession dates from the application of Griffith Davies's formula, expressing the value of an annuity in terms of the annual premium $\frac{1}{p+d}-1$, by taking p as an actually payable premium, and d as the proper remunerative rate of prepaid, or discounted, interest on the capital laid out,—and from the modification of the formula for the value of a reversion, $1-d(1+a)$, by taking a as a practicable annuity value, and d as the remunerative rate; and for this improvement, I take it, we are mainly indebted to Mr. Jellicoe. But however applicable the method thus introduced may be to many cases presented to us in practice, I do not think it is adapted to the solution of the case we are considering.

The example Mr. Jellicoe gives is that of a sum of £1,000 Consols, taken at a value of 85, the proceeds of sale at this rate to be divided between a life tenant aged 70 and a reversioner. Valuing these respective interests by the above-mentioned formulæ, he divides the whole fund, £850, between them, in proportion to the values so arrived at. In justification of this mode of procedure, Mr. Jellicoe says, "it is obvious that the interests are only not the same as they would be were they each selling separately, from the circumstance of their being enabled to offer the whole fund to one purchaser, and thus greatly to increase its value; and as this increased value may be considered as arising from an augmentation in the value of each share, it seems only reasonable that it should be apportioned accordingly. Hence the right mode of procedure in this case would seem to be, to estimate the interests of the life tenant and the reversioner as they would be were they to be sold separately, and to divide between them the proceeds of the fund in the proportions indicated by the estimates thus made. But it has been shown that the market values of the interests in question are properly found by the formulæ $\frac{1}{d+p}-1$ and $1-d(1+a)$ where the current rates for the use of money are taken, and where the two interests are charged, as they should be, with the expense of an assurance in the one case and of an annuity in the other."

Now, what I have first to remark about this mode of settling the question is, that the adoption of the (real or supposed) market values as a standard of equity where no actual sale of either interest in the market is contemplated seems scarcely to be warranted. Surely it is the business of an actuary to say, and he is supposed to hold in his hands the means of saying, what, under equal conditions,—first, as to the average expectation of human life at the given age of the life tenant; and, next, as to the rate of interest the respective shares should amount to, quite independent of the accidents and fluctuations of the market. But setting aside such accidents and fluctuations, which the method does not, as no method but that of actual sale could, take account of, there is a radical default of equity arising out of the fact that not only are different tables of mortality used in the actual granting of the two kinds of benefit involved, an assurance and an annuity, but that different rates of interest are for the most part assumed in the consideration demanded for these respectively, and, worse still, that an addition for expenses is made to the charge for one benefit and not to that for the other. In the method, then, we find an

inequality of conditions applied to the two interests, which must produce results repugnant to any idea that I can form of real equity.

In the next place, I do not doubt that those who advocate the use of Mr. Jellicoe's method would generally admit that the indirect way in which the result is arrived at is a somewhat awkward expedient, to be justified only as a *pis aller*. The sum of the values found does not amount to the fund to be divided. There is a surplus to be appropriated in the ratio of the respective values. Of course, if the values *must* be so determined as to render the appearance of a surplus inevitable, there is nothing else to be done. But everyone will feel that if there is a way of valuing the interests so as to leave no surplus, a disagreeable anomaly of method will have been got rid of. And here let me say, that the existence of a surplus at all seems to me to strike at the very root of the principle on which the method is justified. When it is urged that the fund ought to be divided rateably according to the values of the interests, and that the only way to ascertain the value of a thing is to find out what people are willing to give for it—in other words, what it will fetch in the market—it seems to be forgotten that the question to be decided is, not the values of the interests only, which the market might be permitted to determine, but the equitable division of a fund, with which the market can have nothing to do.

In making these remarks, I would not have it to be supposed that I do not recognize the superiority of Mr. Jellicoe's method, indirect, though it be, to the mode previously much in vogue, that, namely, of determining by some standard deemed proper by the computer the value of *one* of the interests involved, A's or B's, as it might be, and leaving the balance of the fund for the value of the other. Unquestionably Mr. Jellicoe's plan produces results far more nearly approaching to equity than this.

But setting aside for the moment the objection arising from the existence of a surplus, I would revert to the first objection I raised —that based on the inequality of the conditions to which the two interests are subjected by Mr. Jellicoe's method. I think it is fairly open to us to view the question in the following light. Taking the sum to be divided, as in the example quoted, to be £850, and the fixed revenue for A's life to be £30, A being 70 years of age, A is in possession of a certain revenue which he can dispose of as he likes—if he were to invest it as he received it, and likewise invested the interest thus arising, in some easily attainable and

negotiable security, he might reasonably expect to accumulate a fund at 4 per cent compound interest. We may, therefore, assume that the value of his life interest should not be less than £30. a_{70}, and that sum should be set aside out of the fund to be divided, as the minimum to which he can be entitled; then under the equality of conditions which equity requires, a sum should be set aside as the minimum to which B can be entitled, which, accumulated at 4 per cent., would at A's death amount to the fund to be divided —that is to say, B's minimum would be A_{70}.850, A_{70} being taken at 4 per cent. But when these two values had been set aside there would remain a surplus, and the only way of dealing with that would be to divide it rateably, as Mr. Jellicoe does. The final result would be, that A would get £214, and B £636, that is to say, if the Carlisle table were used, which is, I believe, that most generally adopted. By Mr. Jellicoe's method the respective shares come out £190 and £660; and it ought to be remarked that the portion thus assigned to A is less than the bare value of the revenue at 4 per cent., which is £201—A, in fact, gets a sum that would not amount to what he might expect to accumulate at 4 per cent., and B gets what, if invested at only 3¼ per cent., would about reach the sum he would be entitled to at A's death if the fund were left as it is. That, surely, is a result quite inconsistent with equity.

If we are to choose between these two methods, it seems to me that preference should be given to the first. It is true we do not get rid of the anomalous surplus, and in this respect the method is only on a par with Mr. Jellicoe's; but we do get rid of the inequality of conditions. We assume the same standard of life for both interests and the same rate for the improvement of money; and in so doing we fulfil the requirements of equity, not perfectly, it is true, but much more nearly than the assumption of a market value as the standard will enable us to do.

The only way of completely realizing an equitable adjustment is at the same time the only way of arriving at a direct division of the fund, so as to leave no surplus, the sum of the two values being the whole divisible amount. It is obvious that this can only occur when the revenue which constitutes the life income is the exact measure of the rate of interest to be assumed. Let the income be divided by the fund, and the quotient will be the rate of interest at which the value of the life interest is to be determined by multiplying the income into the present value of an annuity, and that of the reversioner by finding the difference between the value of the life interest and the whole fund. That is to say, the

life interest is $S.ia_x$ and the reversion $S(1-ia_x)$. Or, treating S as the value of a perpetuity, it becomes $I\cdot\frac{1}{i}$, and we then have the formula $I\cdot\left(\frac{1}{i}-a_x\right)$, i being in each case $=\frac{I}{S}$, and $I=Si$.

Let us take the same case as before for an example. The income is £30, the fund £850, the former divided by the latter yields a rate a fraction over $3\frac{1}{2}$ per cent. A's interest is therefore the present value of an annuity at age 70 and $3\frac{1}{2}$ per cent., multiplied into £30, that is (again using the Carlisle Table) =£207; consequently B's interest is £643. Here, then, it seems to me that we have the true solution of the problem; there is nothing arbitrary in the way of arriving at it, an accepted standard of human life is appealed to, and a rate of interest used which has not been left to the choice or opinion of the computer, but has been furnished by the actual conditions of the case. There is an objection, obvious indeed, but from a practical point of view by no means formidable, to the adoption of this method—this, namely, that for an absolutely correct determination of cases presented for solution, the actuary should have at hand a table of annuity values at the exact fractional rate which may be represented by the quantity $\frac{I}{S}$. It will, however, at once be seen that there need be no difficulty in obtaining by interpolation a value quite near enough to the exact truth for any practical purpose.

Mr. Jellicoe in his paper makes some remarks, to which, as they may be considered condemnatory of the method I am recommending, I feel bound to refer. "I believe," he says, "the mode of solution most generally adopted is to assume the market rate of interest prevalent at the time, to look upon the fund as a perpetual annuity, and to deduct from the value of it that of the annuity during the life of A (the tenant in possession): that is to say, B's interest would be denoted by the formula $I\cdot\left(\frac{1}{i}-a\right)$ where i and a are taken, say, at 5 per cent. By this rule, the share of A will be £190, and that of B (the reversioner) £660, supposing the stock to realize £850." Now in this there must be some oversight. The formula given does not produce any such result. What *does* bring out the figures quoted is the value of the annuity on A's life, deducted from the whole fund, treated as a capital sum, and *not* as a perpetuity—that is, the formula $S-I.a$. The two formulæ do indeed become identical when, as in my method of

treatment, $i = \frac{I}{S}$, but under no other circumstances. But my reason for referring to this now is, because further on in his paper Mr. Jellicoe condemns the method in these words, "it appears to me to be altogether erroneous, from its dealing with B's interest *as though it were a perpetual annuity.* At A's death, B succeeds to the corpus of the estate, and can dispose of it as he pleases; the assigning to him the perpetuity only is a restriction not justified by the fact. The property he succeeds to is liable to variation in its value, although the income from it be fixed; and this variation must be taken into account." From the oversight I have pointed out it is difficult to appreciate the exact bearing of these remarks—but they would seem to be sufficiently general to involve my way of dealing with the question in the condemnation pronounced, since I propose to treat B's interest as a perpetuity. But the simple fact of treating B's interest as a perpetuity cannot *per se* involve any unfair depreciation of it. If i were taken at a higher rate than that determined by the actual income, that is, than $\frac{I}{S}$, then indeed B's share would be undervalued, as on the other hand, if i were taken at a lower rate, the share would be *over*-estimated; but when i is exactly coincident with $\frac{I}{S}$ a perfectly fair value must result. And that, in point of fact, my method, whether it be abstractedly fair or not, does not unduly depreciate B's interest when compared with Mr. Jellicoe's, the following figures will show:—

Division of a Fund of £1,000 (*at par*), *producing an Income of* £30, £40 *or* £50, *between A the Life Tenant and B the Reversioner.*

1. A being aged 60.

Income.	A's Share.		B's Share.	
	1.	2.	1.	2.
£	£	£	£	£
30	315	336	685	664
40	387	403	613	597
50	447	458	553	542
2. A being aged 70.				
30	214	232	786	768
40	268	287	732	713
50	317	335	683	665

*** The results in Columns 1 are mine—those in Columns 2, Mr. Jellicoe's.

I have before said that the question is not one in which market value can be properly admitted as the standard, because it is not the sale of certain benefits but the division of a fund that is to be effected. When a benefit is put up for sale, the buyer has only to offer such terms as will yield him a proper rate of interest on the capital laid out and insure the return of the capital itself. If A's interest be bought, the purchaser has not at all to concern himself with B's rights; and conversely, the buyer of B's interest does not seek any adjustment with A. But when A and B agree to divide the fund *equitably* between them, each has to receive as much of it as he can get, *saving the rights of the other.* The duration of A's life is the reciprocal measure of both benefits, and we certainly have not to go into the market to determine that. Provided only a proper table of mortality be applied, A's chances of enjoying the income and B's of reverting to the fund will be settled with an equity which there is no disputing.

The only question then remaining to be solved is, what rate should be assumed for the improvement of money? and the fact that any other rate than $\frac{I}{S}$ will not work without producing a result either exceeding or falling short of the sum to be divided, seems to me to be conclusive on this head.

In conclusion, I would point out that unless it can be clearly shown that in some way which I have failed to detect the method I have been recommending contravenes equity, a strong inducement to use it should be found in this—that *merum arbitrium* is excluded, and that by the strict application of a principle there has been established a centre of gravity, so to speak, around which a variety of cases will revolve, at distances varying with the special circumstances of each.

The following account of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

Mr. Sprague welcomed Mr. Baden as a new contributor to the transactions of the Institute. He thought no apology from Mr. Baden was necessary for reopening the subject. Much as Mr. Jellicoe's opinion is respected, they disclaimed a blind deference to any authority however high. He had never himself altogether agreed with Mr. Jellicoe's view of this question. So far as he had understood Mr. Baden's views, from listening to the reading of the paper, he was more inclined to agree with him. He thought that in most cases the proposal for division came from the reversioner, who would say to the tenant for life—"I will make it

worth your while to divide the capital." The least the tenant for life should get under such circumstances was a sum sufficient to buy the same income from the Government or an Assurance Company, with, perhaps, some bonus over. According to this view, the rate of interest at which the capital was employed was of no consequence, and should not govern the transaction. He would suggest to Mr. Baden that it would be well to add to his paper a more extended table of results, such tables being very useful for purposes of reference and comparison.

Mr. A. H. Bailey said Mr. Jellicoe's paper put three cases—first, the case where both the tenant for life and the reversioner are anxious that a division of the fund shall be made; second, where the arrangement is for the interest of the reversioner and not of the tenant for life; and third, the unusual case where it is for the interest of the tenant for life and not of the reversioner. Only the first of these was treated of by Mr. Baden; and to that case Mr. Sprague's remarks hardly applied. It was admitted on all hands that when the change is made for the benefit of the reversioner, the life tenant should receive what would purchase a Government annuity of like amount. For himself, while admiring the clearness with which Mr. Baden's views are stated, he could not agree with his conclusions. It seemed to him that one source of perplexity arose from the use of the words "equity" and "equitable." Mr. Baden's meaning seemed to be abstract justice, but the words were used in a more limited sense, to express the rules and practices of the Courts of Equity; and that was the sense in which he believed actuaries must use the words, because it was to Courts of Equity that these cases were referred for adjudication. It was well known to every one who had been examined in the Rolls Court, or had made affidavits in Chancery on values of life interests or reversions, that the question invariably asked was, "What is the market value?" Therefore the market value was the *equitable* value in the limited sense of the word he adopted. The Court insists that the market value shall regulate their proceedings; therefore Mr. Baden's distinction between the equitable and the market value fell to the ground. But independently of that, he thought market values were to a great extent justifiable on abstract grounds. Mr. Baden had objected to the use of different rates of interest in valuing life interests and reversions. But there is a larger market for life interests than for reversions, because they produce an immediate income, while reversions lock up capital; and hence the rate of interest on life interests is lower. Mr. Baden also objected to Mr. Jellicoe's method on the ground that the assurance premium includes a charge for Office expenses, which the price of an annuity does not; but a Reversionary Office has also expenses to meet, and that is done out of the higher rate of interest. Mr. Baden thought it anomalous that the value of the life interest and that of the reversion together sometimes come to less than the fund. But that is only what occurs in other matters. If a man buys Consols and sells them the next moment, he will not obtain the same terms, because there is a turn in the market. The turn of the market in life interests and reversions is a much larger margin; and when the life tenant and reversioner join, the question is how to dispose of that margin. He could not see a better way than doing what he supposed the Court of Chancery would do—ascertain the market value of each interest, and divide the surplus in the same proportion.

Mr. C. J. BUNYON confessed that notwithstanding Mr. Jellicoe's paper he should have thought there was but one possible method—that which he had himself always adopted, and which was now advocated by Mr. Baden. The question considered by Mr. Baden is "In what way are you to divide the fund where both parties are equally desirous that it should be divided?" In such case he could not see how any question of market value can come in. The market price is what you can get, and that depends on the purchaser. A horse to one man may be worth 150 guineas, to another only 80. He did not conceive there could be any mode of dividing the fund except by ascertaining the values of the two interests at the rate of interest derived from the fund and giving accordingly. If nothing can be spotted as the cause of sale, the interest of the fund exactly measures the interest of the parties, because the lower the rate of interest the larger the share of the reversioner; and it ought to be so, for the lower the rate of interest the greater the security of the reversioner. He entirely agreed with Mr. Baden's use of the term "equitable." How did the Court of Chancery look at these things? It is a rule of the Court that no fund shall be invested so as to benefit either the tenant for life or the reversioner at the expense of the other. Undoubtedly the reversioner had an interest in having every fund kept in the safest possible investment; and that being so, he had a right to say "you shall not invest the fund at any other rate than that it is now producing," and I say that in that case the reversioner is entitled to a larger share of the fund than he would be if he could not help its being invested at a higher rate of interest. One reason for using Mr. Baden's method is that if you go into long calculations of the separate values, and then make a rateable division, you give yourself a great deal of unnecessary trouble. A stronger reason is, that by valuing by Mr. Jellicoe's method you introduce elements of calculation which have nothing whatever to do with the subject. You have to divide a fund according to the equitable interests of the two parties; what has this to do with the rates of premium charged by Assurance Companies? Nothing in the world. The tenant for life takes his money, and does not want to insure his life; why then should the element of insurance be introduced? In the same way the reversioner wants some cash; he does not want to sink money in an annuity. Having ascertained the fair rates of interest and mortality to assume, they should be used throughout the calculation, and the fund divided accordingly.

Mr. AMBROSE SMITH remarked that Professor de Morgan wrote one of his most lucid essays on the points under discussion. In a paper in the "*Companion to the Almanac*," he dealt with the subject as an isolated case beyond the scope of direct calculation by a mortality table, and to be viewed in its actual results only. This method is quite at variance with that advocated by Mr. Bunyon, and would produce a different result from that anticipated by Mr. Baden; inasmuch as valuing the two interests by it, the sum of their values is greater—not less—than the fund. The difficulty therefore was not exactly as Mr. Baden had put it. The case was, A has £30 a year for life, the proceeds of £1,000 Consols; B is entitled to a reversion to £850 at A's death. These were the only two facts. The annuity and the reversion are both certain; therefore, carrying out the method of Professor de Morgan and Mr. Jellicoe, we had only to look at the value of the benefits to the parties. A knows what he would

have to give for his annuity in the market. B knows what single premium he must pay to purchase his reversion. He did not think they need enquire further.

Mr. H. W. Porter had a "case" before him at that time, involving a considerable sum. It was desired by a father and son to bar the entail to an estate; and he was asked to state the values of the father's life interest and of the son's reversion. This was precisely the case under discussion,* one to be determined as between man and man, and not strictly according to market value. There were in use at least four ways of valuing the interests. The first is to ascertain the value of the annuity of the actual rental of the estate, and to give the balance to the reversioner; the second is to use Mr. Griffith Davies' formula, in which the insurance element is introduced; the third is that of valuing the reversion and taking the difference between that and the value of the estate; and the fourth is Mr. Jellicoe's method. Of these Mr. Jellicoe's plan seemed to be much in use, and was a very practical way of meeting the case. He had put down the figures for his case, and the divergence was very great. The difference by Mr. Jellicoe's formula and Mr. Baden's was 2 per cent. only for B's share. The greatest difference was between the formula with insurance but without division of the fund, and Mr. Jellicoe's—viz. 28 per cent., amounting to several thousand pounds. After all it was a matter of judgment how to treat the case. What is wanted in the cases coming before an actuary is the actual market value, and it is the Reversionary Companies in the auction room that really fixed the values of these securities. The discussion of these questions by the Institute of Actuaries was of the greatest importance and use.

Mr. C. Walford had had some experience of these cases. Formerly the rate of interest used in these calculations was always 3 per cent.; but now-a-days we have to deal with quite another class of securities, such as the guaranteed 5 per cent. Indian Railways. He took it for granted that a life interest arising from a fund so invested would be valued at 5 per cent. If the annuity arose out of land, and the rate of interest yielded was uncertain, then some rate which seemed to meet the circumstances should be taken. In fact, when a fixed rate of interest is realized on the fund, that will be the guide; if otherwise, the actuary must use his own judgment.

Mr. E. Smyth thought the only reason why any doubt could arise as to the propriety of Mr. Baden's method was that the market value of each interest is less than the value brought out by taking it as a part of the freehold. He agreed in the main with Mr. Bailey, as to the Court of Chancery having regard to the market value, but doubted whether the Court would decide as to the principle of dividing the dormant or surplus value. What is the reason of the difference between the market value and that brought out by Mr. Baden? It seemed to him to arise from there being less competition for either interest singly than for the fee simple. Another cause is that a person buying a fund absolutely can deal with it as he pleases; but a person buying a life interest is bound to his reversioner, and is therefore under disadvantageous restrictions. In the same way a person buying a

* This case appears to introduce a new consideration, inasmuch as the father and son by joining can acquire the fee, which is of greater value than their interests would be together, if both purchased by a third party.—Ed. *J. I. A.*

reversion may have to resell it as a reversion, and so will be put to certain risks and costs, so that each party damages the other. But if they united their forces and dealt with each other, the disadvantages which caused the difference of price disappear, and with them he thought might disappear the allowances that had been made for them. It was impossible to prove the exact equality of the disadvantages. Such proof became impossible the moment they left the region of pure mathematics; or else the discussions in that room would not be so numerous and lively as they were. He thought that setting off the one against the other did rough justice.

The PRESIDENT had always been inclined to concur in the view taken by Mr. Jellicoe. They must bear in mind that the case Mr. Baden brought before them was clearly one where it was equally the wish and interest of both parties to get at the fund; and therefore Mr. Sprague's observations did not exactly apply. They were told by Mr. Baden that market value has nothing to do with the case. He could not understand that. They were told that the rate of interest yielded by the fund was to be the criterion; but it seemed to him that this had little or nothing to do with the matter. What do the parties ask for? To have a sum of money paid down; and the value of that sum to them depends upon how they can invest it, and therefore the market rate of interest had an important bearing on the transaction. If they are not perfectly agreed, it becomes a matter of bargain and sale; and the party seeking for the division of the fund must offer such terms as will induce the other party to consent. But the case under consideration is that of arbitrating between the two parties in reference to their mutual wishes, and he could not see a better safeguard than to fix upon the market value of both interests. It was true that sale by auction in a particular case is not always a guide to the value of a life interest or a reversion—he had known instances of extraordinary contradictions in that respect; still every actuary could estimate approximately what an interest is likely to fetch in the market, and it appeared to him that the only true way of getting at the division of the sum is first to ascertain, as Mr. Jellicoe supposes, what each would sell his interest for, and then to divide the whole fund in proportion to those values. Suppose that a life tenant and a reversioner had each separately applied to an Office dealing in such securities and ascertained what they could sell their interests for, that then they met and disclosing to each other what they were about, were to say "Why should we allow these people to make a profit out of us? why shouldn't we realize the whole fund and divide it?" The first question then would be "How shall we divide it?" He thought it would actually suggest itself to them to share the whole fund in proportion to what each had agreed to accept for his separate interest. Mr. Baden thought it an anomaly that there should be a surplus. He did not think so. If he had half a house and some one else the other half, and each sold his half separately, the two values would fetch much less separately than the whole house if we joined. The main question was whether you should charge the tenant for life with the premium necessary to recover the capital sunk, or whether you should take a tabulated annuity value; and a similar question arose with a contingent reversion, whether you should charge the reversioner with such a premium as will replace the sum at the death of the tenant for life if the reversioner die first. If they bought a

contingent reversion or a life interest, without being covered by an insurance, they were not making an investment but a speculation. He thought it would simplify the matter to consider the case when the division takes place between a tenant for life and a person entitled to a contingent reversion only. This case might perhaps never arise, but all modes of apportionment should provide for every case that can possibly arise. If the trustees were willing to divest themselves of the fund they must cover themselves by an insurance. Every one would agree that the premium for such insurance should be paid by the reversioner; and if the premium had to be paid by the reversioner in the one case, why not by the life tenant in the other? Each wished to convert an uncertainty into a certainty; the reversioner in case of an absolute reversion was sure of his money at some time or other, but the tenant for life might die to-morrow, and if he wished for certainty he must pay market price for it. Take the case mentioned by Mr. Porter, that is to say, of a man who has a life interest in a landed estate, where it is proposed he should join with the next tenant in tail to bar the entail, so that they may jointly acquire the fee. The tenant in tail, unless he has children, has nothing whatever but a reversionary life interest in the estate (if he has children, he can certainly bar them). There is no doubt then that a life tenant and reversioner can continue to obtain the fee simple of the estate, and thus get an addition to their property at the expense of the person or persons who would succeed to the estate provided no such arrangement were made. In apportioning such an interest you surely would not give the same sum as you would for an absolute reversion which must fall in at some time or other, the only contingency being the lapse of time before it falls into possession. These considerations appeared to him very much in favour of Mr. Jellicoe's plan.

Mr. Baden having taken time to consider the various arguments used in the discussion, requests us to insert the following as his reply to them.

"It will be convenient to divide my remarks into five heads.

"1. It would seem that the proper meaning of the term 'equitable' is open to discussion. It is objected that I use the word 'equity' as an equivalent for 'abstract justice'—whereas, instead of attempting to settle what is abstractedly just, it would have been proper to ascertain and apply the principles upon which the Court of Chancery would deal with the question; and that the method so arrived at, and that only, would properly be called 'equitable.' I have no objection to the phrase 'abstract justice,' except in so far as it may be held to connote something shadowy and vague. What I aim at finding out is, the perfect justice of the case; or, since it may be said that justice admits of no degrees, and therefore that the epithet *perfect* is a pleonasm, I would state my aim to be simply—justice.

"Now the Court of Chancery, as I take it, aims at precisely the same thing, and *because* it does so is styled a Court of Equity. But whether the *practice* of the Court in particular cases is such as to insure an equitable or perfectly just decision is surely open to question. And if the question be

raised, those who take upon themselves to discuss it must set up in their own minds and endeavour to demonstrate to others some standard of equity to which appeal may be made. This is what I have attempted in the case of the division of a fund between the life tenant and the remainder man when both are equally desirous of effecting the arrangement. Either the plan I propose is equitable, or it is not. If not, there is at once an end of the matter; but if it be, then in so far as the principles upon which it is assumed that the Court of Chancery would act, lead to different results, to that extent, I should say, the Court would fail to secure the equity it aims at.

"2. Upon what principle then is the standard of equity to be determined? The principle underlying all equity may be stated in general terms to be what the word itself so clearly points to, absolute equality of conditions. Let us see what these are in the case before us. The life tenant is in receipt of an annual sum so long as he lives; the reversioner will succeed to the fund which produces the income, when the life tenant dies. A has to get an equivalent for his chance of receiving the income one year hence, for his chance of receiving it two years hence, and so forth, until z years, when there is no chance of his being alive any longer; the sum of these equivalents is the value of A's interest. B has to get the equivalent for his chance of receiving the whole fund plus one year's income* one year hence, two years hence, and so on until z years, when by the table it is certain he will receive it; and the sum of these equivalents is the value of his interest. To settle A's chance of receiving the income and B's of receiving the fund, year by year, the same table of mortality must of course be referred to. Here we have absolute equality in the first condition, the determination of the chances of life. But in obtaining a present cash payment in consideration of foregoing the sum which each year they have the probability of respectively receiving, they must each have what, when improved at the same rate of interest will amount at the several periods to the expected claim. Equality of rate is thus secured, but something more is wanted. Not only must the rate be *the same* for both, but *what* it shall be must be determined upon some fixed principle which shall leave no room for choice. Now, there is but one rate in every case which will divide the fund exactly without surplus or deficit, and that is $\frac{I}{S}$, which is, therefore, clearly marked out as the rate to be used; and thus is secured absolute equality in the second condition.

"3. If the proposed standard be, as I maintain it is, a strictly equitable one, it must necessarily exclude every other, and the burden of directly proving any other to be false can hardly be said to rest upon me. But as it may throw further light upon the subject to show how it is that the market standard falls short of the requirements of equity, I will endeavour to do this. The price of any expected benefit in the open market must on the whole, after allowing a certain margin for accident and caprice, depend upon its character as an investment of capital. The buyer will give for it just so much as will yield him a fair rate of interest and secure the return

* It is quite unnecessary to complicate the matter by taking into account half-yearly or quarterly payment of income, and that up to the moment of death. Such details do not affect the principle.

of his outlay. In buying a life interest, as his whole capital may be swept away to-morrow by the death of the tenant, he provides in the terms offered for an insurance against this loss. So, in the case of a reversion, as the life tenant may live beyond the average expectation, and by just so much as he does will the buyer be a loser of the interest on his accumulated capital, the latter offers such terms as will enable him to cover himself by an annuity.* Of course, the buyer of either benefit may decline actually to effect an insurance or purchase an annuity. He may choose to take the risk on himself, either because he deals so largely in such transactions as to secure an average return, or simply because he is of a speculative turn, and goes in to win or lose as fortune may determine. In either case, the fact remains that the market price is upon the whole regulated by the character of the benefit regarded as an investment. But when life tenant and reversioner come together and agree to divide the fund between them, the circumstances of both interests are entirely altered. In conceding an equitable share of the whole to B, A is not laying out money; and to assume that he is, and must regulate his terms accordingly, is to assume the merest fiction. In like manner, B in conceding A's share is not going in for any uncertain investment. A may die to-morrow, it is true, but B has got in hand what if invested on the assumption which at the time it was made was a perfectly fair one, for the average term of a life of A's age at the rate of interest equally determined for both, will really put him in possession of the fund. Neither party is spending money, both are receiving it. Is it equitable, then, to take a standard which is based upon pure fiction, a standard which arises out of a set of circumstances utterly at variance with the actual facts of the case? For my part I cannot conceive what justification there can be for such a course.

"4. But when the market standard has been invoked what does it accomplish? Does it divide the fund? If it does it is only as an exception and by accident; as a rule it will give us two sums which together either exceed or fall short of the fund. If there be a surplus, it is all very well for the parties to say "let us divide it rateably," but what right have they to do so? Each has already got the value of his interest, and the surplus is no man's money, and might be treated as treasure-trove. Might not the ingenious Mr. Lowe introduce a Bill into Parliament providing for the absorption of such excrescences by H. M. Exchequer? The two parties have confessedly to receive the *value* of their respective shares; if there be but one value and that has to be settled by the market, they get by the rateable apportionment either more or less than they ought. But if there be two values corresponding to the two altogether different sets of circumstances, if there be, as I maintain, *either* a market value where there are recipients who are vendors and payers who are purchasers, *or* an equitable value where there are recipients who are not vendors and no purchasers at all, then we have only in this latter case to apply an equitable method and the fund will be exactly divided without surplus or deficit. The rateable division method indeed proceeds on the assumption of a resultant value which is not that of the market and yet the market is appealed to as its foundation. Surely I am justified in calling such a process an anomaly.

* This is on the supposition that the value of the reversion is calculated by Mr. Jellicoe's formula; but we doubt whether that is commonly used.—Ed. *J. I. A.*

And it must be borne in mind that this is not a mere doctrinaire question as to the better of two ways of arriving at the same result; the results themselves differ, and if that of one way be fair that of the other must be unfair. It is said that the rateable division method does "rough justice." Well certainly rough justice is better than none at all, but is not smooth justice better still? Why betake ourselves to inexact means when exact ones are ready to our hand?

"5. The allegation that actuaries are *bound* to frame their opinions in accordance with the practice of the Court, be it right or wrong, is to me quite unintelligible. In the first place the cases which come under the category I am dealing with are not as a rule referred to the Court at all; they are for the most part matters of private agreement or private arbitration. In the next place if they do come before the Court and the Court insists upon ascertaining, not what is the equitable value of any interest, but what is its market value, although of course an actuary would be bound to answer truly the question which is put, he is not bound to think that the Court is pursuing the right method of inquiry in order to arrive at a judgment, and this right method is the sole subject of my investigation.

"When I submitted my views to the Institute, I did not for a moment expect that they would pass unchallenged, but I did expect that those who found themselves unable to assent to them would make some attempt to show in what respect they really involved a failure of equity. I thought it possible that there might be some lurking fallacy in my argument, and that what appeared to me to be an absolute equality of conditions might be shown to imply some essential though perhaps not obvious *in*equality. But unless I am so blinded by a foregone conclusion that I am unable properly to appreciate the reasons of those who have expressed their dissent, this yet remains to be done."

Mr. Porter has kindly furnished us with the following figures, showing

(1) The results of the division of £1,000 sterling, producing £50 per annum, between the tenant for life, aged 45, and the reversioner, in a case *nearly approximating* to that referred to by him;

(2) The results of the division of £850 sterling, producing £30 per annum, the age of the tenant for life being 70;

computed by six different formulas, for comparison with the figures given in Mr. Baden's illustration. He adds, that both these illustrations show that Mr. Baden is the good friend of the reversioner, and seem to indicate that the question of the proper division of a fund between the tenant for life and the reversioner is still far from being satisfactorily settled.

I.

Highest to Lowest Tenant for Life. A £	Lowest to Highest Reversioner. B £	Formulas used.
759	241	Reversion to B by special formula $1-(1-v_5)(1+a)_{3\frac{1}{2}}$, and balance to A.
705	295	Carlisle 4 percent annuity to A, and balance to B.
657	343	Mr. Jellicoe's formula:— Annuity to A by $\frac{1}{1-v_5+P}-1$, using Northampton 3 percent non-participating premium ($P=\cdot 03313$), reversion to B by Carlisle 6 percent; with division of fund in proportion of one share to the other.
650	350	Carlisle 5 percent reversion to B, and balance to A.*
632	368	Carlisle 5 percent annuity to A, and balance to B.*
569	431	Annuity to A by $\frac{1}{1-v_5+P}-1$ as above, and balance to B.
632	368	Mr. Baden's values, identical with the last but one.

* If the annuity be taken as *complete* and the reversion as payable at the instant of death, the values found by these two methods would of course coincide; being in that case £641 to A and £359 to B.—Ed. *J. I. A.*

II.

Highest to Lowest Tenant for Life. A £	Lowest to Highest Reversioner. B £	Formulas used.
320	530	Reversion to B by special formula $1-(1-v_5)(1+a_{3\frac{1}{2}})$, and balance to A.
297	553	Carlisle 5 percent reversion to B, and balance to A.
215	635	Mr. Jellicoe's formula:— Annuity to A by $\frac{1}{1-v_5+P}-1$, using Northampton 3 percent non-participating premium, ($P=\cdot 10017$), reversion to B by Carlisle 6 percent; with division of fund in proportion of one share to the other.
201	649	Carlisle 4 percent annuity to A, and balance to B.
190	660	Carlisle 5 percent annuity to A, and balance to B.
173	677	Annuity to A by $\frac{1}{1-v_5+P}-1$ as above, and balance to B.
207	643	Mr. Baden's values.

On the Risk attaching to the grant of Life Assurances. By Dr. C. Bremiker, *of Berlin. Translated by* T. B. Sprague, M.A., *Vice-President of the Institute of Actuaries.*

(Continued from page 221.)

II.

Having thus, as I believe, demonstrated that life insurance calculations have nothing to do with probabilities, I come back to

the idea of risk. This, as I pointed out at starting, must be taken from the theory of probabilities, or more precisely, from that part of it which has been cultivated since the beginning of this century, by Lagrange, Gauss, Laplace, and others, viz., the method of least squares. In that method is defined the idea of the "*mean error,*" which is considered as the measure of the danger to which we are exposed in a single case. This "*mean error*" is the square root of the sum of all the squares of the errors divided by their number; and the squares of the errors themselves are formed from the deviations of all the single cases from the average or most probable value. In insurances depending upon life and death, the value is also calculated according to the average, so that when all the assured are dead, if the mortality has followed the mean numbers given by the table of mortality, and the additions to the premiums for the expenses of management are disregarded, there will be neither surplus nor deficiency. This average value is the so-called net premium, which may be either a single premium or may be payable for a term of years agreed on beforehand. But we can calculate beforehand from the mortality table all the deviations, or the gains and losses, which can arise from the earlier or later death of the lives assured. Squaring all these deviations, and dividing the sum of the squares by their number, and taking the square root of this sum, we get the value of the mean danger or the risk attaching to a single insurance. For further elucidation some applications of this process will now be given.

III.

Suppose an Insurance Office has received the amount (1) found above, viz.,

$$\mathrm{a} = \frac{\Sigma l'_{x+n}}{l'_x}$$

as the consideration for an annuity of £1 payable in advance, and has to pay £1 at the beginning of every year in which the annuitant is still living. If he dies in the $\overline{m+1}|$th year and consequently receives $\overline{m+1}|$ payments of the annuity, the present value of these payments is

$$1+v+v^2+\ \ldots\ldots\ +v^m = \frac{1-v^{m+1}}{1-v}$$

If now this sum be equal to a, so that the number of years which the annuitant lives after the purchase of the annuity is given by the equation

$$m = \frac{\log(1+v\mathrm{a}-\mathrm{a})}{\log v} - 1 \quad \ldots\ldots \quad (3)$$

then the Office experiences neither profit nor loss. For every other value of m, the Company experiences a profit or loss equal to

$$\mathsf{a}-\frac{1-v^{m+1}}{1-v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

that is to say, a profit if this expression is positive, and a loss if the minuend is the smaller. The number of the persons with whom this contract has been made is immaterial, so that we may take the number to be l_x. Of these, there die in the $\overline{n+1}|$th year after the concluding of the contract, $d_{x+n+\frac{1}{2}}$; and for each of these the Office experiences the calculated gain or loss. Now since n can represent any value we please, from 0 to the highest age,

$$\therefore \quad \Sigma d_{x+n+\frac{1}{2}}\left\{\mathsf{a}-\frac{1-v^{n+1}}{1-v}\right\}^2 \quad \begin{array}{l}\text{from } n=0\\ \text{to} \quad\ n=\infty\end{array}$$

is the sum of the squares of all the deviations. If this sum is divided by the number of all the deaths, or what is the same thing, by the number of all the assured, and the square root then taken, we get the risk, which may be denoted by R,

$$\mathrm{R}=\sqrt{\Sigma\frac{d_{x+n+\frac{1}{2}}}{l_x}\left(\mathsf{a}-\frac{1-v^{n+1}}{1-v}\right)^2} \quad . \quad . \quad . \quad (5)$$

In this expression, n is to have every integer value from 0 to the highest, by which means all the single terms will be obtained. This sum is a minimum with regard to a, as can be easily proved by considering a as variable and determining it so that the sum of the squares is a minimum, when the above value (1) of a will be found. For this purpose the differential coefficient with regard to a must be put equal to 0, consequently

$$\frac{d}{d\mathsf{a}}\Sigma\frac{d_{x+n+\frac{1}{2}}}{l_x}\left\{\mathsf{a}-\frac{1-v^{n+1}}{1-v}\right\}^2=0$$

whence
$$\Sigma d_{x+n+\frac{1}{2}}\left\{\mathsf{a}-\frac{1-v^{n+1}}{1-v}\right\}=0$$

or
$$\mathsf{a}\Sigma d_{x+n+\frac{1}{2}}=\Sigma d_{x+n+\frac{1}{2}}\cdot\frac{1-v^{n+1}}{1-v}.$$

But
$$\mathsf{a}\Sigma d_{x+n+\frac{1}{2}}=\mathsf{a}l_x$$

and
$$\Sigma d_{x+n+\frac{1}{2}}\cdot\frac{1-v^{n+1}}{1-v}=\left\{\begin{array}{l} \quad d_{x+\frac{1}{2}}\\ +d_{x+1+\frac{1}{2}}(1+v)\\ +d_{x+2+\frac{1}{2}}(1+v+v^2)\\ +d_{x+3+\frac{1}{2}}(1+v+v^2+v^3)\\ +\ .\ .\ .\ .\\ +d_{x+n+\frac{1}{2}}(1+v+v^2+\ .\ .\ .\ .\ +v^n)\\ +\ .\ .\ .\ .\end{array}\right.$$

the series being continued until the d's vanish.

If now we sum these terms vertically, then the first vertical row is

$$d_{x+\frac{1}{2}}+d_{x+1+\frac{1}{2}}+d_{x+2+\frac{1}{2}}+\ldots\ldots+d_{x+n+\frac{1}{2}}+\ldots\ldots=l_x$$

the second

$$v(d_{x+1+\frac{1}{2}}+d_{x+2+\frac{1}{2}}+\ldots\ldots+d_{x+n+\frac{1}{2}}+\ldots\ldots)=vl_{x+1}$$

the third $\qquad =v^2l_{x+2}$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

the $\overline{n+1}|$th $\qquad =v^nl_{x+n}$

So that we have

$$\Sigma d_{x+n+\frac{1}{2}}\cdot\frac{1-v^{n+1}}{1-v}=l_x+vl_{x+1}+v^2l_{x+2}+\ldots\ldots+v^nl_{x+n}+\ldots\ldots$$

But the sum on the right hand side can be also written

$$v^{-x}\{l'_x+l'_{x+1}+l'_{x+2}+\text{ \&c. }+l'_{x+n}+\ldots\ldots\}=v^{-x}\Sigma l'_{x+n}.$$

The equation to determine the minimum therefore becomes

$$\mathrm{a}l_x=v^{-x}\Sigma l'_{x+n}$$

from which we get the same expression for **a** as in (I), viz.,

$$\mathrm{a}=\frac{\Sigma l'_{x+n}}{l'_x};$$

so that the sum of the squares under the square root is really a minimum, as we might have anticipated on other grounds.

IV.

In order to obtain also a numerical survey of the question, I will calculate the risk attaching to the grant of a life annuity for some equidistant ages, using as the basis of the calculation the table of mortality which Heym* has with much care deduced from Brune's observations made on the General Widows' Fund of Berlin, with interest at 4 percent. The calculation itself I may well omit since I shall come back to it presently; and I will here only remark by way of anticipation, that in addition to the sums of the living and the discounted numbers of the living and dying which must in all cases be used, the calculation of the expression (5) requires also a special summation by means of which the values for the different ages are obtained with tolerable ease. These values for males are arranged in the following table, in which x denotes the age, **a** the value of an immediate annuity payable in advance, or what is the

* Masius Rundschau, vol. iv., p. 289.

same thing, the value of a yearly payment of 1 in advance, and R the risk.

x	a	R
25	18·467	5·168
30	17·606	5·142
35	16·565	5·206
40	15·417	5·204
45	14·159	5·141
50	12·753	5·015
55	11·235	4·817
60	9·664	4·540
65	8·101	4·176
70	6·694	3·656
75	5·329	2·982
80	3·957	2·237

We here see that the value of the risk in general diminishes as the age increases, but that the ratio it bears to the value of the annuity continually increases.

If the risk is to be calculated for deferred, temporary, or intercepted annuities, the formula (5) remains the same, except that the summation is to be effected between other limits. If, for instance, the annuity has to be paid for the first time after b years, and for the last time after $(c-1)$ years, being consequently paid $(c-b)$ times, its value will be

$$\mathrm{a}_{c-b} = \frac{\Sigma l'_{x+n}}{l'_x} \quad \begin{matrix} \text{from } n=b \\ \text{to } \quad n=c \end{matrix}$$

and the risk is

$$\sqrt{\Sigma \frac{d_{x+n+\frac{1}{2}}}{l_x} \left\{ \mathrm{a} - \frac{1-v^{n+1}}{1-v} \right\}^2}$$

where the upper limit is b and the lower c. The numerical evaluation remains just the same, and the summations made for the calculation of (5) are applicable here also.

We shall endeavour in vain to compare these results with those of other authors. Tetens makes so many distinctions, and has so many kinds of risks, that he can find any value whatever for the risk that suits his purpose. Other authors, as Florencourt,* Littrow,† Baily,‡ Jones,§ do not consider it at all. Raedell‖

* Abhandlungen aus der juristichen Rechenkunst. Altenburg, 1781.
† Lebensversicherungen und Versorgungs-Anstalten. Vienna, 1832.
‡ The Doctrine of Life Annuities and Assurances. London, 1813.
§ On the value of Annuities and Reversionary Payments. London, 1843.
‖ Lebensfähigkeit von Versicherungs-Anstalten. Berlin, 1857.

appears to be of opinion that the risk must be put equal to the mean of all the losses to be expected, contrary to the principles of the theory of probabilities. But altho' he enunciates this principle, he does not keep to it in practice, inasmuch as he finds the risks attaching to the purchase and sale of the same insurance to be different; whereas in every insurance the sum of the present values of all the possible losses is equal to that of all the possible gains. We must not however be surprised that contradictions arise when induction is made to take the place of mathematical demonstration. Nothing but mathematical treatment is capable of drawing right conclusions in cases where it has once fully mastered the subject, and it can all the less be dispensed with when the inner relations of the matter are hidden. It is of little use to parade ostentatiously the theory of probabilities in cases where we have simply to do with the arithmetic mean, if it is neglected when its application ought properly speaking to begin. The arithmetic mean is certainly a deduction from the method of least squares, but it is by no means the method itself; just as Kepler's laws are not the law of gravitation, altho' they were known long before, and are based upon it.

V.

Altho' it is not my object to present special formulas for the calculation of the risk in every branch of insurance, which might easily make the present treatise swell to a large volume; yet it may perhaps serve to elucidate better the course of investigation we have taken, if we explain its application to another kind of insurance of the most frequent occurrence, viz., simple life insurance.

The single premium for such an insurance has been shown above in (2) to be

$$\mathrm{A}=\frac{\Sigma d'_{x+n+\frac{1}{2}}}{l'_x}$$

But o l_x persons of the age x who have purchased this insurance at the price A, there die in the $\overline{n+1}|$th year after entering into this contract $d_{x+n+\frac{1}{2}}$ persons, and for each of these the sum assured 1 is to be paid. If now these sums are discounted back for $n+\frac{1}{2}$ years we get the present value of them

$$d_{x+n+\frac{1}{2}}v^{n+\frac{1}{2}}=v^{-x}d'_{x+n+\frac{1}{2}}$$

Now the Company has received for these insurances $\mathrm{A}d_{x+n+\frac{1}{2}}$ and the profit or loss which it makes on these persons dying in the

$\overline{n+1|}$th year is therefore $d_{x+n+\frac{1}{2}}.(A-v^{n+\frac{1}{2}})$, there being a profit or a loss according as this expression is positive or negative. If now we give n all values from 0 to that corresponding to the highest age, or what is the same thing, from 0 to ∞, and add the resulting quantities, the sum will be equal to 0. But if we take the sum of the squares and divide it by their number, then the square root of this quotient gives the risk, which may be denoted by R′, viz.,

$$R'=\sqrt{\Sigma\frac{d_{x+n+\frac{1}{2}}}{l_x}(A-v^{n+\frac{1}{2}})^2}$$

It may be shown by the same process as was used with (5) that this expression is a minimum with regard to A. Also the numerical evaluation can be reduced to the summations that have been used for (5). I will not here dwell longer on this point, but only illustrate the formula by giving the results which I have obtained by a little calculation based on the same table of mortality and the same rate of interest as the former results. They are contained in the following table, where x denotes the age at which the insurance was effected, A the single premium for an insurance of 1, and R′ the risk.

x	A	R′
25	·28872	·20469
30	·32267	·20367
35	·36414	·20616
40	·40965	·20610
45	·45957	·20367
50	·51497	·19862
55	·57511	·19079
60	·63736	·17980
65	·69933	·16541
70	·75520	·14481
75	·80958	·11811
80	·86472	·08860

Inspection of this table shows that life insurances effected by way of single payment expose the Company to a much greater risk when the lives are young than when they are old; and the same is still the case when the risk is expressed as a percentage of the single premium.

VI.

If a life insurance of £1 is granted in consideration of an annual premium, P, to be paid in advance during the whole of life,

the value of the premiums is $\mathbf{a}$P, because according to (1) the value of such a premium of £1 is $\mathbf{a}$. And since A is the single premium, we have for the determination of P the equation

$$\mathbf{a}P = A,$$

whence

$$P = \frac{A}{\mathbf{a}}.$$

Out of l_x persons of the age x there die in the $(n+1)$th year, $d_{x+n+\frac{1}{2}}$, of whom each has paid the premium P, $(n+1)$ times. The present value of these payments is

$$P(1+v+v^2+\ \ldots\ +v^n) = P\cdot\frac{1-v^{n+1}}{1-v}$$

On the other hand, the value of the £1 assured in consideration of these premiums, discounted for $n+\frac{1}{2}$ years, is $v^{n+\frac{1}{2}}$; and the average profit or loss upon each of the persons dying in the $(n+1)$th year is

$$P\cdot\frac{1-v^{n+1}}{1-v} - v^{n+\frac{1}{2}}$$

Hence we get the risk in this kind of insurance

$$R'' = \sqrt{\Sigma\frac{d_{x+n+\frac{1}{2}}}{l_x}\left(P\cdot\frac{1-v^{n+1}}{1-v} - v^{n+\frac{1}{2}}\right)^2} \quad . \quad . \quad (8)$$

Numerical calculation for different ages at entry x gives the values of R″ shown in the following table, where the values of P are also shown.

x	P	R″
25	·01563	·28544
30	·01832	·29780
35	·02198	·32055
40	·02657	·34428
45	·03246	·37035
50	·04038	·40110
55	·05119	·43732
60	·06595	·47914
65	·08632	·52581
70	·11282	·55712
75	·15193	·57080
80	·21856	·57669

VII.

It may suffice to have shown in these three kinds of insurances how the risk can be calculated in each case. The course of proceeding is always similar, and can easily be extended to

insurances depending on the joint lives of several persons, since the principle is the same. The reduction of the expressions for the risk to definite summations, which may be called their integration, may now and then present some difficulty; but this must be done if we wish to have a formula independent of n, and not to be compelled to make a laborious numerical summation for every separate age. If also we were satisfied with the equations (5), (6), and (8), the analytical connection of the formulas would be entirely lost, being, it must be confessed, much less obvious than in the calculation of the insurances themselves; because the expression for the risk in the different kinds of insurances can take very essentially different forms. We shall presently give some further explanations of this point, so far as is consistent with our object.

VIII.

Hitherto we have calculated the risk only for an annuity of 1 and an insurance of 1. But, denoting the risk by R, and the present value of the annuity by a, if the yearly payment of the annuity is s, its value will be $\mathsf{a}s$, and the risk will be equal to Rs. Just in the same way, if the sum assured is S instead of 1, the single premium for it will be SA, with the risk R′S; and the annual premium will be SP, with the risk R″S. This follows immediately from the fact that all the calculated present values, both of gain and loss, are always proportional to the magnitude of the sum assured or of the annuity. If therefore we have formed for the sum assured, 1, the necessary tables showing the required values for each age, these values have only to be multiplied into the sum assured, in order to get the corresponding numbers. If, for instance, an Insurance Office wished to grant a person 35 years of age a whole life annuity of £200, it would require to be paid for the same two hundred times the value given in (IV), viz., £200 × 16·565 = £3313; and the risk which the Office undertakes would be £200 × 5·206 = £1041. For a life insurance of £1200 on a person 35 years of age, the Office would require either a single payment of £1200 × ·36414 = £437, or an annual premium payable in advance for the whole of life of £1200 × ·02198 = £26·376; and the risk of the Office in the two cases would be £1200 × ·206 = £247, and £1200 × ·321 = £385 respectively, by (V) and (VI).

IX.

If the Company has undertaken a number of insurances at the same time, and we denote the several risks calculated according to

(IV), (V), and (VI) by R_1, R_2, R_3, ... R_r, and the total risk by R_0, then according to the principles of the calculus of probabilities, we have

$$R_0 = \sqrt{R_1^2 + R_2^2 + \ldots\ldots + R_r^2} \quad . \quad . \quad . \quad . \quad (9)$$

This total risk must be covered by the Guarantee Fund of the Company to secure the safety of the assured on the one hand, and a uniform undisturbed progress to the business of the Company on the other hand. The conditions of mutuality, which frequently supply the place of a guarantee capital, have the disadvantage that in unfavourable circumstances the demands on the assured must be pressed too far; while on the contrary, too large a guarantee capital may introduce losses which do not originate from the insurance business of the Company. But both can turn to the very great injury of the assured, and even cause the ruin of the Company.* When therefore we frequently see how one Company claims that the magnitude of its guarantee capital is a special recommendation of it; and another believes that the greatest security and safety are to be found in mutuality; this can only lead us to the conjecture that somewhere or other there is a fundamental deception either intended or induced by ignorance. For the honour of the business and of the Insurance Companies, we will assume the latter to be the case; since in reality the proportion which the guarantee capital should bear to the extent of the business has never yet to my knowledge been the subject of an exhaustive inquiry. This question has a special importance to the Government inspectors who have to satisfy themselves as to the vitality of the Companies before granting them a concession, and consequently must take into their serious consideration the magnitude of the guarantee fund. A prosperous activity can be expected only when all the elements are in proper harmony.

X.

Assume that a Life Insurance Company undertakes insurances to the extent only of £2000 on a single life, reassuring all amounts beyond this, so that, including the smaller insurances, £1200 can be considered as the average amount of its insurances: required to find the amount of the guarantee capital necessary to cover the risk if 10,000 such insurances are granted. If we assume the average age of the lives assured to be 35, then according to (VIII)

* The author explains that a large guarantee capital may cause the ruin of a Company by leading it to embark in financial operations foreign to its legitimate business.

the risk attaching to a single insurance effected by way of annual premium is £385. Then, since R_1, R_2 . . . . are all equal to one another, and to £385, the formula (9) gives the total risk

$$\begin{aligned} R_0 &= \sqrt{10{,}000 \times (385)^2} \\ &= £38{,}500 \end{aligned}$$

If in addition 5000 similar insurances are granted by way of single premium, the risk for each of these is £247 according to (VIII), and the risk attaching to the 5000 will be

$$\begin{aligned} R_0 &= \sqrt{5000 \times (247)^2} \\ &= £17{,}466 \end{aligned}$$

If 1000 annuities of £200 each on the lives of persons 35 years of age are granted in consideration of the payment of the present value, the risk for each of these being £1041 according to (VIII), the formula shows the risk attaching to the 1000 to be

$$\begin{aligned} R_0 &= \sqrt{1000 \times (1041)^2} \\ &= £41{,}443. \end{aligned}$$

And if all these insurances are granted by the same office, then the total risk would amount to

$$\begin{aligned} R_0 &= \sqrt{10{,}000 \times (385)^2 + 5000 \times (247)^2 + 1000 \times (1041)^2} \\ &= £53{,}582; \end{aligned}$$

the total sum assured being over 21 millions.

If, on the contrary, an Insurance Office had granted 1000 insurances of £10,000 each, and 550 of £20,000 each, all by way of annual premium, on the lives of persons 45 years of age, then the total sum assured would be very nearly the same, but the risk would be materially different. Thus, for an insurance of the former kind the risk is $£10{,}000 \times \cdot 370 = £3700$; and for an insurance of the latter kind $£20{,}000 \times \cdot 370 = £7400$; and the total risk is therefore

$$\begin{aligned} &= \sqrt{1000 \times (3700)^2 + 550 \times (7400)^2} \\ &= £209{,}300; \end{aligned}$$

or about four times as great as before.

In general, the risk is greater the less the number of the lives among which the sum assured is divided; and it amounts at last, if the whole sum is assured on a single life of 45, to 37 percent of the sum assured.

It must here not be left unnoticed that the risk hitherto discussed arises only from the contingencies of life and death,

which may happen, and in the long run will happen. The Office undertakes another risk in assuming that it will be always able to improve its funds at the rate of interest employed in its calculations. This risk also can be determined by means of the deviations in the rate of interest that are found to occur in practice. But this is not the object of the present treatise; and we will therefore not consider the point further.

XI.

Altho' the principles according to which the risk attaching to the grant of life insurances may be determined, have been laid down, as I believe, with tolerable clearness, yet it may not be superfluous for the sake of those who wish to make a special study of the subject and calculate numerical values for themselves, if I add a few further explanations as to the integration of the expressions (5), (6), and (8). The principal part of the difficulty always is, that we do not know beforehand the desired form which will be most convenient for numerical evaluation, and we are therefore driven to an undefined analysis of the formula before us, in order to ascertain more clearly its inward connection. If this is perceived, and all the relations to each other of the introduced ideas are clearly explained, then we can with greater certainty decide which are the simplest forms. It will therefore be necessary to start from first principles.

All calculations are based on the table of mortality, with respect to which it is assumed that it correctly represents the mortality, or, according to mathematical conceptions, that the numbers contained in it are the mean values of a very large number of observations. These numbers, which may be denoted by $l_x, l_{x+1}, \ldots l_{x+n} \ldots$, represent the number of persons living at the ages $x, x+1, \ldots x+n \ldots$ out of a number l_0 born at the same time; and the ratios of these numbers are employed to determine how many will be living at any age out of any other arbitrary number, k_x, of persons of the age x. We say for example that after n years there will be living out of those k_x persons, $k_x \cdot \frac{l_{x+n}}{l_x}$, notwithstanding that we very well know that there is but a small probability of this exact number being alive, and that the probability becomes smaller as k_x is less. In saying this, it is not our intention to deal with probability, but only with the consequences that follow from the numbers in the mortality table. Thus, then, these consequences only hold good for an infinitely

large number of persons, and the conclusion above stated is only correct for $k_x=\infty$. When we deal with money values, compound interest is assumed in the calculations, so that if the yearly interest on 100 is equal to I, then £1 after n years increases to $\left(1+\frac{\text{I}}{100}\right)^n$, or v^{-n}, if $\frac{1}{v}$ is put equal to $1+\frac{\text{I}}{100}$. If $\frac{\text{I}}{2}$ is paid half-yearly, or $\frac{\text{I}}{4}$ quarterly, v has a different value, viz. $\left(1+\frac{\text{I}}{200}\right)^{-2}$ in the former case, and $\left(1+\frac{\text{I}}{400}\right)^{-4}$ in the latter case. Hence £1 to be paid or received at the end of n years has the present value v^n; and a sum A under the same circumstances would have the present value $\text{A}v^n$. If now each of l_x persons of the age x has to pay £1 at the beginning of every year in which he is alive, this will make the payments

at the beginning of the first	year	l_x
" second	"	l_{x+1}
....		
" $(n+1)$th	"	l_{x+n}

where n may be any number we choose; and the present value of all these payments is

$$l_x+vl_{x+1}+v^2l_{x+2}+\ \ldots\ +v^nl_{x+n}+\ \ldots$$

which series is to be continued as far as the mortality table exhibits any numbers. If now for each value of n we put $l_{x+n}v^{x+n}=l'_{x+n}$, then the series becomes by substitution

$$v^{-x}\{l'_x+l'_{x+1}+l'_{x+2}+\ \ldots\ +l'_{x+n}+\ \ldots\}=v^{-x}\Sigma l'_{x+n}$$

where the sign of summation denotes that the sum of the l''s is to be taken for which n has the values 0, 1, 2, 3 . . . . , the series being continued until the terms become 0, so that we can take as the upper limit $n=\infty$. All these sums can be easily calculated once for all and tabulated for every value of the argument x, the indefinite n thus disappearing. If the sum $v^{-x}\Sigma l'_{x+n}$ is divided by the original number l_x, we get the average, or arithmetic mean, of the payments made by all the persons; and this average is called the value of an immediate annuity payable in advance. Thus

$$\mathfrak{a}=\frac{\Sigma l'_{x+n}}{l'_x},$$

which is the equation (1).

In the same way, let there be l_x persons of the age x who pay to the Office £ A each at entry, and are to receive £1 each at

death. There die in the $(x+n+1)$th year $l_{x+n}-l_{x+n+1}$, which we denote by $d_{x+n+\frac{1}{2}}$; and since these persons have each on the average lived $n+\frac{1}{2}$ years since their entry, the present value of the payments to be made by the Office is $d_{x+n+\frac{1}{2}}v^{n+\frac{1}{2}}=v^{-x}d'_{x+n+\frac{1}{2}}$. If we give n all values from 0 to ∞, and add, we get the value of all the payments to be made, which is to be equal to the single premiums paid to the Office: whence

$$l_x\mathrm{A}=v^{-x}\Sigma d'_{x+n+\frac{1}{2}}$$

and

$$\mathrm{A}=\frac{v^{-x}\Sigma d'_{x+n+\frac{1}{2}}}{l_x}$$

$$=\frac{\Sigma d'_{x+n+\frac{1}{2}}}{l'_x},$$

which is the equation (2). Here A is called the value of an assurance of £1, and is seen to be the average value of the sums to be paid at the death of all the persons. The sum $\Sigma d'_{x+n+\frac{1}{2}}$ can easily be calculated once for all, and tabulated under the argument x. If the life insurance has been effected by way of annual premium P, then by (1) the value of all these premiums is $\mathrm{P}.v^{-x}.\Sigma l'_{x+n}$, which is to be equal to the former, $v^{-x}.\Sigma d'_{x+n+\frac{1}{2}}$, so that we get

$$\mathrm{P}=\frac{\Sigma d'_{x+n+\frac{1}{2}}}{\Sigma l'_{x+n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From these three equations (1), (2), and (10) can be deduced new equations showing the relation between **a**, A, P, and v.

XII.

If now we substitute in equation (2) the value of $\Sigma d'_{x+n+\frac{1}{2}}$ found from (10), we get

$$\frac{\mathrm{A}}{\mathrm{P}}=\frac{\Sigma l'_{x+n}}{l'_x},$$

But this according to (1) is **a**, so that we have

$$\mathrm{A}=\mathrm{P}\mathbf{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

in which A, P, **a** refer to the same age.

Again, since

$$d'_{x+n+\frac{1}{2}}=\{l_{x+n}-l_{x+n+1}\}v^{x+n+\frac{1}{2}}$$

$$=\left\{\frac{l'_{x+n}}{v^{x+n}}-\frac{l'_{x+n+1}}{v^{x+n+1}}\right\}v^{x+n+\frac{1}{2}}$$

$$=l'_{x+n}v^{\frac{1}{2}}-\frac{l'_{x+n+1}}{v^{\frac{1}{2}}},$$

we have $$\Sigma d'_{x+n+\frac{1}{2}} = v^{\frac{1}{2}} \Sigma' l_{x+n} - \frac{\Sigma l'_{x+n+1}}{v^{\frac{1}{2}}}.$$

But we also have $\Sigma l'_{x+n+1} = \Sigma l'_{x+n} - l'_x$

whence $$\Sigma d'_{x+n+\frac{1}{2}} = \left(v^{\frac{1}{2}} - \frac{1}{v^{\frac{1}{2}}}\right) \Sigma l'_{x+n} + \frac{l'_x}{v^{\frac{1}{2}}} \quad . \quad . \quad (12)$$

This equation expresses the connection between the sums $\Sigma d'$ and $\Sigma l'$. Substituting this expression for $\Sigma d'_{x+n+\frac{1}{2}}$ in equations (2) and (10), and bearing (1) in mind, we get

$$\mathrm{A} = v^{-\frac{1}{2}} - (v^{-\frac{1}{2}} - v^{\frac{1}{2}})\mathsf{a} \quad . \quad . \quad . \quad . \quad (13)$$

$$\mathrm{P} = \frac{1}{v^{\frac{1}{2}}\mathsf{a}} - (v^{-\frac{1}{2}} - v^{\frac{1}{2}}). \quad . \quad . \quad . \quad . \quad (14)$$

With the help of these equations, A and P can be deduced from the annuity a, so that the calculation and summation of the *d''s* become superfluous. Since however A and P by virtue of the equations (13) and (14) are expressed by means of differences, therefore the formulas (2) and (10) are to be preferred; or else a will have to be calculated with more decimal places than would be otherwise necessary, in order that A and P may be known with the requisite accuracy. We find from equation (13) that

$$v^{-\frac{1}{2}} = \frac{1}{\mathsf{a}-1}\left\{\sqrt{\mathsf{a}(\mathsf{a}-1) + \frac{\mathrm{A}^2}{4}} - \frac{\mathrm{A}}{2}\right\}. \quad . \quad . \quad (15)$$

whence the rate of interest can be calculated, if for any age the value of the annuity due, a, and the assurance, A, are given. If, for instance, it were known that according to a certain table of mortality the value of a for age 40 is 15·417, and that of A, ·40965, we should have to make the following calculation:—

$$\begin{aligned} \log \mathsf{a} &= 1{\cdot}18800 \\ \log(\mathsf{a}-1) &= 1{\cdot}15888 \\ \log \frac{\mathrm{A}}{2} &= 9{\cdot}31139 \\ \hline \mathsf{a}(\mathsf{a}-1) &= 222{\cdot}270 \\ \frac{\mathrm{A}^2}{4} &= 0{\cdot}042 \\ \hline \log\left\{\mathsf{a}(\mathsf{a}-1) + \frac{\mathrm{A}^2}{4}\right\} &= 2{\cdot}34696 \end{aligned}$$

$$\begin{aligned} \sqrt{\mathsf{a}(\mathsf{a}-1) + \frac{\mathrm{A}^2}{4}} &= 14{\cdot}9100 \\ \frac{\mathrm{A}}{2} &= 0{\cdot}2048 \\ \hline \log \text{difference} &= 1{\cdot}16748 \\ \log(\mathsf{a}-1) &= 1{\cdot}15888 \\ \hline \log v^{-\frac{1}{2}} &= {\cdot}00860 \\ v^{-\frac{1}{2}} &= 1{\cdot}02000 \end{aligned}$$

We therefore get $\sqrt{1+i} = 1{\cdot}02$, or the half-yearly rate of interest is 2 percent; and this is the actual rate on which the calculations in (IV) and (V) are based. If we wish to avoid the above rather

involved calculation, we can instead of (15) use the following formula of approximation, in which the higher powers of A are neglected,

$$v^{-\frac{1}{2}}=\left(\frac{\mathrm{a}}{\mathrm{a}-1}\right)^{\frac{1}{2}}-\frac{\frac{1}{2}\mathrm{A}}{\mathrm{a}-1}.$$

Applying this formula to the above example, we find the half-yearly rate of interest to be 1·98 percent, which does not materially differ from the truth.

XIII.

The formulas found in the last section may be applied to get expressions for the risk simpler and better adapted for numerical evaluation. The course of proceeding is almost exactly the same in all the cases, so that it will suffice to discuss it for one of them only. For that purpose we may take equation (5) and transform it,

$$\mathrm{R}=\sqrt{\Sigma\frac{d_{x+n+\frac{1}{2}}}{l_x}\left(\mathrm{a}-\frac{1-v^{n+1}}{1-v}\right)^2}$$

The quantity under the square root becomes by expansion of the squares

$$\frac{\Sigma d_{x+n+\frac{1}{2}}}{l_x}\cdot\mathrm{a}^2-2\mathrm{a}\,\frac{\Sigma d_{x+n+\frac{1}{2}}}{l_x}\cdot\frac{1-v^{n+1}}{1-v}+\frac{\Sigma d_{x+n+\frac{1}{2}}}{l_x}\left(\frac{1-v^{n+1}}{1-v}\right)^2$$

The first term

$$=\mathrm{a}^2,\text{ since }\Sigma d_{x+n+\frac{1}{2}}=l_x.$$

The second term

$$=-\frac{2\mathrm{a}}{(1-v)l_x}\left\{\Sigma d_{x+n+\frac{1}{2}}-\Sigma d_{x+n+\frac{1}{2}}v^{n+1}\right\}$$

$$=-\frac{2\mathrm{a}}{(1-v)l_x}\left\{l_x-\frac{1}{v^{x-\frac{1}{2}}}\,\Sigma d'_{x+n+\frac{1}{2}}\right\}$$

$$=-\frac{2\mathrm{a}}{1-v}+\frac{2\mathrm{a}v^{\frac{1}{2}}}{1-v}\cdot\frac{\Sigma d'_{x+n+\frac{1}{2}}}{l'_x}$$

$$=-\frac{2\mathrm{a}}{1-v}\{1-\mathrm{A}v^{\frac{1}{2}}\}$$

$$=-\,2\mathrm{a}^2.$$

The third term

$$=\frac{1}{(1-v)^2l_x}\left\{\Sigma d_{x+n+\frac{1}{2}}-2\Sigma d_{x+n+\frac{1}{2}}v^{n+1}+\Sigma d_{x+n+\frac{1}{2}}v^{2n+2}\right\}$$

$$=\frac{1}{(1-v)^2l_x}\left\{l_x-\frac{2}{v^{x-\frac{1}{2}}}\cdot\Sigma d'_{x+n+\frac{1}{2}}+\frac{1}{v^{2x-1}}\,\Sigma d_{x+n+\frac{1}{2}}\cdot v^{2n+2x+1}\right\}$$

If now we put $$l_x . v^{2x} = l''_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and $$d_{x+n+\frac{1}{2}} . v^{2x+2n+1} = d''_{x+n+\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (17)$$

so that l'', d'' are the doubly discounted numbers l and d; then the last expression becomes by substitution,

$$\frac{1}{(1-v)^2}\left\{1-2v^{\frac{1}{2}}\frac{\Sigma d'_{x+n+\frac{1}{2}}}{l'_x}+\frac{v\Sigma d''_{x+n+\frac{1}{2}}}{l''_x}\right\}$$

$$=\frac{1}{(1-v)^2}\left\{1-2v^{\frac{1}{2}}\mathrm{A}+\frac{v\Sigma d''_{x+n+\frac{1}{2}}}{l''_x}\right\}.$$

Collecting now all the three terms, we get

$$\frac{1}{(1-v)^2}\left\{1-(1-v)^2\mathrm{a}^2-2v^{\frac{1}{2}}\mathrm{A}+\frac{v\Sigma d''_{x+n+\frac{1}{2}}}{l''_x}\right\}.$$

But since by (13), $(1-v)\mathrm{a}=1-v^{\frac{1}{2}}\mathrm{A}$

and $$(1-v)^2\mathrm{a}^2=1-2v^{\frac{1}{2}}\mathrm{A}+v\mathrm{A}^2;$$

we have $$1-(1-v)^2\mathrm{a}^2-2v^{\frac{1}{2}}\mathrm{A}=-v\mathrm{A}^2,$$

by means of which the last sum becomes

$$\frac{v}{(1-v)^2}\left\{\frac{\Sigma d''_{x+n+\frac{1}{2}}}{l''_x}-\mathrm{A}^2\right\}$$

The expression for R is therefore

$$\mathrm{R}=\frac{1}{v^{-\frac{1}{2}}-v^{\frac{1}{2}}}\cdot\sqrt{\frac{\Sigma d''_{x+n+\frac{1}{2}}}{l''_x}-\mathrm{A}^2}\quad . \quad . \quad . \quad (18)$$

The numerical calculation by this formula is now very simple. If the values of a, A, and P, are tabulated, we have only to calculate in addition l'' and d'' by (16) and (17); to form the sums of d'' in the usual way, beginning at the highest age and proceeding to the lower, because for each age x, $\Sigma d''_{x+\frac{1}{2}}=\Sigma d''_{x+1+\frac{1}{2}}+d''_{x+\frac{1}{2}}$; and lastly to divide the sums by the l'''s. We thus get the values of the quotients for each value of x; and if from each of them the corresponding value of A^2 is subtracted, and the square root of the remainder taken, we get for each value of x the value of the square root similarly tabulated. The indefinite letter n has vanished, and the equation (18) can be considered as the integration of equation (5).

XIV.

The reduction of the expressions (6) and (8) can be made in the same way. I will content myself with giving the results, and

leave those who may be interested in the matter to supply the proofs for themselves. I find

$$R' = \sqrt{\frac{\Sigma d''_{x+n+\frac{1}{2}}}{l''_x} - A^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$R'' = \frac{1}{(1-v)\text{a}}\sqrt{\frac{\Sigma d''_{x+n+\frac{1}{2}}}{l''_x} - A^2} \quad . \quad . \quad (20)$$

Hence result some further relations between the various R's. For if we eliminate the radicals, we get the equations

$$R' = (v^{-\frac{1}{2}} - v^{\frac{1}{2}})R \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$R'' = \frac{R}{v^{\frac{1}{2}}\text{a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and if we now eliminate v from these equations, we get

$$R'' = R' + PR \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

XV.

The risk may also be expressed as a percentage of the single premium. Let the risk thus expressed in the case of the three assurances considered be denoted by r, r', r'', so that

$$r = \frac{100R}{\text{a}}, \quad r' = \frac{100R'}{A}, \quad r'' = \frac{100R''}{A};$$

then instead of the relation (23) we get

$$r + r' = r'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

If we express the formula for the total risk in terms of r, r', r''; denoting the values of the single insurances by w's, so that $R_1 = \frac{w_1 r_1}{100}$, $R_2 = \frac{w_2 r_2}{100}$, &c., and the value of all the insurances by w_0, then if μ is the number of the insurances, we have

$$w_0 = w_1 + w_2 + w_3 + \dots + w_\mu \quad . \quad . \quad . \quad (25)$$

and

$$w_0 r_0 = \sqrt{w_1^2 r_1^2 + w_2^2 r_2^2 + w_3^2 r_3^2 + \dots + w_\mu^2 r_\mu^2} \quad . \quad (26)$$

Hence r_0, the ratio of the total risk to the total value of the insurances will become indefinitely small when the number of insurances is indefinitely large. For if we consider each of the quantities under the square root to be replaced by the largest of all, which may be denoted by $w_\nu^2 r_\nu^2$, then evidently

$$w_0 r_0 < w_\nu r_\nu \sqrt{\mu}.$$

Again, if w_λ is the average of all the w's, so that

$$w_0 = \mu w_\lambda$$

then $$r_0 = \frac{w_\nu r_\nu}{w_\lambda} \cdot \sqrt{\frac{1}{\mu}},$$

so that if the value of the largest insurance always bears a finite ratio to that of the average insurance, the risk becomes less as μ, the number of insurances, is greater; and when μ becomes indefinitely large, the risk becomes indefinitely small.

The numerical evaluation of r, r', r'' is contained in the following table,

x	r	r'	r''
25	28·0	70·9	98·9
30	29·2	63·1	92·3
35	31·4	56·6	88·0
40	33·7	50·3	84·0
45	36·3	44·3	80·6
50	39·3	38·6	77·9
55	42·9	33·2	76·1
60	47·0	28·2	75·2
65	51·5	23·7	75·2
70	54·6	19·2	73·8
75	56·0	14·6	70·6
80	56·5	10·3	66·8

Whence it can be seen that as the age increases, the risk attaching to the grant of an annuity becomes greater, but the risk attaching to the grant of an insurance less; and that there is a much greater risk attaching to the grant of an insurance by way of annual premium than by way of single premium.

Dr. Zillmer, in the preface to his work on the theory of life insurance, remarks with reference to the above treatise, first, that Dr. Bremiker has clearly proved that as soon as a fixed table of mortality is assumed for the calculation of life assurance premiums, we have no longer to do with a question of probabilities, but simply with calculations of average; and, next, that Dr. Bremiker has for the first time given a correct definition of risk in conformity with the principles of the calculus of probabilities. This is one of the works referred to by Dr. Kanner in his essay on Risk, which was translated and inserted in this *Journal* for January, 1869, and it will be seen that the views of Dr. Kanner as to the proper measure of risk differ widely from Dr. Bremiker's (see p. 451). In laying before the English reader a translation of Dr. Bremiker's treatise, I need scarcely say that I think very highly of the work; but this is quite consistent with entertaining serious doubt as to the correctness of some of the principles which the author has laid down, and I am accordingly not to be understood as adopting or assenting to his views as to the correct measure of the risk in the grant of life assurances.—T. B. S.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.—Session 1870–71.

First Ordinary Meeting, Monday, 28th November, 1870.

The President in the Chair.

Read and confirmed the minutes of the anniversary meeting, held on the 4th June, 1870.

Mr. W. S. B. Woolhouse was elected an honorary member.

The following gentlemen were elected Associates, viz.:—

Robert Charles Tucker.
John Dawson.
Arthur Francis Burridge.
Ernest Alfred Colquhoun.
Joseph Hardy.
William Haugh.
James Graham.
John Cameron.
Robert Eaton James.
Charles Daniel Higham.
James Martin.

Mr. T. B. Sprague, M.A., read a paper "On Legislation as to Life Insurance and Life Insurance Companies."

Thanks having been voted to Mr. Sprague, the meeting adjourned to Monday, 19th December, 1870.

Second Ordinary Meeting, Monday, 19th December, 1870.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected Associates, viz.:—

George King.
Frank Bertram Wyatt.
James Heron Duncan.
Henry Edward Wilson.
Edward Hall.
Henry Andrew Wilson.
Edward Newton Fuller.
James McGowan.
John Bell Tennant.
George Starling Vinen.

Mr. T. A. Welton read a paper "On the effect of Migrations in disturbing Local Rates of Mortality, as exemplified in the Statistics of London and the surrounding country, for the Years 1851–1860."

Thanks having been voted to Mr. Welton, the meeting adjourned to Monday, 30th January, 1871.

Third Ordinary Meeting, Monday, 30th January, 1871.

Andrew Baden, Esq., in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected Associates, viz.:—

George Humphreys, M.A.
Joseph Dodson Good.

The following was announced to be the result of the Examinations for 1870:—

Matriculation Examination.

Twenty-seven gentlemen presented themselves for this Examination, of whom three withdrew, and eight passed in the following order of merit:—

1. F. B. Wyatt.
2. George King.
3. John Dawson.
4. Charles Higham.
5. R. C. Tucker.
6. J. H. Duncan.
7. E. B. Trew.
8. E. N. Fuller.

SECOND YEAR'S EXAMINATION.

Eleven gentlemen presented themselves for this Examination, and four passed in the following order of merit, viz.:—

1. W. S. Aldis, M.A.
2. T. E. Young, M.A.
3. J. Duncan.
4. J. Burne.

THIRD YEAR'S EXAMINATION.

Two gentlemen presented themselves for this Examination, and one passed, viz.:—

David Carment.

The best thanks of the meeting were given to the Examiners for their recent services.

Mr. Samuel Brown read a paper "On the Rate of Mortality amongst the Natives compared with that of Europeans in India."

Thanks having been voted to Mr. Brown, the meeting adjourned to Monday, 27th February, 1871.

Fourth Ordinary Meeting, Monday, 27th February, 1871.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected members, viz.:—

Fellow.

George E. Cowley.

Associates.

John Prest.
Musgrave Watson.
Arthur William Dixon.
Francis Augustus Cox Hare.

Major-General Hannyngton read a paper "On the use of M. Thomas de Colmar's Arithmometer in Actuarial and other Computations."

Thanks having been voted to Major-General Hannyngton, the meeting adjourned to Monday, 27th March, 1871.

Fifth Ordinary Meeting, Monday, March 27th, 1871.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

Mr. Andrew Baden read a paper "On the Equitable Apportionment of a Fund between the Life Tenant and the Reversioner."

Thanks having been voted to Mr. Baden, the meeting adjourned to Monday, 24th April, 1871.

Sixth Ordinary Meeting, Monday, 24th April, 1871.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.
The following gentlemen were elected Associates, viz.:—

Joseph Muskett.
Robert Welsh.

Mr. Henry Harben read a paper "On Industrial Assurance."
Thanks having been voted to Mr. Harben, the meeting adjourned to Monday, 27th November, 1871.

The Twenty-third Annual General Meeting, Saturday, 3rd June, 1871.

W. B. Hodge, Esq., the President, in the Chair.

Mr. Bailey (Hon. Secretary) having read the circular calling the meeting, and the minutes of the last ordinary meeting, Mr. A. Day (Hon. Secretary) read the Report of the Council and the Statement of Accounts, which were as follows:—

"The Council have much pleasure in submitting to the members a Report of the Proceedings of the Institute during the twenty-third year of its existence.

"The number of members on the 31st March last was 271, as compared with 255 at the corresponding period of the former year. Of these 101 are Fellows and 170 Associates, the increase being entirely in the latter class.

"The financial position of the Institute will be seen on reference to the accompanying accounts. It will be observed that the total funds under the control of the Council now amount to £2,078. 1*s.* 4*d.*; one source of the increase having been the very handsome donation of £200 from the former President, Mr. Brown, which has been invested in the purchase of India Stock. At the same time the ordinary income has been more than sufficient for the current expenditure.

"The following papers have been read during the Session:—

28*th Nov.,* 1870.—'On Legislation as to Life Insurance and Life Insurance Companies.' By Mr. T. B. Sprague, M.A.
19*th Dec.,* 1870.—'On the effect of Migrations in disturbing Local Rates of Mortality, as exemplified in the Statistics of London and the surrounding Country, for the Years 1851-60.' By Mr. Thos. A. Welton, F.S.S.
30*th Jan.,* 1871.—'On the Rate of Mortality amongst the Natives compared with that of Europeans in India.' By Mr. Samuel Brown.
27*th Feb.,* 1871.—'On the use of M. Thomas de Colmar's Arithmometer in Actuarial and other Computations.' By Maj.-Gen. Hannyngton.
27*th Mar.,* 1871.—'On the Equitable Apportionment of a Fund between the Life Tenant and the Reversioner.' By Mr. A. Baden.
24*th April,* 1871.—'On Industrial Assurance.' By Mr. Henry Harben.

"The Council are glad to announce that Mr. Gray has in the readiest manner given his valuable assistance in superintending the calculation of monetary tables based on the recently collected Mortality Experience which were promised in the last report. Considerable progress has been made both

in the computing and printing; and the Council entertain a confident hope that the volume will be published at no distant period.

"Since the last Annual Meeting 'The Life Assurance Companies Act, 1870,' has become law. Short as has been the period during which the Act has been in operation, its good results are already discernible in the internal reforms it is effecting, and in the improved accounts now published by the Offices.

"Observing the continued increase in the class of Associates and in the number of candidates presenting themselves for examination, the Council have been considering whether, in addition to granting certificates of competency, the Institute could not render some assistance to the candidates in their studies, either in the form of lectures, or class instruction, or otherwise. No scheme for this purpose has yet been matured, but believing that such an extension of the functions of the Institute would be acceptable to the younger members, the Council desire at all events to try some experiment with that object."

The PRESIDENT said—"Gentlemen, I have now to move 'That the report of the Council and the abstract of income and expenditure and balance sheet be adopted, entered on the minutes, and printed in the *Journal*.' I think the balance sheet just read must be highly gratifying to everyone who is anxious for the success and prosperity of the Institute of Actuaries. (Hear, hear.) It is quite clear that the funds are amply sufficient to meet all the requirements of the Institute, and that, in fact, they are going on accumulating against future contingencies and liabilities. With regard to the first of those funds —that of £430 for the Mortality Experience—the whole of this will be absorbed by the expenses of the calculations which we are now engaged in with reference to that most important subject, and, of course, the printing of the work. The report says that Mr. Gray has undertaken the superintendence of these calculations, and his name will, I am sure, be a guarantee for their great accuracy. (Hear, hear.) The Institute owes a deep debt of gratitude to Mr. Gray, who has come forward in a most disinterested and public-spirited manner, and taken a great deal of labour upon himself for the sole purpose of forwarding these calculations. (Hear, hear.) I am happy to be able to state that the calculations for the single lives are finished, that a large portion of the joint-lives is completed, and that Mr. Gray is in hopes that the volume containing these results may be published this year. The Council have taken a great deal of pains with regard to this matter, and they have been most ably and warmly seconded by Mr. Gray; and their anxiety has been that the calculations should be of the necessary extent and character, and also that they should be printed in such a manner as would make them most useful for works of reference. (Hear. hear.) I have here some proof specimens of the tables, which Mr. Gray has kindly sent me, and any gentleman looking at them will see that they are printed in excellent type, and in such a way as to make them very easy of reference. That is a point which gentlemen who have been engaged as I have been for nearly 50 years in regard to actuarial tables will know the utility and great importance of. I think everyone will agree that, when this work is completed, we shall have published a set of tables which are unequalled for their utility and accuracy in actuarial science; and if the Institute of Actuaries had conferred no greater advantage on the community than the publication of this work, that alone would be sufficient evidence of its value. (Hear, hear.) With regard to the other funds, I may remark that we only hold some of them in trust, and do not consider them at all available for the general purposes of the Institute; but, as I have said, we have ample funds for all our requirements, and are in such a state of prosperity that the Council, as the report states, have been anxious to vote some portion of the funds for the assistance of some of the younger members in the studies which they pursue with reference to the examinations of the Institute. (Hear, hear.) This is a point which has frequently occupied the attention of the Council;

and if its accomplishment had been an easy matter, or at all in proportion to the anxiety of the Council in regard to it, something would have been done already. Indeed, we were in hopes that we might have done something before the present meeting, but it is a very difficult matter to arrange. It will, however, receive the earnest attention of the Council, and I hope that before long we shall be able to announce to the subscribers a scheme which will be very beneficial to the younger members of our Institute. We shall, I am sure, feel great pleasure in affording to them facilities and advantages which we ourselves—at least I speak for myself—did not obtain in our early instruction, and which we should have been very glad to have obtained. (Hear, hear.) I think these are the only points which require attention with regard to the report before you, and I will now move the motion which I read, and shall be happy to hear any observation which any gentleman may have to make upon the subject."

Mr. Sprague—"I beg leave to second the motion that our worthy President has just made, and in doing so I will not detain you by any very lengthy remarks about the progress of the Institute; but there is one point upon which I should like to say something. It has given me very great pleasure to see that there is a sort of intimation in the report that the Council think the time has now come when the Institute may leave off accumulating money, that it is now sufficiently rich, and that it should therefore devote itself to spending its income in the way that will be of most service in giving benefit to the members, and especially to the younger members. (Hear, hear.) For myself, I have sometimes been inclined to think that the junior members do not get quite so much for their subscriptions as they might; and it is, therefore, extremely gratifying for me to learn that the Council have it in contemplation to give the younger members some further advantages by providing for them a course of public lectures or forming a class to assist them in preparing for their examinations. This will show the members generally that the interests of every class of members, especially that of the associates, are not lost sight of. (Hear, hear.) There is another point which is not mentioned in the report, but which is particularly interesting to the associates—I allude to the Messenger Prizes, which have now been offered for several years. Unfortunately, the result of these offers has not been so good as we could desire on all occasions. We have had the satisfaction and pleasure of finding that on two occasions essays have been sent in which have well deserved the prizes awarded; and the fact of those valuable essays having been obtained is certainly a subject of much gratification. I am myself of opinion, or at all events I entertain a strong hope, that even when essays have been sent in, as has been sometimes the case, which have not come up to the mark, the effect of the prize having been offered has been to direct the attention of some of the younger members of the Institute to the subject proposed, and to lead them to look into it and so benefit themselves, and indirectly the Institute too, altho from the nature of the case we have at present no evidence of the fact. Therefore I rather regret that it was not thought desirable to include in the report this year a statement that we have again offered the Messenger Prize, the subject being the surrender values of policies. [Mr. Bailey—Papers announcing that have been distributed.] I am very glad to learn that. I do not know that I have anything further to say, except to second the motion for the adoption and printing of the report."

Mr. Wallis—"I do not know, Mr. President, whether this is the proper time to mention a matter which I think the Council ought to have constantly before them, and that is the definition of the term actuary; so that it may be known to the public that there are certain persons recognised by the Institute of Actuaries as qualified actuaries. I would suggest whether this might not be done by some distinctive title or letters after a man's name. (Hear, hear.) At present a person can be an F.I.A. who knows nothing whatever about actuarial matters, in respect of his being or having been the secretary of an Insurance Company. Sooner or later I think that will have to be grappled

with, inasmuch as the Life Assurance Companies Act provides that the calculations of an Office are to be entrusted to 'an actuary'; and if this Institute does not say who is an actuary and who is not, I am afraid we shall soon find ourselves put upon the shelf, and I fancy somebody else will become the Institute of Actuaries." (Hear, hear.)

Mr. A. H. Bailey—"I do not think this is the time to discuss this interesting subject, but I may remark that the Institute has spoken trumpet-tongued as to what it considers an actuary—namely, those who pass the three years' examinations; and what it hopes to accomplish is to get legal sanction—it aspires to no more than that—that only those who have passed the examinations shall be deemed competent to practice as actuaries. Until the legislature, in its wisdom, thinks fit to confer upon this or some other body some such power as that, it is impossible to do more than give certificates of competence under the sign manual of its president to those who pass the examinations." (Hear, hear.)

Mr. Wallis—"Don't you think the Institute might make a distinction between those whom it recognises as actuaries and those whom it does not recognise as such? Any gentlemen who has not passed his examination can put F.I.A. after his name, and Mr. A. or Mr. B., who knows nothing whatever of actuarial science, is very glad to put these letters after his name. It is worth considering whether the Institute should not take the bull by the horns, and say that in future no one shall be a fellow who has not passed his examination. In the case of passing the College of Surgeons they are obliged to recognise as qualified all who were in practice in 1858; and I think that for the future this Institute ought to say that a distinction should be made between qualified and unqualified actuaries, especially as it has now got a good surplus and is not in want of money."

Mr. S. Brown—"It is important to draw attention to the fact that formerly the distinction which Mr. Wallis suggests obtained, but under certain circumstances we effected a change. Therefore, it is important we should not do anything hastily." (Hear, hear.)

Mr. C. J. Bunyon—"I quite agree that this is a most important question, and one which cannot too soon have the attention of the Institute. I am very much inclined to agree with the gentleman who first mooted this question. I think it a most objectionable thing that men should be allowed to call themselves Fellows of the Institute who, neither by practice nor theory, have entitled themselves to carry that distinction. In a correspondence which took place between one member of the Institute and Lord Redesdale at no very distant period, an objection was raised by his Lordship that some of the members were not persons whom he would regard as competent actuaries, although at the same time he considered that for a man to be a competent actuary was in itself a proof that he was a man of considerable attainments. But the answer made to his Lordship was to give him a list of persons who had passed the examinations, to show how considerable the number was. The matter, therefore, has been under the notice of those persons who are qualified to judge, and I think the question is one which sooner or later—and I cannot think it can be too soon—must receive the deepest consideration both of the Council and the Institute itself." (Hear, hear.)

The President—"I quite agree as to the importance of the question raised by Mr. Wallis, and I think it would be a great advantage to the public if they could have some test by which they could ascertain the qualification of gentlemen who come forward as actuaries and perform the duties of actuaries. That is a very difficult question to determine at the present time. It is a matter which has been more than once brought under the notice of the Council, and discussed; and I have no doubt whatever that it will come before them again. Whether they will be able to do anything effectual on the subject I am quite unable to say. I am afraid the difficulties which beset it are very great. (Hear, hear.) I quite agree with Mr. Sprague that it is to

be lamented that the subjects which have been offered by the Council for the Messenger Prize have not caused a greater competition amongst the junior members, for we must all feel that the permanent prosperity and success of the Institute, as a scientific body, must depend upon those who are rising up amongst us. (Hear, hear.) Since this Institute was established we have lost several able and excellent colleagues, who have been taken from us by death; there are others who are retiring from active life, and do not take so large a share in the business of the Institute as they did, to its great advantage. We must all yield more or less to time; and it is only on the exertions, intelligence, and talent of the younger members that the Institute can base its hopes of permanent and enduring prosperity and usefulness. (Hear, hear.) I sincerely concur in the wish expressed by Mr. Sprague, that the younger members will come forward; and if they do so, I feel sure they will find it very much to their advantage eventually, although it may be some sacrifice of time and labour, and something withdrawn from enjoyment in their younger days." (Cheers.)

The report was then adopted unanimously.

Mr. D. A. Bumsted and Mr. B. Woods having been appointed scrutineers, a ballot was taken for the election of President, Vice-Presidents, Council, and Officers for the ensuing year, and it was reported that the following gentlemen were duly elected:—

President.

William Barwick Hodge.

Vice-Presidents.

Arthur H. Bailey.
Charles John Bunyon, M.A.
Thomas Bond Sprague, M.A.
J. Hill Williams.

Council.

Marcus N. Adler, M.A.
Andrew Baden.
Arthur H. Bailey.
Samuel Brown.
Charles John Bunyon, M.A.
Edward Cutbush.
George Cutcliffe.
Archibald Day.
Henry Devereux Davenport.
William John Hancock.
Ralph Price Hardy.
*Stewart Helder.
Augustus Hendriks.
William Barwick Hodge.
Charles Jellicoe.
*Claude George Laing.
James Meikle.
Benjamin Newbatt.
William Lewin Newman.
Edward A. Newton, M.A.
William P. Pattison.
*Arthur Pearson.
Henry William Porter, B.A.
Henry Ambrose Smith.
Col. John Thomas Smith.
Thomas Bond Sprague, M.A.
John Stott.
*James M. Terry.
Robert Tucker.
John Hill Williams.

Treasurer.

George Cutcliffe.

Honorary Secretaries.

Ralph Price Hardy.
Edward Algernon Newton, M.A.

The Chairman said the next business was the election of auditors. The three gentlemen who had kindly acted as auditors were Mr. Emmens, Mr. Manly, and Mr. Hopkinson. Mr. Manly, having become a fellow, was no longer eligible to serve in this capacity, but the Institute were much obliged to him for his past services. Mr. Emmens and Mr. Hopkinson were kind

* New Members.

enough to express their willingness to continue their services, and Mr. T. J. Searle would serve in the place of Mr. Manly. He then formally proposed the election of those gentlemen.

The motion was at once agreed to.

Mr. Humphreys proposed "That the best thanks of the meeting be given to the President, Vice-Presidents, Council, and other Officers of the Institute, for their services during the past year."

Mr. Wallis seconded the motion, which was cordially agreed to.

The President—"I am very much indebted to you—and I am sure the Council and Officers are—for the kind notice you have taken of our services during the past year. It is very gratifying indeed to us, and I may say to every one connected with the Institute and with actuarial science, to find that we are going on so prosperously. There are very few societies who have gone on through so long a course of years as this has, with so little to disturb the general course of harmony, and in which all the members co-operate so energetically to bring about success. (Hear, hear.) For myself, I feel very much indebted to you for placing me in the prominent and honourable position you have done, and so far as my humble power extends I shall continue that course which I have adopted in every capacity in which I have been placed in connection with the Institute. (Cheers.) I do not think it necessary for me to make any further remark, except upon one point, and that is with regard to our late honorary secretaries, Mr. Bailey and Mr. Day. The Institute is deeply indebted to them for their valuable and useful services rendered through a long period of time. (Hear, hear.) There is no doubt whatever that the success, the able management, and the harmonious working of the Institute depends in a great measure upon the qualifications of the honorary secretaries; and you all know that Messrs. Bailey and Day possess those qualifications in a high degree. I am sure we must all feel grateful to them for the manner in which they have endeavoured to promote our interests. (Hear, hear.) They have expressed their wish to the Council to be relieved from their duties, and although the Council would have been very glad to have had their services still, yet they did not think it fair, after the great sacrifices these gentlemen have made for the benefit of the Institute, to ask them to forego their resolution. We have elected two gentlemen who will, I believe, make efficient honorary secretaries. Mr. Newton is a gentleman holding an eminent position as an actuary, and the name of Mr. Hardy is historically connected with the Institute. (Cheers.) I have no doubt that they will render us very valuable and useful assistance. (Hear, hear.)

A vote of thanks to the scrutineers having been passed, the President announced that the library would be closed as usual during the month of September.*

* The above report of the proceedings is extracted from the *Insurance Record.*

INSTITUTE OF ACTUARIES.

Income and Expenditure for the Year ended 31st March, 1871.

Dr.

1870–71.	£ s. d.	£ s. d.
Amount of Funds on 31st March, 1870, viz.:—		
Mortality Experience Fund	409 3 9	
Messenger Legacy "	232 16 10	
Hardy Memorial "	200 12 11	
General "	877 13 8	
		1720 7 2
Annual Subscriptions, viz.:—		
68 Town Fellows	214 4 0	
27 Country "	56 14 0	
108 Town Associates	226 16 0	
60 Country "	64 1 0	
		561 15 0
Subscription compounded		31 10 0
Donation from Mr. Brown		200 0 0
Mortality Experience Contributions	10 10 0	
" " Sales	55 6 3	
		65 16 3
Certificate Fee		5 5 0
Dividends on Messenger Legacy (£211. 1s. 10d. Cons.)	6 5 0	
Do. Hardy Memorial (£200 Consols)	5 18 0	
Do. Brown Prize Fund (£95. 15s. 5d. 10½ per Cent East India Stock)	4 18 10	
Do. £488. 18s. 2d. Consols	14 8 0	
Interest on Deposit Account	14 5 10	
		45 15 8
		£2,630 9 1

Cr.

1870–71.	£ s. d.	£ s. d.
Rent	100 0 0	
Salaries	115 0 0	
Journal	153 9 0	
Printing and Stationery	54 17 9	
Ordinary Meetings	16 19 6	
Library	7 12 6	
Repairs	2 7 1	
General Expenses	57 15 11	
		508 1 9
Mortality Experience Expenses		44 6 0
Amount of Funds, 31st March, 1871, viz.:—		
Mortality Experience Fund	430 14 0	
Messenger Legacy "	239 1 10	
Hardy Memorial "	206 10 11	
Brown Prize "	204 18 10	
General "	996 15 9	
		2078 1 4
		£2,630 9 1

Balance Sheet, 31st March, 1871.

	£ s. d.	£ s. d.
Mortality Experience Fund		430 14 0
Messenger Legacy Fund, viz.:—		
£211. 1s. 10d. Consols, cost	203 17 8	
Unappropriated Dividends	35 4 2	
		239 1 10
Hardy Memorial Fund, viz.:—		
£200 Consols, cost	179 14 6	
Unappropriated Dividends	26 16 5	
		206 10 11
Brown Prize Fund, viz.:—		
£95. 15s. 5d. 10½ per Cent East India Stock, cost	200 0 0	
Unappropriated Dividends	4 18 10	
		204 18 10
General Fund		996 15 9
		£2,078 1 4

	£ s. d.	£ s. d.
Union Bank Deposit Account		600 0 0
Three per Cent Consols (£1100)		1008 19 3
East India 10½ per Cent Stock (£95. 15s. 5d. Stock)		200 0 0
Cash, viz.:—		
London and Westminster Bank	241 16 4	
Petty Cash	7 6 9	
		249 3 1
Arrears of Subscriptions		19 19 0
		£2,078 1 4

Examined and found correct, 28th April, 1871.

(Signed) H. W. MANLY, STEPHEN H. EMMENS, J. CLIFFORD HOPKINSON, *Auditors.*

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On a Mathematical Formula to express the Rate of Mortality throughout the whole of Life, tested by a Series of Observations made use of by the Danish Life Insurance Company of 1871. *By* Dr. T. N. Thiele, *of Copenhagen. Translated from the Danish by* T. B. Sprague, M.A., *Vice-President of the Institute of Actuaries.*

THE formula for the law of mortality which I am about to explain and apply to a practical case, is not a merely empirical formula, but is based on a presumed property of the causes of death. The value of the formula is however a question which is in a great measure independent of the correctness of my hypothesis; for it is well known that even a false hypothesis may be of great service. While I shall certainly be very much interested to learn the opinion entertained by actuaries as to the theoretical part of the subject, yet the practical application of the formula is the point to which I attach most importance, and to which I would by preference invite criticism.

I venture the supposition that the causes of death naturally fall into three or four large groups, which can be known by the ways in which they operate upon different ages; three of the groups dividing the whole of life between them, and operating principally, or almost exclusively, upon childhood, middle age, and old age, respectively. The fourth group will accordingly include

the causes of death which operate upon all ages indifferently with about the same force; but it appears to me that such causes of death are of comparatively small importance.

The force of mortality, $\mu(x)$, for a given age x, must therefore be represented by the sum of three (or four) terms, each of which is a function of the age x, so that

$$\mu(x)=\mu_1(x)+\mu_2(x)+\mu_3(x) \quad . \quad . \quad . \quad . \quad (1)$$

Taking $\mu_3(x)$ to represent the mortality of old age, it is natural to choose the form of the function so that that part of the mortality becomes very small for small values of x, but increases rapidly and continuously with it. My choice agrees on this point with the best formulas which have hitherto been used to represent the aggregate mortality at the higher ages, namely, Gompertz's and Makeham's. I put

$$\mu_3(x)=a_3\varepsilon^{-b_3x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ε is the base of the natural logarithms, while a_3, b_3, are two constants to be determined separately for each series of observations. These, as well as all the other constants contained in my formula, always have positive values.

For the mortality of middle age, $\mu_2(x)$, I believe I have found a suitable expression in the law of error. I put

$$\mu_2(x)=a_2\varepsilon^{-\frac{1}{2}b_2^2(x-c)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here are three constants to be determined by reference to experience; namely, c, the age for which this part of the mortality is a maximum; a_2, the amount of that maximum; and b_2, which shows how fast this part of the mortality diminishes as the difference between the ages x and c increases, so that $\frac{2}{b_2}$ is a measure of the space of time in which the mortality of middle age makes its influence felt.

In my first attempt at graduating tables of mortality I assumed the number surviving at each age, out of a given number born, to be represented by the sum of a large number of terms of the form μ_2.

The determination of the true rate of mortality in childhood, $\mu_1(x)$, is a difficult problem, and the more so, because the observations that can be used for the purpose are comparatively few and difficult to submit to strict treatment. However, that problem is of less importance, since insurances are seldom effected at young ages, so that for life insurance purposes we can be well satisfied

with a less accurate approximation. The general conditions which $\mu_1(x)$ must satisfy are that for $x=0$, $\mu_1(x)$ must be extremely large; and that as x increases, $\mu_1(x)$ must diminish rapidly and vanish. The only important previous investigation of this point that I know of is that of Professor Oppermann, explained in the *Insurance Record* for 11th February, 1870. He gives a formula for the force of mortality at young ages, that is up to about 20,

$$\mu(x)=ax^{-\frac{1}{2}}+b+cx^{\frac{1}{2}}$$

This formula obviously assumes that the force of mortality at the instant of birth is infinitely large. The structure of the formula also contains a definite intimation that with respect to the first years of life we shall make more progress by taking the square root of the time, instead of the time itself, as the independent variable of the mortality. It is my conviction that Professor Oppermann has gained for himself lasting credit by this formula. Even if a more natural expression for the mortality at young ages should be found, yet it is in the highest degree improbable that a partial formula will be found better adapted for calculation. This formula is of the greatest value for calculating the force of mortality in the first months and years of life from the rough observations by means of a correction corresponding to differentiation. That the force of mortality just at the beginning of life should be infinitely large, appears at first sight self-contradictory; but I entertain no serious doubt that in reality the fact is so. At any rate, the experiences which I have tested with this object do not contradict that proposition. Unfortunately, however, that formula does not permit of being inserted as the first term $\mu_1(x)$ of my formula, in the same way as Gompertz's formula forms its last term. If anything is done in that way, it will at all events be necessary to reject the third term $cx^{\frac{1}{2}}$ of Professor Oppermann's formula, and to substitute for it the sum of $\mu_2(x)$ and $\mu_3(x)$ as given by (3) and (2). Such a formula, consisting of four terms, the analytical forms of which enable them to represent respectively my four hypothetical groups of causes of death, appears at first sight admissible for my purpose. But in order to agree with the observed mortality of childhood, the constant b in Professor Oppermann's formula would frequently have to be taken as negative; and not to mention that a negative part of the force of mortality is a contradiction in terms, it would be difficult as regards the higher ages to satisfy the observations, if this term had a negative value, unless it were very small. At all events, I am

more inclined to keep only the first term of Professor Oppermann's formula, giving it the more general form

$$\mu_1(x) = x^{-\phi},$$

where ϕ is a function of x which is positive for all positive values of x, and such that $x=0$ gives $\phi=\frac{1}{2}$. A more precise determination of the form of this function would however be premature. It appears to me to be a necessary preliminary step, and also to be of importance for life insurance calculations, that a formula should be formed which can satisfy the series of observations for the whole of life except the earliest part of childhood; and such I hope to produce by putting provisionally

$$\mu_1(x) = a_1 \epsilon^{-b_1 x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so that $$\mu(x) = a_1 \epsilon^{-b_1 x} + a_2 \epsilon^{-\frac{1}{2} b_2^2 (x-c)^2} + a_3 \epsilon^{b_3 x} \quad . \quad . \quad . \quad (5)$$

Here, then, there are in all seven constants to be determined from the observations. It may possibly be sometimes necessary to add to the right-hand side of (5) a constant term answering to the part of the force of mortality common to all ages; but hitherto I have not met with such a case. Where agreement is only required for higher ages than, say, 10 or 20, it may be practicable to omit entirely the term $\mu_1(x)$.

The observations cannot possibly themselves indicate the force of mortality; but they answer more strictly to certain integrals of it: in particular, $l(x)$, the proportion of those who attain the age x out of a number of persons observed from birth, is here of importance.

But $$\log_\epsilon l(x) = \int_x^0 \mu(x) dx \quad . \quad . \quad . \quad . \quad . \quad (6)$$

On $l(x)$ depends the probability of surviving a year, $p(x) = \dfrac{l(x+1)}{l(x)}$.*
But

$$\begin{aligned} \log_\epsilon \frac{l(x)}{l(x+1)} &= -\log_\epsilon p(x) \\ &= \int_x^{x+1} \mu(x) dx \\ &= \frac{a_1}{b_1}(1-\epsilon^{-b_1})\epsilon^{-b_1 x} + a_2 \int_x^{x+1} \epsilon^{-\frac{1}{2} b_2^2 (x-c)^2} dx + \frac{a_3}{b_3}(\epsilon^{b_3}-1)\epsilon^{b_3 x} \quad . \quad . \quad (7) \end{aligned}$$

* It may be useful to remind the reader that most continental writers use the functional form of notation, where English writers are in the habit of using the index, or subscript, form. Thus in Dr. Thiele's paper, $l(x)$ and $p(x)$, which are considered as functions of the age x, mean exactly the same thing as the l_x and p_x commonly used in England.—Ed. *J. I. A.*

If the unit of time is not taken greater than a year, a sufficient approximation will in almost all cases be obtained by writing

$$\log_\epsilon \frac{l(x)}{l(x+1)} = -\log_\epsilon p(x) = a'_1 \epsilon^{-b_1 x} + a_2 \epsilon^{-\frac{1}{2}b_2^2(x-c')^2} + a'_3 \epsilon^{b_3 x} \quad . \quad . \quad (7a)$$

where the right-hand side is of the same form as the expression for the mortality itself, but three of the constants have different values, namely.

$$a'_1 = \frac{a_1}{b_1}(1-\epsilon^{-b_1}),\; c' = c - \tfrac{1}{2},\; \text{and } a'_3 = \frac{a_3}{b_3}(\epsilon^{b_3}-1).$$

In practice it will be more suitable to transform (7a) so that the natural logarithms and powers of ϵ, are replaced by Briggs's logarithms and powers of 10. Thus,

$$\text{Log}\frac{l(x)}{l(x+1)} = -\text{Log}\, p(x) = \alpha_1 10^{-\beta_1 x} + \alpha_2 10^{-\beta_2(x-c')^2} + \alpha_3 10^{\beta_3 x} \quad . \quad . \quad (7b)$$

where $\alpha_1 = a'_1 \text{Log}\,\epsilon,\quad \alpha_2 = a_2 \text{Log}\,\epsilon,\quad \alpha_3 = a'_3 \text{Log}\,\epsilon$

$\beta_1 = b_1 \text{Log}\,\epsilon,\quad \beta_2 = \tfrac{1}{2}b_2^2 \text{Log}\,\epsilon,\quad \beta_3 = b_3 \text{Log}\,\epsilon$

(Log denoting the common logarithm.)

In many important cases, such as the example mentioned below, the observations show for each year of age in how many completed observations of single lives death has occurred within the term of a full year. If D denote the deaths and X the number of observations at the age x, then

$$\frac{X-D}{X} = p(x) = \frac{l(x+1)}{l(x)} \quad . \quad . \quad . \quad . \quad (8)$$

is the observed probability of surviving a year at age x.

From seven trustworthy observations the seven constants of the formula can now be calculated if for each observation we first calculate

$$-\text{Log}\, p(x) = \text{Log}\, X - \text{Log}\,(X-D)$$

then substitute in equation (7b), and determine the constants by trial. Provided that the observations are well divided among the different ages; that is, if 2 fall below the age of 20, 3 at ages between 20 and 50, and 2 at higher ages; that indirect calculation is not very difficult, since changes of α_1 and β_1 exercise material influence only on the values from the two youngest observations; those of α_2, β_2, and c', only on the three middle observations; and those of α_3, β_3, only on the observations from the two highest ages.

But however carefully the observations may be made, it results from the nature of the case, that we must be prepared for such large deviations, that a calculation of the constants based upon observations for single years of age cannot be expected to give even a rough approximation to the true values of the constants. We must therefore of necessity determine the constants by a simultaneous employment of the whole series of observations, each taken according to its weight according to the rules of the calculus of probabilities. The mean error to be expected in the number of deaths D, out of X persons alive, will, if $p(x)$ is the true probability of surviving a year, be

$$\sqrt{p(x)\{1-p(x)\}\mathrm{X}},$$

or will in most cases be not much less than the square root of D itself. But the weights, which in absolute magnitude are equal to $\frac{1}{(\text{mean error})^2}$, can also be expressed as functions of the sought constants of the formula together with the numbers X and the age x. For the number, D, of deaths, the weight is

$$w_x = \frac{1}{p(x)\{1-p(x)\}\mathrm{X}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

We have likewise an exact expression for the weights of the probabilities of surviving a year, that is, of $\frac{l(x+1)}{l(x)}$, as calculated from the observations by means of equation (8),

$$w'_x = \frac{\mathrm{X}}{p(x)\{1-p(x)\}}$$
$$= \mathrm{X}^2 w_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For the weight, however, of the number,—Log $p(x)$, as deduced from the observations and destined for use in equation (7b), we have only approximately, but with sufficient accuracy in almost all cases, except at high ages,

$$w_x'' = w'_x \left\{\frac{p(x)}{\text{Log } \varepsilon}\right\}^2$$
$$= \frac{p(x)\mathrm{X}}{\{1-p(x)\}(\text{Log } \varepsilon)^2} \quad . \quad . \quad . \quad . \quad (11)$$

According to the method of least squares, that system of values of the constants must be regarded as the most probable which gives a minimum value to the sum of the products formed by

multiplying each weight into the square of the difference between the corresponding observed and computed results. The problem of finding this system of values cannot be solved directly; but we may conceive it to be solved indirectly by repeated applications of the method of least squares, using the rules of that method to obtain successive corrections of the assumed values of the constants. But that mode of proceeding also in its strict form is here impracticable, since the uncertainty of the observations renders it so difficult to find a sufficiently correct system of values of the constants to enable us to escape a frequent repetition of laborious calculation.

Instead therefore of investigating the most probable system of values of the constants by the strict but almost impracticable manner, I use the method employed in Astronomy in similar cases, and called the method of Normal Places. This name I have retained in the present case, altho, as nothing local is observed, a modification of the term might be desirable. This method is nothing but such an extension of the method of arithmetical means as makes it applicable to observations of phenomenons that vary during the observation according to any law. It is supposed that we know a function $Q(x)$ of the age x, which shows no considerable deviation when compared with the series of observations $P(x)$, $P(x+1)$, ... $P(x+n)$. The analytical form of this function is of little or no consequence. The best course will be to use a previous adjustment by the same formula; but we may also for groups of adjacent observations use partial adjustments by other formulas; for instance, by an integral rational function of the age; even graphical adjustment is allowable. Then from the observed numbers $P(x)$, $P(x+1)$, ... $P(x+n)$, we subtract the corresponding theoretical value, $Q(x)$, $Q(x+1)$, ... $Q(x+n)$, and compute for each the corresponding weight W_x, W_{x+1}, ... W_{x+n}. Then we calculate $\Delta\xi$ by the formula

$$\Delta\xi = \frac{W_x\{P(x)-Q(x)\}+W_{x+1}\{P(x+1)-Q(x+1)\}+..+W_{x+n}\{P(x+n)-Q(x+n)\}}{W_x+W_{x+1}+....+W_{x+n}} \quad .. (12)$$

Also the corresponding mean age under observation by the formula

$$\xi = \frac{xW_x+(x+1)W_{x+1}+.... +(x+n)W_{x+n}}{W_x+W_{x+1}+....+W_{x+n}} \quad ... (13)$$

Then is calculated the value $Q(\xi)$ corresponding to the age ξ, and $\Delta\xi$ is added, giving

$$P(\xi)=Q(\xi)+\Delta\xi,$$

which may be presumed to be an improved observation (normal place) corresponding to the age ξ; the weight of this normal place being

$$W_x + W_{x+1} + \dots + W_{x+n} \quad . \quad . \quad . \quad (14)$$

Instead of the exact value ξ found from (13) we may almost always safely take the nearest integral age.

When the whole series of observations, divided into suitable groups, has thus been used for the computation of normal places, these are used instead of the observations. Thus a double advantage is gained, the normal places being less in number as well as more trustworthy than the original observations. The determination of the seven constants in formula (7b) requires of course a division of the observation into seven groups, each of which gives its normal place. It is further to be observed that five of the normal places belong to ages below 50, since only two of the constants are important with regard to the higher ages. If, however, the number of lives observed upon was very great, it would be preferable to form a greater number of normal places, say 15 or 20, and then to treat them according to the method of least squares. But in common cases it will suffice to form only seven normal places, Log $\dfrac{l(x)}{l(x+1)}$ being regarded as the observed magnitude, and to determine the seven constants by trial from the seven normal places. When the values of the constants are determined, we have to compare the observed and computed values of Log $\dfrac{l(x)}{l(x+1)}$, to calculate the weights afresh, and to form new normal places, using the same grouping of the observations as before. If the new normal places thus found differ much from the old ones, the whole calculation must be performed again with new normal places. Lastly, it is necessary to investigate how far we may regard the observations as satisfied by the assumed formula, and the calculated constants. For this purpose we are not confined to the common processes, founded on a consideration of the manner in which the errors are grouped (in the table of errors) with regard to sign and magnitude; but since we can calculate *à priori* as well as *à posteriori* the mean error of an observation of given weight, we can use a comparison of these numbers to make an estimate of the goodness of the adjustment. If the weight of an observation calculated by (9), (10), or (11) is 1, then the mean error *à priori* is $=1$; *à posteriori* we find the mean error is

$$\sqrt{\frac{\Sigma w_x(D_x - D'_x)^2}{n-k}},$$

where D_x is the observed number of deaths at age x, D'_x the computed number, w_x the weight according to formula (9), n the total number of observations, and k the number of constants. Hence, in order that the two values of the mean error may not be inconsistent with each other, or that our adjustment may be satisfactory, we must have approximately

$$n = k + \Sigma w_x(D_x - D'_x)^2 \quad . \quad . \quad . \quad . \quad (15)$$

Example.

x	X	D	$-\text{Log}\,\frac{(X-D)}{X}$	x	X	D	$-\text{Log}\,\frac{(X-D)}{X}$
5	1953	32	0·00717	45	1447	22	0·0067
6	2079	28	588	46	1413	27	83
7	2082	13	272	47	1383	18	57
8	2063	5	105	48	1369	16	51
9	2037	12	256	49	1345	20	64
10	1969	10	0·00222	50	1307	17	0·0057
11	1900	15	344	51	1262	21	73
12	1868	3	070	52	1195	22	81
13	1784	9	219	53	1143	23	88
14	1568	7	195	54	1091	21	84
15	1365	8	0·00255	55	1051	27	0·0117
16	1121	7	272	56	928	32	152
17	890	3	15	57	880	19	095
18	691	4	25	58	829	16	085
19	530	3	25	59	793	29	162
20	393	3	0·0033	60	739	30	0·0182
21	290	1	15	61	676	18	117
22	172	3	76	62	634	27	189
23	84	1	52	63	574	24	185
24	91	0	0·0000	64	514	21	182
25	112	0	00	65	474	22	0·0466
26	166	3	79	66	405	23	568
27	213	0	00	67	342	25	730
28	287	2	31	68	305	20	657
29	405	2	22	69	270	16	594
30	506	2	0·0018	70	241	14	0·0581
31	627	4	28	71	203	8	395
32	849	8	48	72	175	16	913
33	931	6	31	73	148	12	812
34	1037	8	37	74	113	10	866
35	1138	10	0·0042	75	86	15	0·1743
36	1207	13	49	76	57	7	·1228
37	1284	11	40	77	44	5	·1137
38	1322	13	45	78	34	8	·2352
39	1386	12	40	79	23	2	·0870
40	1452	13	0·0041	80	17	2	0·1176
41	1461	16	48	81	13	2	·1538
42	1456	20	60	82	9	3	·3333
43	1452	16	48	83	5	2	·4000
44	1447	15	0·0045	84	3	3	1·0000

The series of observations above set out is the same as the Danish Life Insurance Company of 1871 has adopted for the basis of its calculations in the case where the life of a male is assured, the table having been graduated by Professor Oppermann. The fourth column above is calculated from the second and third; and it is to be noticed that for the 20 highest ages the probability of dying at each year of age is tabulated instead of the negative logarithm of the probability of surviving a year.

In order to find the seven constants of my formula answering to the above observations, I have made use of Professor Oppermann's graduated table as the basis for forming the five first normal places for the higher ages, while I have preferred a graphic delineation of the mortality as the basis for obtaining the two of the first set of normal places which related to the youngest ages. With these normal places the seven constants are determined. Then the values following from them are compared with the observations, the weights calculated, a new set of normal places formed, and with these the whole of the above described process was repeated and gave as a result the following set of

(1) $\text{Log}\,\dfrac{l(7)}{l(8)}=0\cdot00350$	observations	for the ages	5– 9
(2) $\text{Log}\,\dfrac{l(13)}{l(14)}=0\cdot00210$	,,	,,	10–18
(3) $\text{Log}\,\dfrac{l(25)}{l(26)}=0\cdot00285$	,,	,,	19–32
(4) $\text{Log}\,\dfrac{l(36)}{l(37)}=0\cdot00412$	,,	,,	32–39
(5) $\text{Log}\,\dfrac{l(45)}{l(46)}=0\cdot00554$	,,	,,	40–51
(6) $\text{Log}\,\dfrac{l(56)}{l(57)}=0\cdot01097$	,,	,,	52–64
(7) $\text{Log}\,\dfrac{l(70)}{l(71)}=0\cdot03278$	,,	,,	65–84

The observation for the age 32 was used with half weight in both the third and fourth normal place. From these normal places I found the following values of the constants:—

$$\text{Log}\,a_1=7\cdot99475 \qquad \text{Log}\,a_2=7\cdot32162 \qquad \text{Log}\,a_3=6\cdot08650$$
$$\beta_1=\cdot07295 \qquad \beta_2=\cdot0014735 \qquad \beta_3=0\cdot03470$$
$$c'=31\cdot209.$$

Hence the formula for the force of mortality becomes

$$\mu(x) = \cdot 02474\,\varepsilon^{-x \times \cdot 17028} + \cdot 00483\,\varepsilon^{-\frac{1}{2}\left(\frac{x-31\cdot 709}{12\cdot 139}\right)^2} + \cdot 0002699\,\varepsilon^{x \times \cdot 7990}$$

The following table of mortality was calculated according to the formula (7b) with the above values of the constants in such a way that Log $l(5)=0$, as in the Danish Life Insurance Company's table. The calculation of new values for the normal places had given the following consistent values:—

$$(1)\ \text{Log}\,\frac{l(7)}{l(8)} = 0\cdot 00353 \pm 0\cdot 00039$$

$$(2)\ \text{Log}\,\frac{l(13)}{l(14)} = 0\cdot 00210 \pm 0\cdot 00024$$

$$(3)\ \text{Log}\,\frac{l(25)}{l(26)} = 0\cdot 00286 \pm 0\cdot 00054$$

$$(4)\ \text{Log}\,\frac{l(36)}{l(37)} = 0\cdot 00413 \pm 0\cdot 00047$$

$$(5)\ \text{Log}\,\frac{l(45)}{l(46)} = 0\cdot 00552 \pm 0\cdot 00038$$

$$(6)\ \text{Log}\,\frac{l(56)}{l(57)} = 0\cdot 01096 \pm 0\cdot 00066$$

$$(7)\ \text{Log}\,\frac{l(70)}{l(71)} = 0\cdot 03287 \pm 0\cdot 00200$$

The calculation therefore did not require to be carried any further.

Graduated Mortality Table.

x	Log $l(x)$	Log $\frac{l(x)}{l(x+1)}$	Log $\frac{1}{w_x}$	D′	D − D′	$w_x(\text{D}-\text{D}')^2$
0	0·03749	0·01008				
1	2741	0856				
2	1885	0729				
3	1156	0622				
4	0534	0534				
5	0·00000	0·00460	1·310	20·61	+ 11·39	6·37
6	9·99540	400	·277	19·07	+ 8·39	4·22
7	9140	350	·219	16·67	− 3·67	·81
8	8790	310	·165	14·70	− 9·70	6·43
9	8480	278	·110	12·97	− 0·97	·07
10	9·98202	0·00253	1·055	11·42	− 1·42	·18
11	7950	234	·005	10·17	+ 4·83	2·31
12	7716	220	0·973	9·43	− 6·43	4·40
13	7496	211	·934	8·63	+ 0·37	·02
14	7286	205	·867	7·40	− 0·40	·02
15	9·97080	0·00203	0·801	6·35	+ 1·65	·43
16	6877	204	·719	5·26	+ 1·74	·58
17	6673	208	·627	4·26	− 1·26	·38
18	6465	214	·529	3·40	+ 0·60	·11
19	6251	221	·423	2·69	+ 0·31	·04
20	9·96030	0·00230	·314	2·07	+ 0·93	·42

Graduated Mortality Table—(continued).

x	Log $l(x)$	Log $\frac{l(x)}{l(x+1)}$	Log $\frac{1}{w_x}$	D′	D − D′	$w_x(\text{D}-\text{D}')^2$
21	5800	240	·201	1·60	− 0·60	·16
22	5560	251	9·995	0·99	+ 2·01	4·07
23	5309	263	·703	0·51	+ 0·49	·48
24	5045	275	·757	0·57	− 0·57	·57
25	9·94770	0·00288	9·866	0·74	− 0·74	·75
26	4482	301	0·056	1·15	+ 1·85	3·01
27	4181	313	·182	1·53	− 1·53	1·54
28	3868	325	·327	2·14	− 0·14	·00
29	3543	337	·492	3·13	− 1·13	·41
30	9·93206	0·00349	0·603	4·04	− 2·04	1·04
31	2857	360	·710	5·17	− 1·17	·27
32	2497	371	·794	6·27	+ 1·73	·48
33	2126	382	·867	7·42	− 1·42	·27
34	1745	392	·919	8·37	− 0·37	·02
35	9·91353	0·00402	0·477	9·57	+ 0·43	·02
36	0951	413	1·028	10·77	+ 2·23	·47
37	0538	424	·066	11·73	− 0·73	·05
38	0114	435	·103	12·81	+ 0·19	·00
39	9·89679	447	·128	13·55	− 1·55	·18
40	9·89232	0·00461	1·161	14·66	− 1·66	·19
41	8771	475	·193	15·76	+ 0·24	·00
42	8296	492	·212	16·48	+ 3·52	·76
43	7804	511	·226	17·00	− 1·00	·06
44	7294	532	·242	17·67	− 2·67	·41
45	9·86762	0·00555	1·276	18·39	+ 3·61	·69
46	6207	582	·268	18·79	+ 8·21	3·64
47	5625	612	·281	19·36	− 1·36	·10
48	5013	646	·299	20·20	− 4·20	·89
49	4367	684	·316	21·04	− 1·04	·06
50	9·83683	0·00727	1·328	21·65	− 4·65	1·02
51	2957	774	·341	22·31	− 1·31	·08
52	2183	826	·344	22·49	− 0·49	·01
53	1357	885	·353	23·04	− 0·04	·00
54	0472	949	·363	23·58	− 2·58	·29
55	9·79523	0·01019	1·363	23·58	+ 3·42	·51
56	8504	1097	·354	23·17	+ 8·83	3·45
57	7407	1182	·362	23·63	− 4·63	·93
58	6225	1275	·368	24·01	− 8·01	2·75
59	4950	1376	·378	24·69	+ 4·31	·18
60	9·73574	0·01487	1·381	24·89	+ 5·11	1·09
61	2087	1607	·374	24·54	− 6·54	1·81
62	0481	1738	·378	24·86	+ 2·14	·19
63	9·68743	1880	·367	24·32	− 0·32	·00
64	6863	2035	·351	23·52	− 0·52	·28
65	9·64828	0·02202	1·348	23·44	− 1·44	·09
66	2626	2384	·312	21·65	+ 1·35	·09
67	0242	2581	·269	19·72	+ 5·28	1·50
68	9·57660	2795	·150	18·99	+ 1·01	·06
69	4865	3027	·228	18·16	− 2·16	·28
70	9·51838	0·03279	1·211	17·52	− 3·52	·76
71	·48559	3551	·168	15·97	− 7·97	4·32
72	·45008	3846	·133	14·82	+ 1·18	·10
73	·41163	4166	·090	13·54	− 1·54	·19
74	·36997	4512	·003	11·16	− 1·16	·13
75	9·32486	0·04887	0·913	9·16	+ 5·84	4·17
76	·27599	5293	·763	6·54	+ 0·46	·04

Graduated Mortality Table—(continued).

x	Log $l(x)$	Log $\frac{l(x)}{l(x+1)}$	Log $\frac{1}{w_x}$	D′	D − D′	$w_x(\text{D}-\text{D}')^2$
77	·22306	5733	·679	5·45	− 0·54	·04
78	·16572	6210	·595	4·53	+ 3·47	2·07
79	·10362	6727	·451	3·30	− 1·30	·60
80	9·03635	0·07286	0·346	2·62	− 0·62	·17
81	8·96349	7892	·256	2·16	− 0·16	·01
82	·88457	8549	·121	1·61	+ 1·39	1·46
83	·79908	9260	9·890	0·96	+ 1·04	1·40
84	·70648	0·10030	9·691	0·62	+ 2·38	11·55
85	8·60618	0·10864				
86	·49754	·11768				
87	·37986	·12747				
88	·25239	·13807				
89	·11432	·14956				
90	7·96477	0·16199				
91	·80277	·17547				
92	·62730	·19006				
93	·43724	·20587				
94	·23137	·22300				
95	7·00837	0·24155				
96	6·76682	·26164				
97	6·50519	·28340				
98	6·22179	·30697				
99	5·91481	·33251				
100	5·58231	0·36016				

D′ is the number of the deaths according to the graduated table of mortality.

The question now arises whether the formula represents the observations with the necessary precision. As far as can be seen by a simple inspection of the number of signs, 35 being positive and 45 negative, with 38 changes from one to the other, there is nothing to prevent a completely favourable judgment; but a severer test is, as already mentioned, afforded by equation (15). Thus we have

$$\Sigma w_x(\text{D}-\text{D}')^2 = 90{\cdot}6$$

If to this we add 7, the number of the constants, there results 97·6, the number of observations being only 80. It might therefore have been expected that the adjustment should have given the sum of the squares of errors less by 17·6 than really is the case. This difference is not very small, but it is however no proof of the adjustment being either wholly or partly at variance with the observations. If in this case we examine the deviations more in detail, we can with good reason maintain that the sum of the squares must be assumed to become a little larger at each adjustment than is usually the case, because the observations for the highest age, 84, exhibit the unusually large product of weight and

square of error $=11\cdot55$, notwithstanding that the circumstances are such that that observation can have scarcely any influence at all on the value of the normal places; the weight of the probability is infinitely small. Nevertheless I believe we ought to presume the presence of a real deviation; for if we examine the observations from the youngest ages, the column $w_x(\mathrm{D}-\mathrm{D}')^2$ shows for those ages many very large numbers; and besides there is a certain approach to regularity in the magnitudes and signs of $\mathrm{D}-\mathrm{D}'$, and besides this proceeds in such a direction as must be expected from the provisional character of the term $\mu_1(x)=a_1\varepsilon^{b_1x}$, which, it must be admitted, gives rather too low a mortality at the youngest ages. To be sure, I have not in other experiences found this deviation show an influence so far from birth as here. The observations here are however very far indeed from being adequate to give any important contribution towards the improvement of the form of the function $\mu_1(x)$. I therefore rely upon the result we have obtained; but if anybody reproaches me for the deviation from observation, I can only ask for a complete acquittal as regards the ages above 10. For the remaining 75 observations, the sum of the squares $+7=79\cdot7$.

It will be of interest to compare this graduation with Professor Oppermann's. The following table gives for every fifth age $\mathrm{Log}\,l(x)$ and $\mathrm{Log}\,\frac{l(x)}{l(x+1)}$ according to Professor Oppermann, and the deviation of the latter from my adjustment.

x	$\mathrm{Log}\,l(x)$	$\mathrm{Log}\,\frac{l(x)}{l(x+1)}$	$T-O$
5	0·00000	0·00280	+0·00180
10	9·98639	263	− 10
15	9·97349	253	− 50
20	9·96090	252	− 22
25	9·94812	264	+ 24
30	9·93446	292	+ 57
35	9·91895	344	+ 58
40	9·90024	429	+0·00032
45	9·87637	563	− 8
50	9·84448	768	− 41
55	9·80039	1077	− 58
60	9·73799	1542	− 55
65	9·64822	2231	− 19
70	9·51779	3256	+ 23
75	9·32704	4773	+0·00115
80	9·04699	7019	+ 267
85	8·63481	·10340	+ 524
90	8·02726	·15250	+ 949
95	7·13092	·22507	+ 1648

A tolerably trustworthy scale for the magnitude of this deviation is had in the mean errors for my normal places, which are for

$x=7$, $\pm\cdot00039$	$x=36$, $\pm\cdot00047$	
$x=13$, $\pm\cdot00024$	$x=45$, $\pm\cdot00038$	$x=70$, $\pm\cdot00200$
$x=25$, $\pm\cdot00054$	$x=56$, $\pm\cdot00066$	

The deviations may therefore be described as large for ages under about 20 and as not unimportant for the rest of life. For a correct judgment hereupon it must not however be overlooked that the learned Professor's graduation advances still less claim to complete agreement with the whole series of observations. Professor Oppermann has verbally requested me to bear in mind in making this comparison, that it was resolved by the Direction of the Life Insurance Company to use this adjustment, notwithstanding the undoubted deviations which the two observations for the youngest ages exhibit. Thus in Professor Oppermann's adjustment the sum of the squares is 129·4, and adding to this the number of the constants, 5, we get 134·4 instead of 80; but the unquestionable deviation here seen falls almost entirely upon the first ages. Omitting, as in my adjustment, the five youngest ages, the sum of the squares becomes 77·7, and $77\cdot7+5=82\cdot7$, which is not too large in 75 observations. It will be remembered that my corresponding number was 79·7, or 3· less. Omitting in both graduations the ten first ages, the sum of the squares + the number of constants, becomes for Professor Oppermann, 74·5, for me 72·8. My graduation appears thus throughout to agree a little more closely with the experience than Professor Oppermann's.

But excluding the observations on the youngest ages of all, the difference is but very small, and both graduations can be considered as good notwithstanding their mutual deviations. Great as these may be, it must be admitted that they are less, and mostly a good deal less, than the deviations which the nature of the observations makes probable, not to speak of the deviations which the future experience of the Life Insurance Company may possibly exhibit.

It follows as a matter of course that the various premiums of the Life Insurance Company would have been rather different if they had been based upon my graduation instead of Professor Oppermann's; but I have had neither the necessary time nor the necessary information to give an opinion upon the financial result of the difference in the graduations. It will however undoubtedly appear that the difference could not be so very great as may be easily supposed on a comparison of the tables of mortality them-

selves, because in calculating the values of annuities the necessary summations will cause the negative and positive deviations in the different ages partly to balance each other. This can be partly seen by a comparison of the following values of the average lifetime for certain ages according to the two graduations, which I have calculated with this view.

Age.	Average Lifetime according to		Difference.
	Oppermann.	Thiele.	
5	52·380	51·924	– 1 percent.
15	45·371	45·246	0 "
25	37·802	37·430	– 1 "
35	30·071	30·067	0 "
45	22·618	22·843	+ 1 "
55	15·912	15·991	0 "
65	10·359	10·271	– 1 "
75	6·203	5·801	– 7 "
85	3·422	3·273	– 4 "
95	1·757	1·658	– 6 "

Hence we can conclude with tolerable certainty that at the younger ages the premiums for the simplest kind of insurances according to the two graduations will also not differ by more than a small percentage. This shows that the observations, few in number as they are, are yet sufficient to use as a basis for calculating insurance premiums. But the magnitude of the premium, as is well known, depends very much upon the rate of interest and the loading. These quantities it is evident cannot be deduced from the mortality of the past. With respect to the surprisingly small addition for safety which the Life Insurance Company is required by law to make, it may be said that even with a future mortality not materially differing from that here investigated, it is not altogether improbable that the loading for safety will prove insufficient. Against the dangers of a real change in the rate of mortality in the country or a possible change in the manner of deciding as to the health of applicants for insurance it furnishes scarcely any security at all.

In conclusion, I must give my thanks to Professor Oppermann, not only because he was the first to strike out the path along which I have endeavoured to proceed, but also because he has favoured me with a series of conversations on the question of mortality which have not been without influence upon this work, notwithstanding that until its conclusion he did not know what

formula I worked with. Every one who knows how important it is to have a problem properly stated, even if the solution is not given, will understand what advantage I have been able to gain from my intercourse with Professor Oppermann.

[Mr. Makeham, having had the opportunity of perusing the foregoing paper while in manuscript, has drawn our attention to a paper on the same subject by the late Mr. Gompertz, and suggested that it would interest our readers to have this paper reprinted, so as to appear with Dr. Thiele's. We have great pleasure in complying with this suggestion, and also in laying before our readers a paper by Mr. Makeham himself, treating of the same questions.—Ed. *J. I. A.*]

On one Uniform Law of Mortality from Birth to extreme Old Age, and on the Law of Sickness. By the late Benjamin Gompertz, Esq., F.R.S.

[Presented to the International Statistical Congress, July, 1860.]

THE honour I have received by the invitation of the President and members for organising the Fourth Session of the International Statistical Congress, to be held during the week commencing the 16th July instant, which the state of my health prevents me having the satisfaction to attend, induces me to offer to the notice of the distinguished members some observations which, I think, may be considered of some importance in connexion with the objects of the Congress; having for many years paid attention to the subject of vital statistics, and to the mode of rendering the information derivable from its pursuit interesting, beneficially and scientifically, to the public, who may gain consoling pecuniary advantage from it, and to the student, who may enjoy the contemplation of the paths of science; because the services which have been obtained by the public from Assurance Societies, and from the proper management of Friendly Societies, have been obtained by the attentive and philanthropic study of the subject, and the mode of pursuing that study depends on statistical inquiries, and, I may say, on deep mathematical reflection,—a reflection which, whilst it may and has been of important service to society at large, may offer many new views in philosophical and mathematical branches of enquiry even quite unconnected with statistics or its objects.

If I recollect rightly, when the Statistical Society of England was founded, I, having been one of the original members who attended at the meeting, by invitation of a learned friend, who was, I believe, either the originator or great and learned promoter in its foundation, it was laid down as a principle that the communications to it should be the result of a mere narration of facts, unencumbered by "theories"; a principle which, though not laid down without reflection, is, in my opinion, one which should not be over strictly adhered to, but should be a guide to its correspondents; not to be kept entirely out of view, but to be allowed a mitigation in the communications; for the object of research is not only to give information of facts, but to draw beneficial and general views; and if generalisations lead to probable theories, they should be regarded as pleasing associates, to be entertained at the feast of knowledge, derived from the related facts, though not allowed to remove from the table facts which are legitimately obtained, but still to have a licence to search into the legitimacy of statements considered as facts. The reason of my venturing to make this observation is, that though part of this paper will be reflections on statistical evidence, it will lead to a beneficial theory, to be tested by that evidence, and may tend to refute the legitimacy of it, and point out a mode of removing that which is not well founded. In the year 1820 I had the honour to have a paper of mine printed by the Royal Society, in their Transactions, on the Analysis and Notation applicable to the Valuation of Life Contingencies, in which, I believe, there are many improvements offered in the mode of investigations respecting the subject, and means shown of meeting difficulties in the enquiry not then known to the public; and I have since had the honour of having a paper of mine, written in a letter to my late learned and esteemed friend, Francis Baily, Esq., inserted in the Transactions of that Society, in the year 1825, which contained a theorem relative to human mortality, which has received and does still receive much flattering attention from scientific gentlemen usefully devoted to this branch of mathematics. My health has prevented me from publishing more on the subject since then. But my mind at intervals, up to about the year 1845 or thereabouts, and I am unable to say how much before, has been engaged on the subject of what is called Vital Statistics; and I have been for above two years past, though much prevented by the state of my health, attempting to write a supplement to these two papers, which supplement is not finished, nor can I reasonably hope that it will be shortly finished, so as to

enable me very early to present it to the Royal Society,—an honour which I am anxiously desirous of having. I am, therefore, led to suppose that some hints relative to this interesting subject may not be considered unimportant.

In my paper of 1825, representing by L_x the number of persons living at the age x out of L_0 persons born x years previously, I stated (but not with the same letters) the equation $L_x = A\,.\,\overline{B}|^{q^x}$; in which A, B, q were supposed to represent constant quantities, or at least were shown to differ very little from constants, for a very long term of years, for instance, about 50 years, but differing a little for length of term, and from one locality to another, and A and B and q being so related to each other, that supposing them to be constant, we should have $A \times B = L_0$, the number at birth.

But in making the investigation, I did not pretend that A and B were absolutely constant; they were determined by a random selection from three distant periods of age, from a statement of the number of persons who will be living at different ages, out of a certain number of persons stated to have been born. And therefore as $L_x = A\,.\,\overline{B}|^{q^x}$ will not perfectly, during the whole term of life, express the facts, if A and B are taken constant, it is evident that they should not be considered absolutely constant; and it is evident that their values will be found somewhat to differ, by the difference of periods of the three selections of ages; and, in fact, in the 10th article of that paper I pointed out that A, B, and q, by this method, which I may, in order to fix the importance of it in the mind, call the "Vital Rule of Three," have each different values, according to the different periods of selection; and I do not on that account fall in with the views of a gentleman who, since my paper has been published, seems to fancy that there are three distinct periods, between which there are distinct laws of mortality. By some gentlemen, who have set much value on the theorem, obtained by one of the same Vital Rules of Three (as there are four necessary, as will be hereafter explained, to obtain the one continuous and uniform law of mortality, from birth to at least the age of 100), it has been merely considered as a mode of smoothening down the irregularities found in other modes of forming tables of mortality, and as such, even were the theorem used for that object alone, it would be of much value, because there being a very limited set of tables expressing the values of annuities on joint lives, there becomes in consequence a necessity to interpolate values for different combinations from those of the tables, by, for instance, taking first and second differences of a series of

terms proceeding irregularly, so that very gross errors may occur in critical cases. But from the investigation respecting the data of different tables of mortality, and a very particular attention to Mr. Milne's Carlisle table, with respect to the mortality occurring in the first months after birth till the age of one year, when compared with my attempt to explain the law of mortality, I place much confidence in the formula, λL_x standing for the common logarithm of the number of the persons living at the age x, out of the number L_o of persons born x years previously, giving

$$\lambda L_x = \text{constant} + ke^x - \underset{1}{k} . \underset{1}{e}^x . x - nq^x - P_x;\ P_x \text{ being put} = \theta . (\omega)|^{\pi^x . \overline{x-u}};*$$

where all the values on the right-hand side of the equation, except x, are constant, including θ, ω, π and u; and θ very nearly, if not exactly = unity; but I think most probably not exactly equal to it.

And with regard to parts of the formula, the part ke^x is at birth $= k$, and decreases regularly as x increases, and becomes before the age of 20, and is ever after, in the tables I have examined, perfectly insignificant; the term $\underset{1}{k}\underset{1}{e}^x . x$ is of no value at birth, but increases in a very short time to a maximum of significant value; but in less than 12 months, in the table I have examined, decreases to perfect insignificance. The term nq^x is $= n$ at birth, and continually increases with the increase of x, to the remotest age. The term P_x is perfectly insignificant at birth, and till an age but a few years below 60, though, for some purposes in the valuation of annuities, its then extremely insignificant value will require attending to; and it then continually increases with the age till far beyond the age of 100, and then decreases into insignificance.

And I consider the equation quoted, namely—

$$\lambda L_x = \text{constant} + ke^x - \underset{1}{k}\underset{1}{e}^x x - nq^x - P_x,$$

sufficiently interesting for my endeavour to discover the cause of the equation's existence, and the following is my attempt to do so.

But I first observe that the equation itself may be put in the form—

$$L_x = \text{constant} \times \overline{A}|^{e^x} \times \overline{B}|^{\underset{1}{e}^x . x} \times \overline{C}|^{q^x} \times \overline{D}|^{P_x},$$

by an easy transformation, where A, B, C, D, e, $\underset{1}{e}$, q are constant.

* In a paper read before the Royal Society in June, 1861, the author gives "what he considers an improved formula" for expressing the law of mortality, viz.:

$$\lambda L_x = C\beta^x + k\varepsilon^x + \underset{1}{k}\underset{1}{\varepsilon}^x - \lambda^{-1}(\iota^x . \lambda q_0 . \overline{x-h}) + \mu\nu^x$$

But in consequence of all the factors but $\overline{C}|^{q^{x}}$ being, in the tables I have examined, between the ages of 20 and 60, so nearly constant, that the difference from it may be neglected, the equation may between those limits be put into the form constant $\times\,\overline{C}|^{q^{x}}$, where C and q are constant, as I stated (with a different letter for C) in my paper in the Philosophical Transactions for 1825. But from birth to twelve months, it would stand $L_x = \text{constant} \times \overline{A}|^{e^{x}}\ \overline{B}|^{1^{e.x^{x}}}\ \overline{C}|^{q^{x}}$; from twelve months to 20 years it would stand $L_x = \text{constant} \times \overline{A}|^{e^{x}}\ \overline{C}|^{q^{x}}$. For some years before 60 it would stand, and continually after, $L_x = \text{constant} \times \overline{C}|^{q^{x}} \times \overline{D}|^{Px}$.

And contemplating on this law of mortality, I endeavoured to enquire if there could be any physical cause for its existence.

And setting out with the supposition, that life requires certain powers of integration in the material of its necessary organization to be kept up; and that these powers may be divided into two portions, the one which I call the principal and fundamental part, and the other a distinct and an auxiliary part to keep up the integration; and that there are powers in opposition to them, to destroy the necessary powers of integration of the materials of organization necessary for life; and that should the latter prevail, life would be destroyed. And to show the result of this supposition, I will use the terms physical functions to be performed by the physical functionaries, whatever these may be, whether the heart, lungs, liver, &c., or some other organ of the living frame, for I do not pretend to say which are the functionaries acting, whilst I endeavour to draw from statistical data, mathematical functions which are to express for the age x, the number L_x of persons who are living out of the number L_o, which were born x years previously; and calling F the resulting force to destroy life (that is, the necessary power of integration of the material of organization necessary for life) which remains of the destroying forces after the auxiliary force for maintaining life is deducted; that is, putting F for the intensity of that excess due to each individual of the number of persons L_x of the age x, the total force to produce destruction among them all, or, in other words, the quantity of death, or loss of life among them, will be $F.L_x$, which, multiplied by $\dot{x}$, the fluxion of time, must be equal to $-\dot{L}_x$, that

is, the fluxion of death, which is — the fluxion of life, the fluxion of life being $\dot{L}_x$; but if m be put for the Napierian logarithm of 10, the equation which gave the value of λL_x, will give the equation Napierian logarithm of $L_x = m(\text{const.} + ke^x - k_1 x e_1^x - nq^x - P_x)$; and if this be put into fluxions, we shall have, by dividing by $\dot{x}$—

$$\frac{\dot{L}_x}{L_x\dot{x}} = m^2k.\lambda e.e^x - m.k_1.e_1^x.(1+m.\lambda e_1.x) - nm^2.\lambda q.q^x - \frac{m\dot{P}_x}{\dot{x}}$$

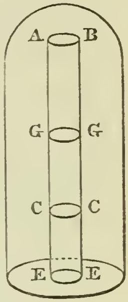
λ standing to express the words "common-logarithm of," or if to simplify, we put $nm^2\lambda q = \text{unity}$, and put $\frac{k}{n\lambda q} = A$, $\frac{k_1}{mn\lambda q} = B$, $\frac{1}{mn\lambda q} = C$, the expression will stand

$$\frac{\dot{L}_x}{L_x\dot{x}} = A.\lambda e.e^x - Be_1^x.(1+m\lambda e_1.x) - q^x - C.\frac{\dot{P}_x}{\dot{x}};$$

but the equation $\frac{\dot{L}_x}{L_x} = -F.\dot{x}$, gives $\frac{\dot{L}_x}{L_x\dot{x}} = -F$; consequently F, the resulting force to destroy life or produce death, is $= -A\lambda e.e^x + Be_1^x(1+m\lambda e_1.x) + q^x + C.\frac{\dot{P}_x}{\dot{x}}$

And, considering these four portions of the value of F separately, I shall first pay attention to the force q^x, which, as q is above unity, will increase with x, and not decrease, as the density of the air in the receiver of an air-pump, by x equal strokes of the piston, but in indirect proportion, which I mention because the indefinite mode of expression I by some chance used in the article of my paper in the Philosophical Transactions of 1825, in the comparison I made of that force to the density of air in the receiver which remained after x strokes, may seem to express that q^x is in direct proportion to the reduced density, which is impossible when x increases, if q be greater than 1; and I observe that in the other forces, such as $A\lambda e.e^x$, where e is less than 1, the force might be expressed properly by stating it to be directly as the diminished density in the receiver by the strokes of the piston. This force q^x is a force to destroy life, or, in other words, a force diminishing the effect of the forces which preserve life. But though I thought the comparison an apt one to relieve the mind in search of a reason why the tables introduced the expression q^x, I had no wish to bind the theory of mortality to having sprung from such a cause, for there are a variety of actions which may arithmetically lead to the same numbers. But the aptitude of the comparison may be

explained, even in the case of q being greater than 1, by exhausting a receiver, as we may by the following mode get a condensation $=q^x$ of air in a tube, by the number x of strokes, of an exhausing piston; though the common condensing air-pumps, by introducing air of a fixed density, only condense in arithmetical progression. The tube A B E E is open at the top, and firmly fixed to the bottom of the receiver of an air pump, and has a valve opening inside to let in the air of the atmosphere at pleasure. The part C C E E is separated from the other part by an air-tight cork, moveable with perfect freedom up and down; the cork and the air in the receiver and all the tube has the common density of the air of the atmosphere, D, then by x strokes of the piston the air in the receiver and upper part of the tube is reduced to the density $\frac{D}{q^x}$, and the cork will rise to G G, so that the air in the part G G E E may also have the density $\frac{D}{q^x}$; then open the valve to let in the fresh air; and then, since before the fresh air rushes in, we had $\frac{D}{q^x} \times EG = D \times CE$, it follows that $q^x = \frac{EG}{CE}$; but now the air being of the density D, and at the height E G, drive down the cork to the space C C, and the density will become in the space C C E E $= D \times \frac{EG}{CE} = Dq^x$.



With respect to the portions $-A\lambda e.e^x$, and $B.\underset{1}{e}^x\,(1+m\lambda \underset{1}{e}\,x)$ of the force F, the first is conservative, and the second destructive whilst $1+m\lambda\underset{1}{e}.x$ is affirmative; but they both speedily become of small effect. The first of these in the Carlisle mortality becomes, by its fast decrease as x increases in the Carlisle table of mortality quite insignificant before the age of 20, and continues to decrease with the increase of x. The second decreases much quicker, and becomes insignificant, in the Carlisle tables, for instance, in less than twelve months. The first and second both commence at birth; the first with a force $=-A\lambda e$, which is conservative, notwithstanding that as e is less than 1, λe is negative; because, however small x may be, the fluent of $-A\lambda e.e^x.\dot{x}$ becomes $-\frac{A\lambda e.e^x}{m\lambda e}$, and would be $-\frac{A.e^x}{m}$, m being $=2{\cdot}3026$. The second force becomes the fluent of $\dot{x}.B\underset{1}{e}^x.(1+m\lambda\underset{1}{e}\,x)$, generated from $x=0$,

and is $= \mathrm{B}.x.e_1^x$, and is always destructive. The force itself diminishes to insignificance in the Carlisle mortality in less than twelve months, and becomes absolutely 0 in about $8\frac{1}{3}$ years, that is, when $x = \frac{1}{-m\lambda e_1}$, and then increases with a negative effect till $x = \frac{2}{-m\lambda e_1}$ to an insignificant small maximum, and then decreases to still increased insignificance. With respect to the portion $\mathrm{C}\frac{\dot{\mathrm{P}}x}{x}$, it is a force of perfect insignificance, but destructive, till an age a few years short of 60, continually increasing to an age beyond 100 by several years, till it becomes a maximum, and then decreases in its effect.

But to render the theory of mortality more complete than the state in which the above observations leave it, I think it necessary to show that, though the formula above would appear to require that all individuals of a birth in the same locality should have ultimately the same length of life, and to indicate that each lost an absolute and equal portion of his life between two given ages, and still continued to live on with the mutilated portion of life remaining, which would evidently be a most absurd conclusion; for the formula only shows that out of L_a lives of the age a there would be living L_{a+x} only, x years after; therefore, to reconcile the formula, or rather theory, with the case of nature, and to show how it comes to pass, that, consistently with the theory, different persons live to different ages, I will commence by quoting the following observations from Art. 4. of my paper in the Philosophical Transactions of 1825:—"It is possible that death may be "the consequence of two generally co-existing causes; the one "chance, without (*i.e.* independently of) predisposition to death, "or deterioration, and the other a deterioration, or an increased "inability to withstand destruction. If, for instance, there are a "number of diseases, to which the young and old are equally "liable, and likewise which should be equally destructive to the "young and old, &c." And first I observe, that the force represented by q^x, which, with the increase of x, is an increasing force, in consequence of q being greater than 1, is a destructive force, and that it may be expressed by the density of the air indirectly in the receiver of an air-pump, which had lost by x equal strokes of the piston a portion of its density; and I observe that if there are preservative and destructive forces of life, whilst the preservative forces are greater than the destructive forces, the quantity

q^x (which is indirectly as the density) whilst it increases, diminishes the effective force by which life is preserved, and therefore the increase which q^x receives as x increases, is a decrease of effective preservative force.

And to show that the formula does not require all persons of a birth to live to the same age whatever the causes may be on which the formula depends, suppose, instead of considering the terms ke^x, $\underset{1}{k}x\underset{1}{e}^x$, nq^x, &c., as they stand in the formula, we take a function Ap^x on supposition that it belongs to the formula, as by substitution of the elements it could be made to express either, as, if p be greater than 1, it might stand for q; if p be less than 1, it might stand for e, or for e_1; and supposing the different individuals of a birth were called 1, 2, 3, 4, &c., and that their portions referable to the term Ap^x were $A_1p_1{}^x$, $A_2p_2{}^x$, $A_3p_3{}^x$, &c., all different or not from each other, which supposition supposes only that Ap^x can for all values of x be $=A_1p_1{}^x+A_2p_2{}^x+A_3p_3{}^x+$&c., and then it would appear that the formula does not require all individuals of the same birth to live to the same age. Now, if p_1, p_2, p_3, &c., were all equal to each other, the supposition would only require A to be $=A_1+A_2+A_3+A_4$, &c., without requiring A_1, A_2, A_3 to be equal to each other; and if p_1, p_2, p_3 differed so little from each other that, though they were different, some would be but a small matter greater than some approximate value p, and some a small matter less, in such sort that p might be approximately taken for them, it would follow that Ap^x might be taken for an approximate result of the effect on the whole number of persons of which L_0 consists, though owing to the effect which is various on the individuals.

And with respect to the influence of chances, as there are evidently many chances by which individuals are affected. If the chance affecting a person named 1 (suppose), relative to the power $A_1p_1{}^x$, affected the co-efficient so as to increase or diminish it in a given ratio, then will that individual A_1, with respect to his vitality, be differently affected than other individuals of his age; and it is easy to point out how chances may occur which would introduce a variability in different years for that individual's life from that of other persons of that age.

And other hypothesises may be made besides those assimilated to the exhaustion and condensation of air in the air-pump, to render the formula to appear a natural result; for terms such as Ap^x in the portion of the force affecting vitality might be produced by effects such as that which would increase the value of land by allowing it to cultivate itself by its seed, or by causes which may increase

population, or to effects which would impoverish land not cultivated, by allowing the seed of weeds to reproduce.

The comparison from the age of birth for every year till the age of one hundred, appears very satisfactory; but in Milne's table there are given for the ages of birth, one month, two months, three months, six months, and nine months, also the mortality; and it is interesting and instructive to make the comparison of the formula $\lambda L_x = \text{constant} + ke^x - nq^x - P_x$ with the statement in Mr. Milne's table, and for this reason I place below the means of comparison.

Living.	At Birth.	At 1 Month.	2 Months.	3 Months.	6 Months.	9 Months.	12 Months.
By Milne,	10,000	9,467	9,313	9,226	8,970	8,715	8,461
deaths in		533	687	774	1,030	1,285	1,539
By formula,	10,000	9,827	9,620	9,511	9,107	8,757	8,462
deaths in		173	380	489	893	1,243	1,538

To judge of the comparative results the more clearly, it will be well to compare the deaths by Milne's table with the deaths of the formula which occur in like number of months. And it appears that while, at the age of twelve months and nine months, there is a great proximity, that to the age of six months the formula gives less deaths by about ten per cent., but that during the first month there are only about one-third of the deaths which occur in Milne's table. And reflecting more particularly on the large number of deaths which in Mr. Milne's table take place in one month in comparison with the deaths which in that table take place in the following months, I have found that the formula for λL_x stands in need of an additional term, $-\underset{1}{k}.x.\underset{1}{e}^x$, in which e is such a fraction that in a few months its effect will be insignificant, and that the formula by this correction will stand

$$\lambda L_x = \text{constant} + ke^x - nq^x - \underset{1}{k}.x.\underset{1}{e}^x - P_x;$$

and for the Carlisle mortality I find $\underset{1}{k} = \cdot 39242$; and $\lambda \underset{1}{e} = \bar{4}.34652$, and for comparing the result of this corrected formula we have living:—

Living.	At Birth.	At 1 Month.	2 Months.	3 Months.	6 Months.	9 Months.	12 Months.
By Milne,	10,000	9,467	9,313	9,226	8,970	8,715	8,461
deaths in		533	687	774	1,030	1,285	1,539
By formula,	10,000	9,467	9,313	9,253	9,046	8,747	8,462
deaths in		533	687	747	954	1,283	1,538

x Age.	L_x Carlisle Formula.	Dec.	L^1_x Milne Formula.	Dec.	x Age.	L_x Carlisle Formula.	Dec.	L^1_x Milne Formula.	Dec.
0	10,000	1,538	10,000	1,539	51	4,327	74	4,338	62
1	8,462	828	8,461	682	52	4,253	75	4,276	65
2	7,634	513	7,779	505	53	4,178	76	4,211	68
3	7,121	163	7,274	276	54	4,102	78	4,143	70
4	6,958	146	6,998	201	55	4,024	79	4,073	73
5	6,812	122	6,797	121	56	3,945	82	4,000	76
6	6,690	74	6,676	82	57	3,863	84	3,924	82
7	6,616	56	6,594	58	58	3,779	86	3,842	93
8	6,560	47	6,536	43	59	3,693	89	3,749	106
9	6,513	41	6,493	33	60	3,604	93	3,643	122
10	6,472	38	6,460	29	61	3,511	96	3,521	126
11	6,434	37	6,431	31	62	3,415	100	3,395	127
12	6,397	38	6,400	32	63	3,315	105	3,268	125
13	6,359	35	6,368	33	64	2,210	110	3,143	125
14	6,324	37	6,335	35	65	3,100	116	3,018	124
15	6,287	38	6,300	39	66	2,984	122	2,894	123
16	6,249	39	6,261	42	67	2,862	127	2,771	123
17	6,210	39	6,219	43	68	2,735	133	2,648	123
18	6,171	40	6,176	43	69	2,602	140	2,525	124
19	6,131	41	6,133	43	70	2,462	146	2,401	124
20	6,090	42	6,090	43	71	2,316	150	2,277	134
21	6,048	42	6,047	42	72	2,166	153	2,143	146
22	6,006	44	6,005	42	73	2,013	156	1,997	156
23	5,962	45	5,963	42	74	1,857	157	1,841	166
24	5,917	45	5,921	42	75	1,700	156	1,675	160
25	5,872	47	5,879	43	76	1,544	135	1,515	156
26	5,825	47	5,836	43	77	1,389	153	1,359	146
27	5,778	49	5,793	45	78	1,236	145	1,213	132
28	5,729	50	5,748	50	79	1,091	138	1,081	128
29	5,679	50	5,698	56	80	953	170	953	186
30	5,629	52	5,642	57	81	823	118	837	112
31	5,577	53	5,585	57	82	705	109	725	102
32	5,524	53	5,528	56	83	596	102	623	94
33	5,471	55	5,472	55	84	494	81	529	84
34	5,416	56	5,417	55	85	413	73	445	78
35	5,360	57	5,362	55	86	340	65	367	71
36	5,303	57	5,307	56	87	275	55	296	64
37	5,246	59	5,251	57	88	220	44	232	51
38	5,187	60	5,194	58	89	176	31	181	39
39	5,127	61	5,136	61	90	145	30	142	37
40	5,066	62	5,075	66	91	115	30	105	30
41	5,004	63	5,009	69	92	85	20	75	21
42	4,941	64	4,940	71	93	65	16	54	14
43	4,877	65	4,869	71	94	49	11	40	10
44	4,812	66	4,798	71	95	38	9	30	7
45	4,746	67	4,727	70	96	29	8	23	5
46	4,679	68	4,657	69	97	21	5	18	4
47	4,611	70	4,588	67	98	16	4	14	3
48	4,541	70	4,521	63	99	12	3	11	2
49	4,471	72	4,458	61	100	9	..	9	
50	4,399	72	4,397	59					

The Vital Rule of Three is applicable very satisfactorily to sickness prevailing in the different districts, in all England, and in the whole of Scotland, given in the Statistical Journal for December 1845, by the ingenious F. G. P. Neison, Esq., in his well-digested

article, and by the ingenious Charles Ansell, Esq., in his valuable work on Friendly Societies, published under the superintendence of the Society for the Diffusion of Useful Knowledge, of whose committee I had the honour to be a member,—a society which has rendered great service to literature, and which had the advantage of having the Right Honourable Lord Brougham for its excellent Chairman; and as I think much praise is due to the working members who procured the data from the different societies, and much praise is due to Mr. Ansell for his digest of the facts, and to Mr. Neison, the writer of the article in the Statistical Journal alluded to, I feel that my impression of their merits should be in this place expressed to the Congress. And I also feel that much gratitude is due to the Registrar-General* for his able reports, which are likely to add much advantage towards the development of this useful branch of statistics.

The exact law of mortality, or a very near approach to it, being expressed analytically, gives a vast scope to the analyst of means to solve most intricate questions respecting vital statistics. When the supposed law was that of equal decrements, occurring annually in the number of living from some particular age, originating from birth, and proceeding in the same exact law to old age, our powers of obtaining any moderate accuracy to the solution even of very simple problems, was extremely small. The late excellent Mr. William Morgan, the actuary of the Equitable Assurance Office, with whose friendly acquaintance I was favoured in my early years, saw the defect of adopting that incorrect hypothesis, and made his calculations of the values of annuities from a table which appeared to him to represent the mortality; but when certain contingencies were considered with respect to order of survivorship in the cases, he adopted the hypothesis of uniform and equal decrements, which being adopted, would give the result, that if two persons, though they were of different ages, had died, it would be an equal chance which should have died first,—a curious and useful theorem, if that law could be adopted as a true or near approximation for a long term of years, though the same law of uniform decrement did not prevail for both lives, provided that for the term for one life the decrement remained constant, and for the other life for the same term the decrement remained constant; but

* But having mentioned the name of Registrar-General above, I consider that I ought to state, that though my health did not permit me to respond in the manner I should much have liked, to his courteous letter to me of 1845, I should have felt it a duty to offer my aid to his laudable wish to cause his useful labours to be aided by the labour of others.

as this is not accurate in any possible continuous and uniform function of mortality, in my paper of 1820 I investigated,—which is a curious problem,—the problem what uniform and continuous function would give that law between two lives, and I found it to be $L_a = A + B\,C^a$, and no other, where A, B, and C are constant; but if C were taken infinitely small, or were = unity, it would become the law of equal decrements, but otherwise it would be the law of a geometrical progression, increased by a constant quantity.* But that these observations may not appear irrelevant, I observe that a law, which I call the real law of mortality, shows that the hypothesis of equal chances of survivorship very proximatively obtains for a long term of years; a useful hypothesis in cases where there are some lives concerned connected with conditions of order of survivorship between them; but all cases of even as few as three lives, in which one of them is to be assured, by Mr. Morgan's method or by the slight improvements and some amendments of Mr. Baily and Mr. Milne, the theorems are tremendously intricate, and I am inclined to say useless, when the calculation is to be made from the value of annuities given by the combination of only two lives. Therefore, in my paper of 1825 I gave a new mode of considering the problems, leading to comparatively simple methods, and gave an easy method of approximating to the value of an annuity of a great number of joint lives, by tables approximating to the value for terms to be added together to any degree of accuracy, though for different tables of mortality, so that a great part of the difficulties was in a great degree met. But the possession of the true, or very approximately true, single and continuous law of mortality from the age of birth, without any interruption of the uniformity, gives a vast command over cases of very great complication of contingencies, whether simple or connected with any proposed order of survivorship; but to give the full advantage of that law, derived from statistical statements, I found it necessary to introduce what I think may be called a new branch of the mathematics, which I denominate formulized or analytical arithmetic, involving what I call formulized or analytical logarithms and anti-logarithms; but as hints rather than abstruse mathematics seem more adapted for the

* Referring again to this subject in his paper of June, 1861, the author says,—"But "that this law" (*i.e.* the law of equal chances of survivorship) "cannot accurately exist "for any possible continuous law of mortality, I have proved in my paper of 1825, "unless of the form $L_a = e' - e''.e^a$, where e', e'', e, are constant quantities at pleasure, "and a the age, and which in an extreme case of e differing infinitely little from unity, "is reducible into the form $L_a = g' - g''a$, if $g' = e' - e''$, and $g'' = e''\varepsilon$,—e and e'' being "infinitely large, differing from each other by a finite quantity, and ε infinitely small, "and in consequence g'' a finite quantity."

discussion of the Congress, I think I need not regret that I find it better to reserve that portion of the subject for the Royal Society, for which learned body the paper I was writing was and is intended, should it not be thought these few hints I have the honour of preparing for this Congress should disentitle it to be received, which I do not feel it will be thought to do, as the body of the argument will be in that paper; and therefore I feel I am not departing from my duty as a Fellow to give these few hints to this distinguished Congress, as no doubt every Fellow of the Society feels with me, that it is a pleasure to aid in promoting the important objects of the meeting; and that therefore the Royal Society is indirectly promoting its own views by allowing slight hints of papers to be offered it to be given to the Congress, especially in cases like mine, where indisposition may delay the presentation of the paper itself to the Society. But I hope to lay before the Congress a sufficient portion of the tables, calculated from the formula, to enable it to judge if my hints are entitled to its attention, without prejudicing my paper which is intended for the Royal Society in its title for reception.

Formula for calculating the Tables of Mortality,

$$\lambda L_x = \text{constant} + ke^x - \underset{1\,1}{ke^x}.x - nq^x - P_x;\text{—where } P_x = \overline{\omega}|^{\pi^x.x-u}.$$

	u	λk	λe	$\lambda \underset{1}{k}$	$\lambda \underset{1}{e}$	λn	λq	$\lambda \pi$	$\lambda \lambda \omega$	Constant.
Carlisle	90·37	$\bar{1}$·2310	$\bar{1}$·76774	$\bar{1}$·59375	$\bar{4}$·34652	$\bar{2}$·75526	·0126	$\bar{1}$·98952	$\bar{1}$·50837	3·88631
Northampton	901·31	$\bar{1}$·43172	$\bar{1}$·72758	No data.	No data.	$\bar{1}$·11526	·011213	$\bar{1}$·9954	$\bar{1}$·15125	3·92650
Sweden	86·437	$\bar{1}$·28502	$\bar{1}$·7918	No data.	No data.	$\bar{2}$·87042	·01296	$\bar{1}$·99608	$\bar{1}$·03727	3 87142
Deparcieux ..	86·21	$\bar{2}$·99	$\bar{1}$·8415	No data.	No data.	$\bar{1}$·3323	·006005	$\bar{1}$·99293	$\bar{1}$·2250	3·19130

With respect to sickness, representing by S_x the amount of weeks of sickness at the age x to any person, I find by the Vital Rule of Three applied to statistical statements a similar form to express the powers to preserve health to apply to all ages very nearly between 20 and 60; so that the same remarkable piston law, with respect to the exhaustions of the variable power (added to a constant power) to preserve health, and oppose to the power to support constant power to destroy health, also appears to be both a valuable physiological fact, and a useful theorem. λ representing the character to express logarithm of, $\lambda S_x = \lambda A + B e^x = \lambda A + \overset{-1}{\lambda}(\lambda B + x \lambda e)$, $\overset{-1}{\lambda}$ being put for anti-logarithm of; or we

may express the law by $S_x = A.C^{e^x}$, C being the number whose Log. is B;

Selected Ages.	—	—	—	—
25 45 65	Ansell, all England.	λ A = $\bar{1}$·72778 B = ·04755 Sum = $\bar{1}$·77533	λ B = $\bar{1}$·6772	λ e = ·020433
25 45 65	Scotland.	λ A = $\bar{1}$·66237 B = ·02594 Sum = $\bar{1}$·68831	λ B = $\bar{2}$·41351	λ e = ·02433
25 45 65	Ansell, Town.	λ A = $\bar{1}$·65112 B = ·11040 Sum = $\bar{1}$·76152	λ B = $\bar{1}$·04297	λ e = ·02818
25 45 65	Ansell, Town.	$\bar{2}$·92937 ·51218 Sum = $\bar{1}$·44155	λ B = $\bar{1}$·70942	λ e = ·0165265
25 45 65	C. Districts.	λ A = $\bar{1}$·84121 B = ·03010 Sum = $\bar{1}$·87131	λ B = $\bar{2}$·	λ e = ·009206
35 50 65	City.	λ A = $\bar{1}$·84121 B = ·03010 Sum = $\bar{1}$·87131	λ B = $\bar{1}$·15573	λ e = ·0301685
30 40 50	Rural District.	λ A = $\bar{1}$·85616 B = ·10703 Sum = 1·96319	λ B = $\bar{2}$·02951	λ e = ·0301685

Age.	S_x All England. (Ansell).		S_x Scotland.		S_x Three Districts. (Ansell.)		S_x Town Districts.		S_x City Districts.		S_x Town.		S_x Rural.		Age.
	Formula.	Statement.	Formula.	Statement.	Formula.	Statement.	Formula.	Statement.	Formula.	Statement.	Formula.	Statement.	Formula.	Statement.	
20	..	..	·585	·585	·79746	·8398	·77168	·8564	·8690	·5659	·51519	·8564	·7927	·8387	20
25	·762	·762	·585	·585	·8744	·8744	·8649	·8640	·9650	·9650	·63019	·8649	·8259	·8630	25
30	..	..	..	..	·9507	·9107	·99236	·8794	1·0909	1·1059	·99236	·8794	·87555	·8753	30
35	·9431	·964	·7009	·675	1·0622	·9236	1·1836	1·0112	1·25992	1·2372	1·0114	1·1014	·956	·8991	35
40	..	..	..	..	1·2298	1·1808	1·43542	1·3669	1·4894	1·4663	1·3340	1·2669	1·0677	1·0677	40
45	1·327	1·327	·962	·962	1·4939	1·4939	1·83227	1·8323	1·8126	1·8128	1·8515	1·8323	1·2588	1·2537	45
50	..	..	..	..	1·9332	1·9603	2·46173	2·5559	2·2806	2·3821	2·5559	2·5559	1·5894	1·5896	50
55	2·2920	2·443	1·6713	1·821	2·7166	2·7047	3·6455	3·3029	2·9837	2·3036	3·7393	3·302	2·2165	2·3260	55
60	..	..	..	..	4·2623	4·1657	5·4196	4·9132	4·0858	4·4975	5·7087	4 913	3·526	3·8531	60
65	5·498	5·498	4·399	4·400	7·7505	7·7505	9·1387	9·1387	5 9028	5·9028	9 1387	9·1387	6·829	7·6535	65
70	10·201	11·683	9·143	10·761	17·0993	14·0397	..	..	9·0747	9·9610	19·071	24·0134	17·130	14·1949	70
	Selected Ages. 25, 45, 65.		Selected Ages. 25, 45, 65.		Selected Ages. 25, 45, 65.		Selected Ages. 25, 45, 65.		Selected Ages. 25, 45, 65.		Selected Ages. 35, 50, 65.		Selected Ages. 30, 40, 50.		

S_x being the weeks of sickness in one year.

And, in conclusion of this paper, I wish to add, that having had the pleasure to read the excellent address of His Royal Highness, who honoured the meeting by accepting the invitation to be the President, I feel a gratification in finding that my endeavours to draw a theory from statistical data was not deviating from the intentions of the Congress, and hope that the international union of the great talents of different nations will be a stimulus for international friendship, sufficient to promote the happiness of humanity, and prove that there is more honour and real advantage to be gained by the calm, patient, and peaceful pursuit of science, than there is to be gained by gunpowder and the sword.

[Mr. Makeham writes to us—"This paper, which, I believe, is "very little known, was drawn up by Mr. Gompertz for the International Statistical Congress, during the preparation of his more "elaborate contribution to the Royal Society in June, 1861. It "contains much interesting matter, not included in the last-"mentioned paper,—relating more especially to the physiological "basis of the author's theory of mortality. The omission to "incorporate this matter in his later paper arose doubtless from a "fear of infringing the rule of the Royal Society against the "admission into its 'Transactions' of anything previously published. That it did not arise from any wish on the author's part "to suppress any of the opinions expressed in the accompanying "paper, is sufficiently proved by the fact that my copy was presented to me by Mr. Gompertz in July, 1861, and, therefore, "after his paper had been read before the Royal Society."—Ed. *J. I. A.*]

Explanation and Example of a Method of Constructing Mortality Tables with imperfect data; and of the Extension of Gompertz's Theory to the entire period of Life. By W. M. Makeham, *Fellow of the Institute of Actuaries.*

I HAVE recently had occasion for a special purpose to apply the formula $L_x = dg^{q^x}s^x$ to a case which, I think, presents points of sufficient interest to justify me in laying it before the readers of the *Journal of the Institute of Actuaries.*

The chief difficulties encountered in the construction of the following Table have arisen, first, from the extreme paucity of the

data for adult ages,—the whole number of deaths observed above the age of 14 being 27 only,—and secondly, from the want (hitherto) of a simple and satisfactory method of adapting the law of mortality prevailing in adult life to the observations for infancy and childhood. I proceed to explain the mode in which these difficulties have (as I venture to think) been overcome.

In my paper on "The Law of Mortality" (see *Journal of the Institute of Actuaries*, vol. xiii p. 347) I stated that I had found the constant q, in the formula above quoted, to be nearly the same in different observations. The average value of the common logarithm of q in the best observations appears to be ·04, very nearly. Thus in

$$\left.\begin{array}{l}\text{Brown's Clergy Observations}\\ \text{Experience of 17 Offices}\\ \text{Peerage Mortality (both sexes)}\\ \text{American Life Table (Males)}\end{array}\right\}\log q=\left\{\begin{array}{l}\cdot0391\\ \cdot0407\\ \cdot0419\\ \cdot0397\end{array}\right.$$
$$(\text{Average}=\cdot0403)$$

These variations among *different* Tables are in fact not much greater than in different parts of the same Table,—as may be seen from the fact that the value of $\log q$ deduced by Mr. Woolhouse from the Experience of 17 Offices, by using a different set of ages, is ·03956 (*Journal*, vol. xv p. 405). Mr. Woolhouse also gives the following comparison between the constants for the "new" and "old" experience respectively,—to which I have added a column showing the ratio of the difference to the mean value.

	New (H^M).	Old.	Ratio of Difference to Mean Value.
$\log q$	·03947	·03956	·23 per cent.
a	·006147	·006619	7·39 „
b	·0000988	·0000972	1·63 „

If the values of q instead of $\log q$ be compared, it will be found that the ratio of the difference to the mean value is ·022 per cent. only,—a perfectly insignificant quantity. I may add that Mr. Woolhouse, in constructing his Mortality Table according to the formula above mentioned, takes $\log q = \cdot04$ as a sufficient approximation to the true value.

These considerations have induced me, in the construction of the following Table, to assume $\log q = \cdot04$, instead of deducing a value from the very limited data available for the purpose; and I believe that by the help of this assumption I have succeeded in

obtaining a satisfactory exponent of the true law of mortality prevailing among the body subject to observation,—a result which certainly could not have been attained in this case by the ordinary methods of constructing Mortality Tables.

Let E_x denote the number actually exposed to risk *for a complete year* following any given age, x, (or the number of "years of life" observed between the ages x and $x+1$), and d_x the corresponding number of deaths. We shall then have d_x: $E_x = F_{x+\frac{1}{2}}$ (approximately) $= a + bq^{x+\frac{1}{2}}$ $\therefore$ $d_x = aE_x + bE_xq^{x+\frac{1}{2}}$, and therefore

$$d_n + d_{n+1} + \ldots + d_{n+t-1} = a(E_n + E_{n+1} + E_{n+t-1}) + b(E_nq^{n+\frac{1}{2}} + E_{n+1}q^{n+\frac{3}{2}} + \ldots + E_{n+t-1}q^{n+t-\frac{1}{2}}) \quad (1)$$

$$d_{n+t} + d_{n+t+1} + \ldots \textit{ad inf.} = a(E_{n+t} + E_{n+t+1} + \ldots \textit{ad inf.}) + b(E_{n+t}q^{n+t+\frac{1}{2}} + E_{n+t+1}q^{n+t+\frac{3}{2}} + \ldots \textit{ad inf.}) \quad (2)$$

Put

$$\begin{aligned}
D &= d_n + d_{n+1} + \ldots + d_{n+t-1} \\
D' &= d_{n+t} + d_{n+t+1} + \ldots \textit{ ad inf.} \\
P &= E_n + E_{n+1} + \ldots + E_{n+t-1} \\
P' &= E_{n+t} + E_{n+t+1} + \ldots \textit{ ad inf.} \\
Q &= E_nq^{n+\frac{1}{2}} + E_{n+1}q^{n+\frac{3}{2}} + \ldots + E_{n+t-1}q^{n+t-\frac{1}{2}} \\
Q' &= E_{n+t}q^{n+t+\frac{1}{2}} + E_{n+t+1}q^{n+t+\frac{3}{2}} + \ldots \textit{ ad inf.}
\end{aligned}$$

Then from equations (1) and (2) we may deduce the following values of the two constants a and b, viz.:

$$a = \frac{DQ' - D'Q}{PQ' - P'Q}, \text{ and } b = \frac{D'P - DP'}{PQ' - P'Q}.$$

From this it will be seen that not only may the whole of the available data be brought to bear upon the determination of the two constants a and b, but the elements involved are the *actual numbers observed*, and not merely the ratios of the deaths to the living as in ordinary cases. In reference to the first-mentioned point I may be allowed to quote the following passage from my letter in the *Journal* (vol. xiv p. 244) bearing upon this point. "And herein lies the great importance of the discovery of general "laws, viz. that by their aid we are enabled to economize our facts, "and, by bringing them all to bear upon a single point, compel "them, as it were, to yield us information which we should "otherwise be unable to obtain." I may also refer to my observations on the same subject, in vol. xiii p. 348,—the justice of which is, I think, strikingly illustrated by the case in hand.

In deducing the values of a and b by the formulas just given, I have in the present case taken $n=14$ and $t=10$, by which we get 15 deaths in our first series, and 12 in the second. We thus obtain

$$a = \cdot0073292$$
$$\log b = \bar{4}\cdot07380$$

and by means of these values, and that of $\log q = \cdot04$, the following table of F_x, or force of mortality at each age, has been computed.

Force of Mortality (F_x).

x	F_x	x	F_x	x	F_x	x	F_x
14·5	·00780	30·	·00930	45·5	·01553	61·	·04151
15·	·00782	30·5	·00939	46·	·01592	61·5	·04312
15·5	·00785	31·	·00949	46·5	·01632	62·	·04481
16·	·00787	31·5	·00959	47·	·01674	62·5	·04658
16·5	·00790	32·	·00969	47·5	·01719	63·	·04843
17·	·00792	32·5	·00981	48·	·01765	63·5	·05036
17·5	·00795	33·	·00992	48·5	·01814	64·	·05239
18·	·00798	33·5	·01004	49·	·01865	64·5	·05451
18·5	·00801	34·	·01017	49·5	·01918	65·	·05674
19·	·00804	34·5	·01031	50·	·01974	65·5	·05907
19·5	·00808	35·	·01045	50·5	·02033	66·	·06150
20·	·00811	35·5	·01059	51·	·02094	66·5	·06406
20·5	·00815	36·	·01075	51·5	·02158	67·	·06673
21·	·00819	36·5	·01091	52·	·02225	67·5	·06953
21·5	·00823	37·	·01107	52·5	·02295	68·	·07246
22·	·00827	37·5	·01125	53·	·02369	68·5	·07553
22·5	·00831	38·	·01144	53·5	·02446	69·	·07875
23·	·00836	38·5	·01163	54·	·02527	69·5	·08211
23·5	·00841	39·	·01184	54·5	·02612	70·	·08564
24·	·00846	39·5	·01205	55·	·02700	70·5	·08933
24·5	·00851	40·	·01227	55·5	·02793	71·	·09319
25·	·00857	40·5	·01250	56·	·02890	71·5	·09724
25·5	·00863	41·	·01275	56·5	·02991	72·	·10148
26·	·00869	41·5	·01300	57·	·03098	72·5	·10591
26·5	·00875	42·	·01327	57·5	·03209	73·	·11056
27·	·00882	42·5	·01355	58·	·03326	73·5	·11542
27·5	·00889	43·	·01384	58·5	·03448	74·	·12052
28·	·00897	43·5	·01415	59·	·03576	74·5	·12585
28·5	·00904	44·	·01447	59·5	·03710	75·	·13144
29·	·00912	44·5	·01481	60·	·03851		
29·5	·00921	45·	·01516	60·5	·03997		

It follows, necessarily, from the method adopted in the determination of the constants, that if the deaths be computed upon the numbers observed, by the calculated Table of F_x, the result should coincide with the actual number of deaths observed for the sum of the first series, ages $14\frac{1}{2}$ to $23\frac{1}{2}$, and also for the sum of the second series from $24\frac{1}{2}$ to the end. With the view, therefore, of testing the accuracy of the process of the construction of the Table F_x, and also of comparing the general results with the

original observations, I have computed the second Table showing the "computed" and the "actual" deaths in juxtaposition. It should be mentioned that as the original observations were made for each *half year* the "actual" deaths of the accompanying Table are double the number of those of the original observations.

Comparison of "Computed" with "Actual" Deaths.

Age.	Exposed to Risk for a complete Year.	Deaths during the Year.			Age.	Exposed to Risk for a complete Year.	Deaths during the Year.		
		Actual.	Computed.	Sum of both Columns.			Actual.	Computed.	Sum of both Columns.
14–15	382·25	4	3·0		45–46	15·5	..	·2	
15–16	386·5	6	3·0		46–47	11·5	2	·2	8
16–17	440·5	..	3·5		47–48	7·	..	·1	
17–18	481·75	2	3·8		48–49	13·	..	·2	
18–19	458·	8	3·7	30·0	49–50	13·	..	·3	
19–20	486·75	2	3·9		50–51	7·5	2	·2	
20–21	386·25	2	3·2		51–52	8·	..	·2	
21–22	260·75	2	2·1		52–53	6·	..	·1	
22–23	227·5	4	1·9		53–54	13·	..	·3	
23–24	225·	..	1·9		54–55	4·	..	·1	
24–25	209·75	2	1·8		55–56	5·	..	·1	
25–26	186·5	..	1·6		56–57	2·5	..	·1	
26–27	184·	..	1·6		57–58	8·	..	·3	
27–28	143·75	4	1·3		58–59	9·5	2	·3	
28–29	132·25	2	1·2		59–60	3·5	..	·1	
29–30	137·5	2	1·3	14·0	60–61	1·			
30–31	104·	..	1·0		61–62	2·	..	·1	2
31–32	100·25	..	·9		62–63				
32–33	99·25	..	1·0		63–64				
33–34	83·5	2	·8		64–65				
34–35	75·	2	·8		65–66				
35–36	64·25	..	·7		66–67				
36–37	72·5	..	·8		67–68	1·	..	·1	
37–38	69·	..	·8		68–69	2·	..	·2	
38–39	60·25	..	·7		69–70	2·	..	·2	
39–40	69·5	2	·8		70–71	4·	..	·4	
40–41	55·	..	·7		71–72				
41–42	57·5	..	·7		72–73				
42–43	46·5	2	·6		73–74				
43–44	52·25	..	·7		74–75				
44–45	28·75	..	·4		75–76				

In the foregoing process of determining the law of mortality from the age of 14 upwards, it has not been necessary to consider any special deviations from the normal law which at particular ages the observations may present; indeed the very limited extent of our data would alone preclude our taking such deviations into account even if it were otherwise supposed desirable. In the observations for ages below 14, however, it is imperatively necessary to adopt a different course,—the normal law of mortality prevailing in adult life being altogether inadequate (without some modifica-

tion) to represent with any tolerable degree of accuracy, the actual mortality of infancy and childhood.

The deviations or disturbances to which at certain ages the normal law of mortality is subject, may originate either in artificial or in natural causes. In the case of assured lives, a strongly marked instance of the former is found in the very light mortality prevailing for some years after selection. By the elimination of persons actually suffering from disease, an important constituent portion of the normal rate of mortality—viz. the mortality from chronic diseases—is for a time annihilated; and it is only after a sufficient period has elapsed to allow of the development of the normal proportion of such cases that the law of mortality can assume its normal character. By examining the results of our observations we may easily determine when this normal character has been attained—at any rate approximately—and if we construct the Mortality Table according to the normal law from this point to the extremity of life, we can then continue it backwards to the age of selection, and thus by a comparison of the results so obtained with those of the original observations, we may see the precise effect of the disturbing element in this case. If x denote the age at which, practically, the effect of selection is found to cease, and if the number living at this age be made the same in the two Tables, then at all younger ages the number-living in the Table representing the normal law will exceed those in the Table representing the actual experience, and if the latter be subtracted from the former, age for age, we shall have a gradually diminishing residual series, which vanishes, or, rather, becomes insignificant at age x. The series thus obtained by the subtraction of the "actual" from the "normal" Table—or rather the adjusted series which is substituted for it—I designate the "complementary" series of the Mortality Table. It represents in fact the survivors from age to age among the lives eliminated by the *real* or *efficient* action of the process of selection.

Let us now, however, suppose a case where the lives assured are admitted without medical examination, so that the effect of selection would be *against* the Office. Here the results may be expected to be precisely the reverse of those just considered. The mortality at first would probably greatly exceed that of a normally constituted body, in consequence of the undue proportion of deteriorated lives which would be introduced; and it would continue in excess until this extraneous class became exhausted, and the whole body had attained a normal character. As in the preceding case, the point

at which this normal character was attained would be (approximately) shown by an examination of the experienced mortality; and starting from this point in forming our normal Table for the remainder of life, and then working backwards to the age at entry (as in the former case), we should be able to ascertain precisely the effect of the adverse selection by *subtracting,* in this case, the "normal" from the "actual" series. The residual series thus obtained would also (as in the former case) consist of a gradually diminishing series, which becomes insignificant at the point at which the effect of selection is supposed (practically) to have ceased. This residual series, representing the extraneous body *introduced* by the action of selection, I designate the "supplementary" series of the Mortality Table.

These two series, the "supplementary" and the "complementary," will be of such a character as to admit of being expressed with more or less fidelity by the formula $L_x = dg^{q^x}s^x$, but with different values of the constants d, g, and s from those of the normal series; and thus we shall find that the law of mortality expressed by the function in question can be without difficulty adapted to cases which, upon a superficial view of the matter, would seem to be quite irreconcilable with that law.

I propose to extend the application of the method derived from a consideration of the *artificial* disturbance resulting from selection, to the analogous case of a disturbance arising from natural causes. The cases we have hitherto considered may be termed cases of "simple disturbance," one cause only being supposed in operation at a time. It is necessary, however, in order to enable us to deal with the case which we have in hand, now to consider cases of a "complex" description, as for instance where one disturbing cause is followed or accompanied by another of an opposite character. The resulting residual series in this case may be conceived to be of a kind which attains a maximum somewhere about the middle term, and gradually vanishes at each end. It is indeed with a series of this character that we shall have most frequently to deal; and it is a consideration of some importance that the same formula $L_x = dg^{q^x}s^x$ is perfectly adapted (as we shall find) to express a series of this particular form.

The following Table shows (1) the number under observation *for a complete year* at each age from 0 to 14 years, (2) the number of deaths corresponding thereto, and (3) the ratio of the latter to the former.

Age.	Exposed to Risk for a complete Year.	Deaths.	Ratio.	Age.	Exposed to Risk for a complete Year.	Deaths.	Ratio.
0–1	440·	20	·04545	7– 8	1475·75	8	·00549
1–2	869·25	24	·02761	8– 9	1358·25	10	·00736
2–3	1014·	18	·01775	9–10	1234·	4	·00324
3–4	1065·5	14	·01314	10–11	1144·25	12	·01049
4–5	1288·5	12	·00931	11–12	1043·5	2	·00192
5–6	1440·25	14	·00972	12–13	816·	6	·00735
6–7	1479·	8	·00541	13–14	550·75	2	·00363

E_x denoting the number in the second column (*i.e.* the number under observation for a complete year), d_x the number in the third column, we have upon the supposition that E_x remains constant during the year $d_n = E_n \int_n^{n+1} F_x dx \quad \therefore \; d_n : E_n = \int_n^{n+1} F_x dx = -\log_e p_n$ ($\log_e p_n$ being the Nap. log. of the probability of living a year). By this expression the numbers-living, according to the actual experience as contained in the second column of the following Table have been computed:

Age.	Number Living.		Residual Series.
	Actual Experience.	Normal Law of Adult Life carried back to Birth.	Col. 2.—Col. 3.
0	11828	11120	+708
1	11302	11037	+265
2	10994	10955	+ 39
3	10801	10873	− 72
4	10660	10792	−132
5	10561	10711	−150
6	10459	10631	−172
7	10403	10551	−148
8	10346	10471	−125
9	10270	10392	−122
10	10237	10313	− 76
11	10130	10234	−104
12	10110	10156	− 46
13	10036	10078	− 42
14	10000	10000	− 0
0–14	158137	158314	−177

I have given the sums of the several columns, as they tend to confirm the coincidence pointed out by Mr. Meech, in reference to the American Life Table, in the Massachusetts Life Assurance Report for the year. Should further investigation prove it to be a general law that the sums of series of "Numbers Living" are equal,

Mr. Meech's discovery may prove of importance in the construction of Mortality Tables. A consequence of such a law would be that the sum of the negative terms of the residual series (contained in the last column) should be equal to that of the positive terms; and also that *at birth* the expectation of life during the period of childhood is affected in the inverse ratio of the radices of the columns.

We have now to adjust the "residual" series according to the formula $dg^{q^x}s^x$. It is evident, however, that the whole of the terms cannot be expressed as one series, in consequence of the change of sign which occurs between ages 2 and 3, showing that we have a case of "complex" disturbance to deal with. Let us commence with age 6, where the negative portion attains a (numerical) maximum. Taking intervals of 3 years we have the three terms, 172, 122, and 46, to determine the three constants of the formula (q being, as before, assumed to be $\log^{-1}\cdot 04$) by means of which we get the entire series contained in the third column of the following Table,—and which will form the "complementary" series of our final Mortality Table.

Age.	First Residual Series.	Complementary Series.	Second Residual Series. (Col. 2.—Col. 3.)
0	+708	− 91	+799
1	+265	−111	+376
2	+ 39	−132	+171
3	− 72	−150	+ 78
4	−132	−164	+ 32
5	−150	−172	+ 22
6	−172	−172	+ 0
7	−148	−163	
8	−125	−145	
9	−122	−122	
10	− 76	− 95	
11	−104	− 69	
12	− 46	− 46	
13	− 42	− 28	
14	− 0	− 15	
15	..	− 7	
16	..	− 3	
17	..	− 1	

The preceding process is an exact repetition of the first. Completing the series in the third column from the age of 6 upwards until it becomes insignificant and then working back to birth, we deduce the second residual series contained in the last column by subtracting the complementary from the first residual series. This latter series (*i.e.* the *second* residual series) has now to be adjusted in the same way, and will then form our supplementary series, completing the adjusted Mortality Table.

In adjusting the second residual series I have taken the three first terms for determining the constants, in order that in our final Mortality Table the numbers living at ages 0, 1, and 2, shall coincide with those of the original observations. The following is the complete series, side by side with the original figures for comparison.

Age.	Second Residual Series.	Same Adjusted. (Supplementary Series.)
0	799	799
1	376	376
2	171	171
3	78	75
4	32	32
5	22	13
6	0	5
7	..	2
8	..	1

This series, like each of the others, is of course adjusted by the formula $dg^{q^x}s^x$; and thus we see that one simple law suffices for the construction of the whole Table from birth to extreme old age. The importance of this fact will be at once seen when it is considered that the integral of the above expression, and also that of the same expression multiplied by the value of £1 certain due at the expiration of the time x, and, further, of the product of any number of terms of the same form, can all be reduced to the form $\int \epsilon^{-v}v^{n-1}dv$; and, therefore, that a complete Table of this last-mentioned integral will enable us to obtain the values of all annuities without the aid of special Tables for different rates of interest and mortality. As the method herein explained of giving effect to the deviations from the normal law enables us to follow the original Table with almost any degree of accuracy that may be desired, from birth to extreme old age, a Table of the integral in question seems to be the only thing wanting to give us that complete command over the practical branch of the science of life-contingencies which we already possess in the theory of the subject. Such a Table I have had for some time past in course of construction, and hope shortly to be able to lay it before the public.

In conclusion, I have only now to beg attention to the following Tables; the first containing the component parts of the Mortality Table for infancy and childhood (with a comparison with the

original figures), and the second the complete adjusted Table to the age of 75 years.

Age.	Normal Series.	Supplementary Series.	Complementary Series.	Resulting Mortality Table.	Original Mortality Table.
	(+)	(+)	(−)		
0	11120	799	91	11828 − 526	11828 − 526
1	11037	376	111	11302 − 308	11302 − 308
2	10955	171	132	10994 − 196	10994 − 193
3	10873	75	150	10798 − 139	10801 − 141
4	10792	32	164	10659 − 107	10660 − 99
5	10711	13	172	10552 − 88	10561 − 102
6	10631	4	172	10464 − 74	10459 − 56
7	10551	2	163	10390 − 64	10403 − 57
8	10471	1	146	10326 − 56	10346 − 76
9	10392	0	122	10270 − 52	10270 − 33
10	10313	..	95	10218 − 53	10237 − 107
11	10234	..	69	10165 − 55	10130 − 20
12	10156	..	46	10110 − 60	10110 − 74
13	10078	..	28	10050 − 65	10036 − 36
14	10000	..	15	9985 − 70	10000 − 104
15	9922	.	7	9915 − 73	9896
16	9845	..	3	9842 − 76	
17	9767	..	1	9766 − 76	
18	9690	..	0	9690	

Adjusted Mortality Table.

Age.	Numbers Living.	Decrements.	Age.	Numbers Living.	Decrements.	Age.	Numbers Living.	Decrements.
0	11827·83	525·94	26	9068·00	79·04	52	6468·63	146·82
1	11301·89	307·48	27	8988·96	79·57	53	6321·81	152·80
2	10994·41	196·21	28	8909·39	80·21	54	6169·01	159·05
3	10798·20	138·93	29	8829·18	80·93	55	6009·96	165·56
4	10659·27	107·52	30	8748·25	81·76	56	5844·40	172·28
5	10551·75	87·96	31	8666·49	82·70	57	5672·12	179·19
6	10463·79	73·97	32	8583·79	83·76	58	5492·93	186.22
7	10389·82	63·45	33	8500·03	84·96	59	5306·71	193·33
8	10326·37	56·18	34	8415·07	86·29	60	5113·38	200·42
9	10270·19	52·47	35	8328·78	87·77	61	4912·96	207·42
10	10217·72	52·39	36	8241·01	89·42	62	4705·54	214·20
11	10165·33	55·31	37	8151·59	91·24	63	4491·34	220 65
12	10110·02	60·06	38	8060·35	93·23	64	4270·69	226·65
13	10049·96	65·30	39	7967·12	95·42	65	1044·04	232·02
14	9984·66	69·91	40	7871·70	97·82	66	3812·02	236·60
15	9914·75	73·29	41	7773·88	100·44	67	3575·42	240·23
16	9841·46	75·41	42	7673·44	103·29	68	3335·19	242·71
17	9766·05	76·51	43	7570·15	106·38	69	3092·48	243·86
18	9689·54	77·03	44	7463·77	109·72	70	2848·62	243·50
19	9612·51	77·24	45	7354·05	113·34	71	2605·12	241·46
20	9535·27	77·38	46	7240·71	117·23	72	2363·66	237·61
21	9457·89	77·49	47	7123·48	121·41	73	2126·05	231·84
22	9380·40	77·68	48	7002·07	125·89	74	1894·21	224·07
23	9302·72	77·91	49	6876·18	130·67	75	1670·14	
24	9224·81	78·22	50	6745·51	135·75			
25	9146·59	78·59	51	6609·76	141·13			

The Hon. Elizur Wright on Net-premium Valuations with reference to the American Insurance Law.

THE *Insurance Times* of New York having reprinted the greater part of Mr. Sprague's paper *On the proper method of estimating the liability of a Life Insurance Company under its policies*, and Mr. Makeham's letter which appeared in the same number of this *Journal*, the Hon. Elizur Wright has thought it desirable to explain the aim of the American Insurance Law in a letter to that periodical, from which we make the following extract. We do this not only on account of the general interest with which Mr. Wright's views will be read, but still more because he corrects a misconception as to the object and purpose of the American Life Insurance legislation.—ED. *J. I. A.*

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* * * No one that I know of prescribes a net valuation as the best, or, by itself, any method at all of ascertaining the financial prospects of a company, or objects to a gross valuation, unless it is to be used for the purpose of deceptively mixing up and confounding future with existing assets. * * * Mr. Sprague has shown a misconception of the purpose of the net valuation applied by law to companies doing business in Massachusetts.

He says :—

> "In America the net-premium method of valuation is proving to be inapplicable for a different reason. The members of the Institute are well aware that by the laws of the State of Massachusetts a valuation is made yearly by the Insurance Commissioner of the liabilities of all Life Insurance Companies doing business in the State. The basis of this valuation has hitherto been the 'Experience' table of mortality with four per cent. interest; but with the increasing competition and the tendency of newly-formed Companies to reduce as far as possible the premiums, it is found that the premiums charged by some Companies very little exceed, or even fall short of, the net premium used in the valuation. In this latter case it is a manifest absurdity to give a Company credit in the valuation for a net premium actually greater than that it will receive; and in the former case it is little less absurd to give it credit for a net premium which shows a margin quite insufficient to defray the necessary expenses of management."

If it were the purpose of the State to compare the financial prospects of different companies with each other, or with the mathematical results of any set of assumptions it might think reasonable, so as to divine in regard to the question of future solvency, it would, of course, be very absurd to neglect the premiums actually charged on the policies and payable in the future. The State has no purpose of the sort. It does not undertake to divine at all. If a life insurance company were

actually insolvent, that is, if it could not liquidate matured claims, it would be treated as other debtors in like cases are. As long as it is not thus insolvent it is allowed to go on receiving premiums and paying losses, whatever may be its prospects as to the future. But it is not allowed to make any new contracts, except on the condition thus expressed in the statute :—

"When the actual funds of any life insurance company doing business in this Commonwealth are not of a net cash value equal to its liabilities, counting as such the net value of its policies according to the 'Combined Experience' or 'Actuaries'' rate of mortality, with interest at four per centum per annum, it shall be the duty of the Insurance Commissioner to give notice to such company and its agents to discontinue issuing new policies within this Commonwealth until such time as its funds have become equal to its liabilities, rating its policies as aforesaid."

This does not pretend to be a test of solvency or a dictation of premiums, but simply a prudential check or safeguard. Any new company, whether with high or low premiums, may spend the whole of them in obtaining new business or in other ways, and go on while its guarantee capital can fill the gap, and make its net assets equal the net value of the policies as calculated by the Commissioner. When this gap fails to be filled the company is not necessarily insolvent. A good actuary, with Mr. Sprague's scientific and exhaustive method of valuation on not unreasonable assumptions, might be able to show not only that it would probably meet all its future liabilities, but even by and by come up to the State's standard of reserve. The State, however, as a matter of prudence, thinks best to restrict it to the contracts already made. In this no injustice is done, for if the future premiums are high enough it will by and by regain a legal reserve. And a high reserve, as Mr. Sprague knows very well, does not enhance the cost of insurance to the *average* policyholder. On the other hand, if the future premiums are too low to secure ultimate and final solvency, the company is arrested before the evil has become very serious. The State does not in this way prevent all possible evil, but it does effectually prevent one evil which has been exceedingly common in the mother country: the sacrifice of new members to fill the financial deficits of the old.

Again, whatever absurdity there may seem to be in giving a company credit for premiums greater than it will receive, it is to be observed that the State assumes a much lower interest on the reserve than it will probably earn, so that premiums slightly below the net ones credited will be able, by virtue of the extra interest, in due time to come up to the requisition of the State. And seeing that the lower the premiums are the stronger the reason is for

making the reserve as large as that required, this theoretical absurdity is converted into a very effective practical means of enforcing the necessity of economy and thrift on such companies. The State wishes to interfere with the business of corporations as little, and as little arbitrarily, as possible. It does not, therefore, dictate premiums or prohibit business on those it deems too low, but establishes a prudential check, against which any company will bring up, if either by too low premiums or too large expenses it does not at any time have on hand a certain calculated sum over and above all matured claims. Such a prudential check could not be the same for all without being a *net* valuation. To apply a six or seven per cent. net valuation to companies that charge the *maximum* premiums, instead of being to them a check on expenses or dividends, would seem to be just the reverse; and also by reducing the savings bank feature of the business in relation to the insurance done by the company, it would, in regard to dividends, favour the short lives at the expense of the long ones.

On the other hand, to apply a six or seven per cent. net valuation to the companies charging *minimum* premiums, though it might be free from the absurdity of giving credit for premiums larger than will be received, would require less reserve than a four per cent. valuation, and thus enhance the danger, if the premiums *are* too low. Obviously in applying a four per cent. valuation to these companies, the State is under a sort of logical obligation, after giving them credit for four per cent. net premiums, to debit them with the present value of the deficiency of their actual premiums. All this, and more too, if it were its purpose to compare the financial prospects or future resources, in regard to the indemnities pledged, the ability to pay expenses, and the probable surplus of the different companies. But practically this is of no consequence, or less than the cost of making the charge, for a company without guarantee capital, charging premiums not loaded above the net at four per cent. sufficiently to pay its working expenses, could not by any possibility proceed more than a year or two under a four per cent. net valuation, though nothing should be charged against it for the above-named deficiency. And a company having a guarantee capital would be very careful not to risk it on premiums which were not pretty sure, by virtue of economy and an interest higher than the State's assumption, to bring its reserve ere long fully up to the State's requirement, without counting in the said capital.

To illustrate: Suppose the premium charged at forty for a life policy of £1,000 is only £23·30, whereas the net premium

credited by the State is £23·68. Here would be a deficiency of £·38 per annum, to say nothing of the absence of any provision for expenses. Logically it may be admitted that in valuing such a policy at four per cent. net the State shall debit the company with $\cdot 38 \times a_{40} = 6\cdot 12$, in addition to the net value of the policy. But if it is reasonably certain that the company will realize six per cent. on its investments, we may allow ten per cent. out of the £23·30 for expenses, and the remainder, £20·17, by a "combined experience" accumulation formula at six per cent., will amount at the end of thirty years to £616·01, or £70·67 more than the four per cent. net value of the policy at that time. As the value of the £·38 deficiency at that time is only £2·78, there is a net excess over the State's requirement then of £67·71.

The Life Association of America.

We have received through the courtesy of Mr. William Barnes, late Superintendent of the Insurance Department of New York, and now "Consulting Counsel and Actuary," a pamphlet of 63 pages, purporting to be a report on the condition and prospects of the above Insurance Company for the year 1870, but which may with perhaps greater propriety be called a review of the general position of Life Insurance in America, England, Germany, and France.

This report is so full of interesting information that we are certain our readers will be glad to have a somewhat detailed account of it laid before them.

The Life Association of America appears to be an Insurance Company established in the year 1868, with its head-quarters at St. Louis in the State of Missouri, and to have succeeded in the short space of "2 years 6 months and 15 days" in acquiring a business of £9,131,148. We have no information whatever supplied us as to the expense at which this remarkably large new business has been acquired, but the report indicates clearly some of the means which have been used to push the business. Thus, pages 51–63 of the pamphlet are filled with the names of Presidents, Vice-Presidents, Treasurers, Attorneys, Medical Examiners, Managers, and Directors, with a statement of the number of "other Trustees" of the Association appointed in each of the following Departments:—

Home Department		450	other Trustees.
Alabama	"	112	"
Arkansas	"	22	"
California	"	50	"
Colorado	"	12	"
Iowa	"	14	"

Indiana Department	64	other Trustees.
Kentucky „	52	„
Kansas „	168	„
Louisiana and Texas Department	329	„
Maryland Department	67	„
Macon (Georgia) Department	20	„
Michigan Department	35	„
Middle Tennessee Department	—	„
Mississippi Department	49	„
New York „	118	„
North-West Missouri Department	78	„
North Carolina Department	11	„
Northern Illinois „	235	„
North Nebraska „	7	„
Ohio Valley „	39	„
Southern Illinois „	210	„
South Nebraska „	17	„
West Tennessee „	73	„
West Virginia „	68	„
West Pennsylvania „	60	„

—together 2360.

This list concludes with a note—"Besides the above, about "100 Trustees have been qualified at different places where "organizations are being perfected, but of which no publication is "made." The Association thus appears to be one that appeals to local influence as distinguished from the corporations that have their chief offices in New York and other Eastern States. Consistently with this, it appears to be part of the policy of the Association to lend its funds on mortgages of property in the various States where it has organizations, and thus to encourage local enterprise and trade. The amounts so advanced are stated as follows:

"The Association has now invested in bonds and mortgage, or note and deed of trust on real estate, the following amounts:

	£
"Louisiana and Texas Department	36,439*
Kansas „	6,263
Illinois „	20,190
Alabama „	13,280
West Virginia „	1,260
Colorado „	800
Indiana „	1,300
Tennessee „	8,237
Maryland „	3,040
Kentucky „	2,800
Iowa „	600
Florida Branch	200
Ohio Department	2,804
Nebraska „	3,823
Georgia Branch	800
Mississippi „	900
South Carolina Branch	800
Arkansas Department	3,200
Home (Missouri) „	66,312
Total	£173,048"

* In reprinting Mr. Barnes's figures, five dollars have been taken as equal to £1 sterling.—Ed. *J. I. A.*

As was to be expected, investments so widely made produce much more than the ordinary rate of interest, the above sum being invested at the following rates of interest:—

"At 15 per cent. per annum		£ 800
" 12 " "		17,052
" 10 " "		125,552
" 9 " "		14,060
" 8 " "		10,264
" 6 " "		5,320
	Total, Dec. 31, 1870	£173,048

"which, payable semi-annually, makes the rate over ten per cent. yearly: "average rate 9·83 per cent. per annum.

"[The investments at fifteen per cent. are in the State of Colorado.]"

It is obvious that if investments producing such rates of interest are attended with only slight risk of losing the capital, the effect must be very considerably to reduce the prime cost of insurance. Mr. Barnes tells us that the New York Companies realize on the average only 6 or 6½ percent interest, in consequence of the law restricting their investments to bonds and mortgages on unencumbered real estate within the State of New York; and he gives at full length petitions presented by the chief officers of the various Life Insurance Companies to the Legislature, praying for an amendment of the said law. The petition presented in 1869 urges the change for the following reasons:—

"1. *Because the restriction it imposes is entirely unnecessary and superfluous.* * * * * * * *

"2. *Because it is impolitic and unwise.* * * *

"3. *Because it is materially injurious and morally prejudicial.* * * * * * *

"4. *Because it is unfair, unjust and short sighted.* * *

"5. *It is illiberal, selfish, ungrateful and contrary to the spirit of the age, and of American institutions.* New York as the Empire State, and New York City as the Metropolis of the western world, and the gate of commerce, should be the last to incarcerate capital, or display a want of enlightened reciprocity. We gather our wealth from every source, every land, and obtain by a liberal outlay manifold returns. It is not therefore our part to retrograde to cramping, feudal, effete systems of restriction, but rather to excel others in generous breadth of mind and enterprise, and to advance and keep pace with the requirements and progress of our country and age."

That presented in 1870 is of sufficient interest to be quoted at greater length.

"*To the Honorable, the Legislature of the State of New York:*

"We respectfully solicit the honorable body, the Senate and Assembly of New York, to take into consideration the loss and injustice suffered by

the life insurance companies organized in New York, through the operation of the law enacted in 1853, which restricts the investment of their funds in bonds and mortgages on real estate to this State and a radius of fifty miles around the city of New York, and we earnestly petition that the said law be amended, for the following reasons:

"1. *Because the considerations which led to the passage of this act no longer exist, and its operation now tends to defeat the very purposes which it was originally intended to promote.* Eighteen years ago it was, perhaps, expedient to confine corporate bodies to investments within the prescribed area, as those beyond it had not then so fixed and determinate a value, and as first liens on unencumbered real estate were at that time easily obtained within the limits specified. But security is now offered on property in other States, to which no exception can be taken, as it has acquired a perpetual value admitted by all, while the vast concentration of capital in the metropolis and State of New York is continually lessening the number of investments lawful and eligible to life insurance companies, so that these corporations which depend so much for their prosperity on the interest earned by their assets find their income diminished and their safety endangered by the law designed for their protection.

"2d. *Because it is unfair to our customers and injurious to ourselves.* There is now no region in the world where capital can be more securely and profitably placed than the West, and none in which our insurance companies have reaped a more bountiful harvest, and yet the latter are, after drawing for many years a large portion of their income from this section, compelled to refuse the loan of one cent to its citizens on first liens on real estate of the most unquestionable value. This ingratitude and want of reciprocity are producing their natural fruit. We are losing our best customers West and South. The prejudice and opposition excited against New York life insurance companies by their exclusiveness, have strengthened the hands of their rivals and obliged the former to sustain their competition at a great disadvantage. Countless insurance companies without capital, merit, or stability, have consequently sprung up in other States, and trading upon the local partiality and exclusiveness for which we ourselves have set the example, and the spirit of retaliation our illiberality has aroused, have deceived and plundered the public, cut down the gains of our solid companies, and brought disgrace and suspicion upon the business of life insurance.

"3d. *Because it is contrary to the policy of the Empire State, and unworthy of the American people.* The metropolis of America and the Empire State of this great republic should be the last to shut up its capital from the States that deal freely and liberally with us, and from whose development and prosperity the energy and enterprise of our citizens reap the richest and most constant rewards. The contributions that flow to us from this and every other quarter, and sustain our commercial greatness, can only be arrested by an oppressive system of restriction, and they naturally demand on our part the practice of the most liberal, just, and enlightened reciprocity. Our life insurance companies would be enabled, by loaning capital in the great western commercial centres, to exercise a powerful influence there to our advantage and to the benefit of our customers. Your honorable body must therefore perceive the impolicy of refusing the privilege for which we petition, especially as it is possessed by

Connecticut and Massachusetts insurance corporations, which, therefore, enjoy every facility for becoming more popular and successful than their competitors from New York. But our life companies are by this restrictive and obstructive law not only shorn of the local influence they ought to have in the West, but are also thus deprived of the solid and palpable gains of a much higher rate of interest, which the rapid growth and increasing prosperity of that section securely and uniformly yield to capitalists investing in bonds and mortgages on real estate worth fifty per cent. more than the amount loaned thereon. We, therefore, respectfully claim that New York companies should be allowed to profit by these advantageous circumstances as do the companies of New England, and at the same time that they enrich themselves by this harvest they should share both in the gain and glory of building up the West.

"4th. *Because it enhances the cost of life insurance, diminishes and imperils its benefits to widows and orphans, and thus proves a great detriment to the public weal.*" * * * *

We need scarcely inform our readers that we consider these arguments unanswerable, and that we think they furnish another illustration of the impolicy of placing upon Insurance Companies any legal restrictions as to the investment of their funds and the manner of conducting their business.

The Balance Sheet of the Association for 31 Decr., 1870, is as follows:—

"Assets.	£
"Loans on Bond or Note and Mortgage, including interest due and accrued thereon	178,048
Premium Loans and Liens	166,693
Premiums in course of collection, &c.	85,576
Deferred Premiums by actual calculation	25,439
Cash in Bank at Home and Department Offices	33,080
Stock and other loans on collaterals, and interest	21,502
State, City and other Stocks owned, and interest	15,375
Due for re-assurances, office furniture, &c.	2,900
Total Assets, including accrued interest	£528,613

"Liabilities.	£
"Re-insurance Fund, (American ex. $4\frac{1}{2}$)	439,334
Losses unpaid, unadjusted and contested	24,175
Cost of collecting outstandings, &c.	9,627
Total Liabilities	£473,136
Surplus, Dec. 31, 1870	55,477
	£528,613"

Thus showing an amount of realized assets, which, according to English ideas, is a very handsome amount of accumulation for the short period the Office has been in operation. At the same time we must not omit to notice that the "Premium Loans and Liens" and "Deferred Premiums" form apparently, according to English notions, an unduly large proportion of the total assets. This,

however, is probably to be explained by the Company being forced to adopt a particular line of policy in consequence of the necessity of calculating its "Re-insurance Fund" according to certain data prescribed by law.

Pages 18–50 are principally devoted to the "Relative progress "of the Life Association of America, as compared with American, "British, German and French Companies," which is illustrated by sixteen elaborate tables, from which the following are extracts.

"Table No. I.—Compiled by William Barnes, showing the names of all the different Life Insurance Companies transacting business in the State of New York, which have obtained an amount of risks in force equal to, or exceeding those of the Life Association of America on the 31st day of December, A.D., 1870, and the relative periods of time consumed in reaching the 'line' of the Association.

Name of Company.	Location.	Period of Time consumed in reaching this amount.		
American Companies.		Yrs.	Mos.	Dys.
1 Life Association of America......	St. Louis	2	6	15
2 Continental Life	New York	4	0	8
3 Equitable Life Assurance Society	„	7	2	16
4 North-Western Mutual	Milwaukee, Wis.....	9	9	8
5 St. Louis Mutual	St. Louis	12	1	18
6 Knickerbocker	New York	14	9	9
7 Ætna Life	Hartford, Conn.	14	10	10
8 Connecticut Mutual	„ „	17	3	16
9 Charter Oak	„ „	17	9	2
10 Phœnix Mutual	„ „	17	10	18
11 Mutual Benefit	Newark, N. J.......	18	2	15
12 Manhattan	New York	19	4	29
13 Mutual Life	„	20	5	10
14 New York Life	„	20	8	13
15 New England Mutual	Boston, Mass.	22	4	2
Total	.. ..	219	4	9
Average..................	.. ..	14	7	20

Mr. Barnes adds—"The table shows conclusively that no "other company has ever, in any country or at any time, been "so successful as the Life Association in obtaining business." Table II. contains a list of the remaining thirty-seven Companies doing business in the State of New York, which have been in operation for various periods, ranging from 2 years 6 months 29 days to 25 years 6 months 29 days, without obtaining so large an amount of business as the Life Association of America. Table III. shows "the names, locations and officers and amount of risks in "force, including Bonus additions, in the British Life Insurance

" Companies, as compared with the Life Association of America, " together with the respective periods of time consumed in reaching " these amounts, and also the number of companies amalgamated " with the existing companies, with their periods of corporate " existence prior to such amalgamations, arranged in their relative " order of priority, as indicated by the largest amount of risks in " force." It will not be necessary to make any extracts from this table; but we may simply remark that of 88 Companies included in it, 8 only exhibit a larger amount of risks in force and bonus additions than the Life Association of America.

Mr. Barnes makes the following remarks on the business of life insurance in England:—

" England has the honor of being the native home of modern Life Insurance, which really had its origin with the incorporation of the Equitable Society, in 1762. Since that period the science of life contingencies, and the practice of the business, have been sedulously cultivated by trained and accomplished scholars, and able and thorough executive officers and managers. Notwithstanding these advantages, the reticence of the officers as to their standing and affairs, and the low rates of interest prevailing, have so far countervailed their other advantages, as to allow the American Companies to gain a marked superiority over them, although Life Insurance cannot be said really to date in this country before the organization of the Mutual Life of New York, in the year 1842.

* * * * *

" Even at this late date, it is with difficulty that reliable statistical information can be obtained as to the actual condition, status and business of the British Companies. The first great need in England, toward the re-establishment and maintenance of the business on a solid and permanent foundation, is reliable and specific annual statements from all the companies, somewhat in the American or German form.

* * * * *

" As a matter of course, some of the British Companies have already reached their maximum, as well in the number of insured lives, as in assets; some occupying a sort of stationary position, the new entrants about balancing the lapses and deaths. A perusal of the Table [No. III.] of British Life Insurance Companies, will inspire the managers and agents of the Life Association with new hope and impetus, as they scan the results of so many years of labor by their co-workers in Great Britain. Indeed, all American Companies can draw therefrom lessons of wisdom, and abundant encouragement."

Table IV. deals similarly with the German Life Insurance Companies, 39 in number, which have been in operation for various periods from one year to forty-two, but not one of which apparently has obtained the amount of business of the Life Association of America.

" One of the most economically managed and best of any of the Life Insurance Companies in the world is the Gotha Life Insurance Bank, of

Gotha, Germany, for many years under the extraordinarily able and superior management of Finanzrath G. Hopf, who, in addition to his practical duties as general manager, is an accomplished scholar, reliable economist, and accurate statistician; alive as well to questions of general public importance as to those immediately affecting the business, the best interests of which he equally promotes and honors by consecrating his life to its service.

" This company in 1869 had reached about the same amount of risks in force as the Association on the 31st day of December, 1870; but it had attained this position by forty-two years of steady corporate growth. The total amount of risks in force in the German companies approached nearly £60,000,000 in the year 1869, as the result of five hundred and seventy years of corporate growth, considering the whole thirty-nine institutions as one company. It must be recollected, however, that the average German policy is only £131, or less than one-fourth of average American policies. In fact the average of German policies is less than the above figures, as many German companies return the number of persons instead of policies, so that several policies on one life in a German company only count as a single policy."

Table V. is similarly devoted to the French Life Insurance Companies, 10 in number, and of ages ranging from 4 years 6 months to 50 years 8 days.

" Life Insurance has never obtained any strong foothold in France. The shadow of the celebrated *l'Ordonnance de la Marine* of 1683, which prohibited any insurance on lives, seems to have darkened the eyes of the French for nearly two hundred years, and it has only been within a few years that new vigor has been infused with [*sic*] the Companies by the enthusiastic labors and writings of economists and scholars like Eugène Reboul, Lamartine, M. and Edmond Maas, Edmond About, Michel Chevalier, E. Levasseur, F. Versigny, Ernest Beauvisaal, and many others. At the breaking out of the late war, the Life companies in France were assuming positions of strength and importance. Annuities and Tontines have, however, been more favored by the genius and tastes of the French, than the self-sacrificing plan of the Life policy, which flourishes more naturally on British, German or American soil.

" It will be seen by the above Table No. V., that only two French Companies, the *Générales* and *La Nationale*, exceed in Life risks the business of the Association, one being more than half a century old, and the other nearly forty years of age. Nearly £40,000,000 are in force in the French Corporations, the fruits of two hundred and thirty one years of corporate existence. In addition, however, the sum of £729,779 of various forms of annuities was in vigor, for which the capital sum deposited must have been over £6,000,000. This annuity business is, I presume, larger than that of all the other Life Insurance Companies in the world.

" The *Générales* and *La Nationale*, it will be seen by the Table, transact more than half of the whole Life Insurance and Annuity business of the Stock Companies in France.

" The Mutual Life Insurance Corporations are few in number and comparatively unimportant, and their statistics are not accessible."

We come now to Table VI., which is as follows:—

"Table No. VI.—Compiled by William Barnes, showing the relative net increase or decrease during the calendar year 1870, of the amount insured or risks in force in American Companies (doing business in New York), as compared with the Life Association of America, including old policies revived and reversionary dividends added to policies, during the year, and excluding re-insurance, both at the beginning and end of the year.

Name of Company.	Location.	Net Increase of Risks in force, year 1870.	Net Increase No. Policies year 1870.	Name of Company.	Location.	Net Increase of Risks in force, year 1870.	Net Increase No. Policies year 1870.
1 Life Association of America	St. Louis	£4,000,188	3,867	39 Widows and Orphans	New York	£266,099	484
2 Continental	New York	2,650,810	7,407	40 Atlantic Mutual	"	260,223	874
3 Equitable	"	1,949,801	2,862	41 Government Security	"	248,900	296
4 New York	"	1,846,889	4,121	42 New Jersey Mutual	Newark	248,673	783
5 Hope Mutual	"	1,704,049	4,259	43 Brooklyn	New York	246,654	775
6 Empire	"	1,293,102	3,294	44 Universal	"	230,516	252
7 Metropolitan	"	1,218,514	5,644	45 Merchants	"	227,920	528
8 Union Mutual	Augusta, Me.	1,192,025	2,575	46 Hahnemann	Cleveland	226,728	785
9 Charter Oak	Hartford	1,176,184	2,778	47 World Mutual	New York	211,048	621
10 North-Western Mutual	Milwaukee	1,115,606	3,291	48 Homœopathic Mutual	"	197,122	567
11 Mutual Life	New York	1,148,904	3,209	49 North American	"	184,198	915
12 Phœnix Mutual	Hartford	1,105,176	3,151	50 Mutual Benefit	Newark	180,543	976
13 St. Louis Mutual	St. Louis	1,041,235	3,358	51 New York State	New York	163,617	624
14 Anchor Life	Trenton, N. J.	914,699	1,868	52 Hercules Mutual	"	132,620	503
15 Massachusetts Mutual	Springfield	885,911	1,739	53 Western New York	"	121,480	315
16 Security	New York	844,618	1,602	54 National	"	113,867	681
17 Globe Mutual	"	840,077	1,848	55 Safety Deposit	Chicago	93,640	204
18 Connecticut Mutual	Hartford	781,828	3,628	56 State Mutual	Worcester, Mass.	66,100	199
19 Penn Mutual	Philadelphia	760,215	1,232	57 Berkshire	Pittsfield, Mass.	52,182	215
20 Empire State	Waterto'n, N. Y.	605,160	1,512	58 Economical Mutual	Providence, R. I.	51,697	337
21 Republic	Chicago	587,299	1,176	59 American Tontine	New York	26,542	60
22 Commonwealth	New York	559,724	1,540	60 American Popular	"	26,187	89
23 National of U. S.	Wash'ton, D. C.	520,623	1,126	61 Amicable Mutual	"	14,882	136
24 Germania	New York	515,409	1,391	62 Standard	"	13,027	156
25 John Hancock	Boston	491,439	1,177	Total increase (62 Companies)	.. ..	37,684,414	90,165
26 Eclectic	New York	447,683	1,101				
27 National	Montpelier, Vt.	384,567	553			Decrease.	Decrease.
28 Union Central	Cincinnati	371,709	609	63 New York Life and Trust	New York	£5,000	10
29 Travelers	Hartford	368,621	1,191	64 Home	"	28,525	157
30 Hartford Life and Annuity	"	361,702	959	65 United States	"	32,969	96
31 Provident Life and Trust	Philadelphia	349,980	686	66 Washington	"	228,928	906
32 Asbury	New York	348,773	833	67 Guardian Mutual	"	258,101	265
33 International Life and Trust	Jersey City, N. J.	292,016	594	68 Ætna	Hartford	402,706	2,825
34 Excelsior	New York	288,185	948	69 Manhattan	New York	435,519	369
35 Craftsman	"	287,310	622	70 Knickerbocker	"	919,646	1,561
36 National Capital	Wash'ton, D. C.	284,037	201	71 New England	Boston	974,987	1,332
37 Connecticut General	Hartford	276,461	419	Total decrease (9 Companies)		£3,286,381	7,521
38 Mutual Protection	New York	269,420	419				

Net increase (71 Companies), £34,398,035; of which the Life Association's share was £4,000,188, or between one-eighth and one-ninth of the entire net increase of new business of all the seventy-one Companies transacting business in the State of New York, during the calendar year 1870.

"In adding to the old business during the current year, the material and substantial test is, of course, the NET INCREASE of new business. A Company's position is obviously not bettered by writing twenty millions of new policies and having the same amount from various causes cease to be in force during the year. The *Net*, therefore, instead of the gross increase, should be considered in estimating the probable future of a Company. This touchstone proves the Life Association to be without a rival in any country. * * * *

"It is probable that the older large companies have nearly reached their maximum of the amount of risks in force, although not in assets. The inevitable lapses will soon equal if not exceed the new entrants. It may be well worthy of consideration whether one Board of Directors and one set of officers can properly supervise and care for more than a hundred thousand policy holders. Their best energies could be more judiciously exercised in critically examining and husbanding the resources of such enormous moneyed corporations, rather than in entering the field with younger companies, and by reckless and ruinous competition squandering resources which belong to policy holders of many years standing, who have furnished the pabulum for the building up of such powerful organizations. Something is also due by the way of example to the general public and corporate interests of the business. Nine companies show a decrease instead of an increase in the amount of business in force at the end of the year, as compared with the prior year. This result is in part abnormal and unusual in this country. Companies adopting the Massachusetts non-forfeiture law will probably find it a more difficult task to retain their policy holders. Some gentle force or stimulus is wanted with many men to enable them to keep up their premiums year after year, and unless these incentives are used in some form, the inclination is to lapse their policies. This natural tendency should not be encouraged, but counteracted by all legitimate means. The old fashioned and beneficent life policy, with equal annual payments, has already been so trimmed and modified that its originators, the fathers of assurance, would not be able to recognize the modern instrument as the child of their creation. While premiums have remained stationary, or been reduced, the increased expenses and rise in values incident to the war have abnormally burthened the companies, and thus burthened, still further weights are added by increased taxation, legislation, and other interpolations into the contract or policy, which if not discontinued may sap or undermine the sacred assets, which should be religiously kept in reserve, in order to honorably meet all contracts at maturity, whether payable to this or a future generation. Competition is now so sharp and zealous between the different companies, that all safe and prudent conditions and advantages will be voluntarily offered by the companies, to the full extent of their ability consistent with solvency. Indeed, I fear that in many cases too much may be, if it has not already been, offered. Not that I fear ultimate insolvency of the companies honestly and fairly managed, but rather immediate weakness, according to the American standards of solvency, which are the most strict and severe of any country where life insurance is practised.

"Unless under very extraordinary circumstances, the provisions of the life policy should be left free and untrammeled by legislation. Its various clauses and details can be more safely trusted, as in other cases, to the

contracting parties, than to the varying moods and conflicting opinions of forty different legislative bodies.

"The corporate privileges of Life Insurance Companies are not monopolies, but free to all citizens willing to organize companies under general acts, and if the policies which judicious officers, in their best judgment and discretion, now issue are not liberal enough in their provisions, any thirteen men can organize a new company, and then issue such kinds of policies, and with such liberal conditions, as they desire and approve."

"Table No. VII.—Compiled by William Barnes, showing the Net Increase or Decrease during the year 1870, of the amount insured or risks in force in several British Companies, as compared with the Life Association of America.

The statistics of the other British Companies could not be obtained.

Name of Company.	Location.	Net Increase of Risks in force year 1870.	Net Increase of No. of Policies year 1870.
Life Association of America	St. Louis ..	£ 4,000,188	3,867
1 National Provident	London	651,293	336
2 Scottish Provident	Edinburgh ..	392,283	711
3 Equity and Law	London	322,722	71
4 Economic	" ...	279,770	106
5 British Equitable	"	197,517	574
6 Law Union	"	156,299	128
7 London and Provincial Law........	"	152,685	110
8 National	"	101,759	44
9 Mutual	"	95,262	175
10 Westminster and General*	"	77,144	*
Total Increase (10 Companies) ..	.. ..	£2,426,734	2,255
11 Scottish Provincial	Aberdeen ..	Decrease. £ 385,854	*
Total Net Increase (11 Companies)	.. ..	£2,040,880	2,225

* Number of Policies could not be ascertained.

"The statistics of the British Companies are so imperfect, and hitherto obtained in such a clipped and distorted state, as to render the preparation of comparative tables a task of much difficulty, and in many respects of impossible performance. Of the hundred or more companies, I have been enabled to give the net increase or decrease of eleven only, as above exhibited."

[As far as we have been able to verify the figures in the above table it appears to give the "Net Increase or Decrease" during the year 1869, and not 1870. In other respects, also, we believe the figures must be accepted with some caution.—Ed. *J. I. A.*]

"TABLE No. VIII.—Compiled by WILLIAM BARNES, showing the relative Net Increase of the Amount Insured or Risks in force in the German, Austrian and German-Swiss Life Insurance Companies during the year 1869, as compared with the LIFE ASSOCIATION OF AMERICA.

	Name of Company.	Location.	Net Increase of Risks in force year 1869.	Net Increase No. Policies year 1869.
			£	
	Life Association of America	St. Louis	4,000,188	3,867
	German Companies.			
1	Life Ins. Bank for Germany	Gotha........	648,228	2,065
2	Germania	Stettin	564,663	7,318
3	Life Insurance and Savings Bank ..	Stuttgart	434,438	2,940
4	Generali and three others..........		420,000	6,000
5	Leipzig	Leipzig	388,108	1,961
6	Frederic William	Berlin........	293,605	2,490
7	Basle Life	Basle	281,254	1,500
8	General Home	Carlsruhe	260,277	1,851
9	North German Mutual............	Berlin	256,857	2,848
10	Anchor	Vienna	250,249	1,300
11	Concordia	Cologne	244,939	900
12	Berlin Life	Berlin........	243,233	1,882
13	German Life	Lubeck	225,789	1,658
14	Teutonia..	Leipzig	209,860	3,403
15	Magdeburg	Magdeburg....	187,579	1,156
16	Nordstern	Berlin........	171,741	1,099
17	German Life Pension and Annuity ..	Potsdam	167,743	3,114
18	General Annuity	Stuttgart	159,097	1,534
19	Prussian Life	Berlin........	121,613	1,182
20	Janus	Vienna	107,825	1,127
21	Providentia	Frankfort-on-M.	101,549	736
22	General Railroad Ins.	Berlin........	96,949	686
23	Janus	Hamburg	95,969	787
24	Thuringia	Erfurt........	88,376	667
25	Swiss Annuity	Zurich	86,634	463
26	Riunione Adriatica di Sicurta	Trieste	85,955	940
27	Iduna	Halle	85,123	1,486
28	Patria	Vienna	67,476	1,289
29	Frankfort Life	Frankfort-on-M.	63,000	800
30	Donau	Vienna	50,444	539
31	Life Ins. and Saving	Schwerin	48,468	372
32	Hypothecation and Exchange	Munich	26,228	69
33	Bremen Life	Bremen	13,607	283
34	Hanover Life	Hanover	13,440	147
35	General Annuity	Darmstadt	3,132	31
36	General Insurance Co.	Brunswick	1,321	30
	Total Net Increase (39 Companies)..		£6,564,769	56,653

"The Net Increase of all the thirty-nine German Companies during the year 1869, was not much in excess of the Gross Increase in risks of the Life Association. The average German policy is, however, about one-tenth as large as the average policy of the Life Association. One thing is favorable; that the German increase of new business is steady and progressive; and with the proverbial economy of management and the small rates of expenses, the institution is doubtless destined to increased future usefulness and success in the new Empire of Germany."

"Table No. IX.—Compiled by William Barnes, showing the relative Net Increase and Decrease of the Amounts Insured or Risks in force, including Annuities (immediate, deferred and contingent), and excluding Tontines, in several French Stock Life Insurance Companies, during the calendar year A.D. 1869, as compared with the Life Association of America.

Name of Company.	Net Increase in Life Business during the year 1869.	Net Increase in Annuities during the year 1869.
	£	£
Life Association of America .. (1870)	4,000,188	
French Companies.		
1 Compagnie d'Assurances Générales	1,893,565	40,147
2 La Nationale	1,655,955	17,476
3 Le Phénix	823,719	288
4 L'Union	566,907	3,108
5 Le Monde	259,354	2,444
6 Caisse Générale des Familles	235,031	3,460
7 Caisse Paternelle	202,962	3,471
Total Net Increase (7 Companies)	£5,637,493	£70,394

"With only ten Stock Companies in France the Net Increase of new business was about equal to all of the thirty-nine German Companies. The Générales and La Nationale each show an increase about equal to the Equitable of New York. The brilliant promise for the future of Life Assurance in France has been temporarily but not permanently destroyed. The Government has not hitherto given to the public any official report of the various companies, and statistics are gleaned with more or less embarrassment from the reports of the officers and the '*Comptes-Rendus*,' given at the annual meeting of the shareholders, which are generally held in the month of April and published in the Insurance Journals of the following summer."

"Table No. X.—Compiled by William Barnes, showing the relative Gross Increase during the calendar year of 1870, of the amount insured or risks in force, including old policies revived and reversionary dividends in American Companies (doing business in New York,) as compared with the Life Association of America, and not deducting Policies which have ceased to be in force from any cause whatever, during the year, and excluding reinsurance both at the beginning and end of the year.

Name of Company.	Location.	Gross Increase of Risks in force year 1870.	Gross Increase No. of Policies year 1870.
		£	
1 Equitable Life	New York......	8,059,160	10,063
2 Mutual Life	"	6,737,607	12,463
3 Life Association of America	St. Louis	6,065,546	5,818
4 New York	New York......	5,428,399	9,925
5 Connecticut Mutual........	Hartford, Conn...	5,317,108	10,608
6 Continental	New York.....	5,049,888	12,025
7 Ætna	Hartford, Conn...	4,341,095	11,217
8 Charter Oak..........	" "	4,023,697	8,785
9 Phœnix Mutual	" "	3,905,672	9,093
10 North-Western Mutual	Milwaukee, Wis.	3,635,752	9,348
11 St. Louis Mutual..........	St. Louis	3,354,759	6,604
12 Knickerbocker	New York......	3,149,384	5,627
* * *	* *	* *	* *

"From the foregoing table, No. X., it will be seen that only two companies in the United States, the Equitable and the Mutual Life of New York, exceeded the Life Association of America in the gross amount of new business during the year 1870. In both cases the additions made to old policies during the year by reversionary dividends are included: in the case of the Equitable, such additions amounted to £491,902; and in the Mutual, to £50,509. Excluding the New England (no returns) the total gross increase of new business during the year in those companies transacting business in New York State was £113,603,264. This is more than was transacted in all the world besides. It should be borne in mind, however, that endowments, long and short, constitute a larger percentage of American than of European business, and the short endowments soon pass off a Company's books by normal termination."

"Table No. XI.—Compiled by William Barnes, showing the Gross Increase of the amount insured, or risks in force in several British Life Insurance Companies during the year 1870, as compared with the Life Association of America.

	Gross Increase Risks in force year 1870.	Gross Increase No. of Policies year 1870.
Total Gross Increase (74 Companies)	£ 23,944,495	49,038

"The gross increase of the Association in new risks was equal to one-fourth of the above (Table No. XI.) seventy-four British companies; so that three other companies like the Association would have done all the new business of all these corporations. * * * * Many of these English and Scotch companies are institutions managed on thoroughly scientific and correct business principles, and are as sound and solvent as the Bank of England. Such companies should unhesitatingly make full exhibit of their condition and affairs, and thus show to the public their abundant competency to fulfill all their obligations at maturity."

Tables XII. and XIII. deal similarly with the business of the German and French Offices.

"Table No. XII.—Compiled by William Barnes, showing the Gross Increase of the amount insured or risks in force by the German, Austrian and German-Swiss Life Insurance Companies, during the year 1869, as compared with the Life Association of America.

	Gross Increase Risks in force, year 1869.	Gross Increase Policies year 1869.
Total Gross Increase (39 Companies, treated as 36). .	£ 11,397,785	95,696

"The Gross Increase of the new business of the Life Association, by the above table, No. XII., is seen to have been more than one-half of the whole thirty-nine German Companies, covering a population nearly double that of the whole United States.

"In American returns, the policies made out and issued from the office, whether accepted and paid for or not by the assured, are included on both sides of the account, viz., in the new issues and in those which ceased to be in force during the year,—in fact, if this entry on each side could be eliminated, it would be plainer for statistical purposes. I am not aware whether German returns include such entries, but presume that they do not on either side of the account. The apparent gross increase and decrease of American business is considerably affected by the policies returned as 'Not Taken,' as can be seen by the detailed statements of the terminations in their annual reports."

"Table No. XIII.—Compiled by William Barnes, showing the relative Gross Increase of the amount insured or Risks in force, including Annuities (immediate, deferred and contingent), and excluding Tontines, in the French Stock Life Insurance Companies, during the calendar year 1869, or 1868, as compared with the Life Association of America.

	Gross Increase in Life Business year 1869.	Gross Increase in Annuities year 1869.
	£	£
Total Gross Increase (10 Companies)	8,271,416	88,699

"The whole of the French stock companies, aside from their annuity and tontine transactions, write yearly only about one-third more life and endowment risks than the Life Association of America. The *Générales* and *La Nationale* transact more than half of the whole business."

Tables XIV., XV., XVI. are devoted to the average amount of the policies in force. Table XIV. shows "the average amount of "each Policy in force, including additions, in American Life "Insurance Companies, transacting business in the State of New "York, which had over one thousand Policies in force on the "31st day of December, 1870." The average amount of each policy ranges from £1,171 in the Life Association of America to £286 in the Metropolitan.

"The average amount of each policy in force is an important and very decisive feature in the business of the Life Association. The Equitable, the Mutual Benefit, and the Mutual Life, rank next to the Association in this respect. These companies also carry much larger risks on a single life than the Association,—at least double, and probably more than double the amount. Policies covering large amounts are generally considered in the Directors' room as being practically better risks than smaller policies, principally for the reason that a man of means can escape contagion, travel in case of necessity, and employ the best medical skill and suitable and appropriate remedies for any disease with which he may be attacked. In the same amount insured, although the mortality may be less in cases of large policies, yet the mortuary record will doubtless show occasional and more violent deflections in the curve of actual deaths, as the lesser number of risks would not be so well graduated as the larger number.

"The average amount of each policy in force, including additions, in the seventy-one companies transacting business in the State of New York, is the sum of £545.

	£
"In forty-six of the strongest British companies, with large bonus additions	555
In the thirty-nine German companies	131
In the ten French companies (life policy)	399
In the French companies (annuity bond)	134

	Number.	*Amount.* £
The total amount of risks in force in all the American companies transacting business in the State of New York is (71 companies) 1870	748,045	407,927,285
In 88 British companies, 1870	not fully returned.	345,170,004
In 39 German companies, 1869	456,144*	59,738,444
In 10 French companies (life policies)	90,060	36,752,000
Total, 208 companies	1,294,249	849,587,733
In 10 French companies (annuities)	26,815	720,000

"The number of policies in the British companies cannot be obtained from their returns, and are therefore not included in the above aggregate.

"It seems to be evident from the above figures that the Life Insurance companies of the United States have now a larger amount of outstanding risks in force than those of the United Kingdom. Doubtless the average amount insured per head, according to the population, is now larger in the United States than in any other country."

"TABLE No. XV.—Compiled by WILLIAM BARNES, showing the average amount of each policy in force, including additions, in British Life Insurance Companies, according to their reports for 1870, as compared with the LIFE ASSOCIATION OF AMERICA.

Name of Company.	Location.	Average Amount of each Policy.	No. Policies in force.
		£	
Life Association of America	St. Louis	1,171	7,799
British Companies.			
1 Equitable	London	2,055	3,785
2 Law	"	1,533	6,687
3 Equity and Law	"	1,530	2,190
4 University	"	1,382	1,503
5 London and Provincial Law	"	1,344	2,049
6 Rock	"	1,341	4,013
7 London Life Association	"	1,275	5,792
8 Guardian	"	1,085	4,094
9 National	"	1,029	2,112
10 Universal	"	968	3,206
11 Economic	"	924	9,682
12 Clergy Mutual	"	878	5,341
13 Metropolitan	"	863	4,620
14 Scottish Widows' Fund	Edinburgh	816	19,000
15 London Assurance Corporation	London	796	6,116

* In the German statistics, the returns of some companies cover the number of persons,—in others, number of policies.

Table No. XV.—(*continued.*)

Name of Company.	Location.	Average Amount of each Policy.	No. Policies in force.
16 Friends' Provident	Bradford	777	3,828
17 Scottish Equitable	Edinburgh	655	10,302
18 English and Scottish Law	London	652	4,850
19 Law Union	"	649	2,314
20 North British and Mercantile	Edinburgh	637	13,272
21 Atlas	London	617	6,199
22 Union	"	583	4,450
23 City of Glasgow	Glasgow	581	6,835
24 Mutual	London	577	4,092
25 Crown	"	575	7,697
26 Liverpool, London and Globe	Liverpool	548	12,760
27 Provident	London	517	9,496
28 National Provident	"	514	19,621
29 Northern	Aberdeen	473	8,217
30 Scottish Amicable	Glasgow	469	11,627
31 Scottish Provident	Edinburgh	468	14,523
32 Royal	Liverpool	467	14,292
33 West of England	Exeter	454	8,949
34 Caledonian	Edinburgh	402	3,683
35 Gresham	London	395	23,945
36 Scottish Provincial	Aberdeen	393	8,439
37 London and Lancashire	London	392	2,381
38 Reliance	"	332	5,665
39 Briton Medical and General	"	291	24,629
40 British Empire Mutual	"	239	11,370
41 Provident Clerks	"	207	11,004
42 United Kingdom Temperance	London Bridge	186	31,090
43 British Equitable	London	178	15,614
44 Western Counties and London Mutual	Plymouth	155	2,198
45 Post Office (Gov't.)	.. ..	76	2,168
Nos. and Total Average Amounts of Policies	.. ..	£31,278	385,700

"Average Policy of the 45 Companies, considered as separate Companies .. £695
Amount of Average Policy of the above 45 Companies, considered as one Company, (385,700 Policies, insuring £214,124,324) 555

"If the statistics of the other British Companies could be obtained, the average amount of the British policy would undoubtedly be considerably diminished, as the above table includes companies which write very large policies on the lives of the nobility and leading men of business."

"The average policy in German Companies is the smallest of any of the Life Insurance nations, being only £131. This small average almost classes them with industrial policies in other countries; but their persistency exceeds that of the large average policies in the United States. The Life Association has many German Trustee Directors and policy-holders; the transplanted German-American, however, readily falls into the habit of taking heavy policies in this country.

"In compiling the above tables, the pound sterling has been converted into dollars at five dollars each, the franc into dollars at five francs per dollar, and the thaler at seventy cents each.

"It is not impossible, indeed it is probable, that there are some errors

in the above sixteen tables; but special care has been taken to ensure accuracy, and the figures may be considered as substantially reliable. The sources of information were various, and sometimes inconsistent or contradictory; and I have, under the circumstances, tabulated the best statistics accessible, trusting that my labor upon them will prove to be of some interest, not only to policy-holders in the Life Association, but to other companies and to the general public.

" On a careful inspection of the above sixteen tables, it will be readily seen that, however much life insurance in this country may have declined, as compared with itself in the most favorable prior years, yet, when contrasted with the same institutions in other countries, our progress is rapid and unprecedentedly successful. Our smaller companies should therefrom derive sufficient hope and consolation, and not enter into an insane and wild crusade to bribe agents and the public into their organizations. With reasonable economy, and by maintaining themselves in positions of undoubted solvency, honorable careers of future usefulness are as certain as the progress of years and the ravages of death.

" On arriving at normal maturity, the first and greatest need of a healthy and well-organized young man is an equally sound and healthy wife. He then becomes a perfect human being, and an integral portion of society, and manly growth and illimitable progress and happiness are attainable. * * * * After marriage, as speedily as possible should come the home, be it ever so poor or humble. Through it and by it only are individuals and the nation nurtured, strengthened and sustained. Then should come the life policy, in some form, as certainly as the fire policy,—the one to perpetuate the existence of the home against the calamities of fire, and the other to provide for its support and maintenance, if death should abnormally come to the husband and father, by whose labors, as society is now organized, the family are mainly sustained and prospered. The best time to insure is generally in the younger ages, when premiums are low and the future dividends can increase insurance or reduce the annual payments of premium with advancing years and declining powers of labor.

" On the first view it may seem very difficult to trace any connection between the fighting power of a nation and the extent of its life insurance business. The close observer and analogist may, however, discover that to the degree that the character of a people tends towards life insurance, it leans in the direction of patriotism and bravery. The love of fatherland, of wife, and children, and home, leads naturally and equally to sacrifices on the field of battle and in the daily struggles and conflicts of civil life.

" At the beginning of the late war, Germany had over three hundred thousand life and endowment policies in force, and France less than one hundred thousand."

PRACTICAL QUESTIONS.

" Required to find the value at 5 percent interest, of an annuity on the
" life of the last surviver of five persons aged respectively 66, 65, 63, 62,
" and 59."

This may be readily done by the application of Mr. Woolhouse's formula given in the *Journal*, vol. xi. p. 321.

We shall have $a = m\mathrm{S} + \dfrac{m-1}{2} - \dfrac{m^2-1}{12}(\mu+\delta)$

where $\mathrm{S} = p_{\mathrm{x},m}v^m + p_{\mathrm{x},2m}v^{2m} + \ldots.$

X denoting the last surviver of the five given lives.

Putting $m=5$, we get

$$a = 5\mathrm{S} + 2 - 2(\mu+\delta)$$

where $\mathrm{S} = p_{\mathrm{x},5}v^5 + p_{\mathrm{x},10}v^{10} + \ldots.$

It is however to be noticed that $\mu=0$ in all cases where the last surviver of a combination of lives is in question. Thus, when we have two lives, $\mu_{\overline{xy}} = \mu_x + \mu_y - \mu_{xy}$, which $=0$, since $\mu_{xy} = \mu_x + \mu_y$.

Now $p_{\mathrm{x},5} = 1 -$ probability that 66, 65, 63, 62, and 59 will all die within the next 5 years.

$$= 1 - \frac{l_{66}-l_{71}}{l_{66}} \cdot \frac{l_{65}-l_{70}}{l_{65}} \cdot \frac{l_{63}-l_{68}}{l_{63}} \cdot \frac{l_{62}-l_{67}}{l_{62}} \cdot \frac{l_{59}-l_{64}}{l_{59}}$$

Taking the Carlisle table of mortality,

$l_{66}-l_{71}=617$ the log of which is	2·790285	$\log l_{66}=$	3·461499
$l_{65}-l_{70}=617$ „	2·790285	„ $l_{65}=$	3·479719
$l_{63}-l_{68}=620$ „	2·792392	„ $l_{63}=$	3·514282
$l_{62}-l_{67}=624$ „	2·795185	„ $l_{62}=$	3·530840
$l_{59}-l_{64}=606$ „	2·782473	„ $l_{59}=$	3·573915
	13·950620		17·560255
	$-$17·560255		
	$\bar{4}$·390365		
$\therefore\ 1-p_{\mathrm{x},5}=$	·000246		

Proceeding in this way we obtain the values of $1-p_{\mathrm{x},10}$, $1-p_{\mathrm{x},15}$, &c., these values being given in the second column of the following table.

n	$1-p_{\mathrm{x},n}$	$p_{\mathrm{x},n}$	$\log p_{\mathrm{x},n}$	$\log v^n$	$\log p_{\mathrm{x},n}v^n$	$p_{\mathrm{x},n}v^n$
5	·000246	·999754	$\bar{1}$·999893	$\bar{1}$·894054	$\bar{1}$·893947	·78333
10	·009930	·990070	·995666	·788107	·783773	·60782
15	·093340	·906660	·957444	·682161	·639605	·43612
20	·337220	·662780	·821369	·576214	·397583	·24979
25	·668904	·331096	·519954	·470268	$\bar{2}$·990222	·09777
30	·899018	·100982	·004244	·364321	·368565	·02337
35	·974588	·025412	$\bar{2}$·405039	·258375	$\bar{3}$·663414	·00461
40	·994685	·005315	$\bar{3}$·725503	·152428	$\bar{4}$·877931	·00075
45	·999735	·000265	$\bar{4}$·423246	·046482	$\bar{5}$·469728	·00003
			$\delta_5=$ ·04879			2·20359 = S
			2			5
			·09758			11·01795
						+ 2
						13·01795
						− ·09758
						12·92037 = a

Hence $$a_{\overline{66,65,63,62,59}}=12\cdot920$$

For the purpose of comparison, we have also calculated the value by the more usual method.

(1) By finding $a_{\overline{62,59,51}}$, 51 being the single age equivalent to the last survivor of 66, 65, and 63.
(2) By finding $a_{\overline{66,65,48}}$, 48 „ „ 63, 62, and 59.
(3) By finding $a_{\overline{66,59,50}}$, 50 „ „ 65, 63, and 62.

The following are the results:—

	Value of Annuity.
Application of Mr. Woolhouse's formula	12·920
$a_{\overline{\overline{66,65,63},62,59}}$	13·311
$a_{\overline{66,65,\overline{63,62,59}}}$	13·524
$a_{\overline{66,\overline{65,63,62},59}}$	13·526

[We take this opportunity of reminding our readers that we shall be at all times glad to give insertion to solutions of questions of practical interest. —Ed. *J. I. A.*]

HOME AND FOREIGN INTELLIGENCE.

SCOTTISH NATIONAL INSURANCE COMPANY.

Commenced Life Business in 1843.

* * * * *

II.—Business of the last Four Years.

5. The following were the number and amount of new Life Policies issued during the

Four years to 15th May 1864,	1420	.	£ 672,435.
Do. do. 1868,	1841	.	£1,037,520.

6. To the position of the Fire Business it is scarcely necessary to allude in detail, and the Directors will only repeat the statement made in last Report, that for twenty years at least there has been no one year in which this Branch has not paid all its own losses and expenses, contributed to the general expenses of the Company (thus lessening the charge on the Life Branch), and yielded, over and above, a satisfactory profit.

7. The average rate of interest actually yielded by the Company's investments was, during the

Four years to 15th May 1864,	.	£4 7 5	per cent. per annum.
Do. do. 1868,	.	4 12 6	„ „

If income-tax had been reckoned as a part of the Company's expenses, instead of being dealt with as a deduction from interest, and if interest upon the cost of the premises used for the business of the Company in

Edinburgh and elsewhere had been taken credit for, the average yearly return during the last four years would have been £4, 15s. 6d. per cent.

III.—Ascertainment of Profits.

8. According to the present constitution of the Company, it became the duty of the Directors, at the close of this last financial year on 15th May 1868, to ascertain what profits had been realized in the Life Branch during the four years since the last Division in 1864. The necessary calculations occupied the attention of the Manager and other officers of the Company for more than a year.

9. The principles followed out on the present occasion, as in all previous Investigations, have been these—

The calculations are founded on the well-known "Carlisle" Tables of Mortality, which have long been used by most Offices in Scotland, as well as elsewhere, and have recently been adopted by more than one Office of importance which had hitherto used other Tables.

In the great bulk of the calculations, embracing more than nine-tenths of the Company's obligations, future interest is reckoned on at not more than *three* per cent.; $3\frac{1}{2}$ per cent. being the rate assumed in the remaining cases.

Every claim and expense up to the period of Investigation is written off, and deducted from the Company's assets.

In valuing the future Premiums, the *nett* premiums only are reckoned on. If in valuing the future receipts the "loadings" were to be included, the effect would be to reckon as a present asset what ought to be reserved for future charges and profits.

10. The whole Policies issued since the commencement of the Life Business in 1843 have amounted to .	£3,901,137	5	9
The Policies which have been cancelled or surrendered, or which, having been granted for fixed terms, have now expired, or which have been paid on death, have amounted to	1,508,685	10	2
Leaving, . .	£2,392,451	15	7

as the amount of Policies in force at 15th May 1868.

11. The obligations contained in these Policies are of course future and contingent, and subject to the receipt of future Premiums. The object of the present inquiry is to ascertain what sum the Company ought to have on hand *now* in order to provide for these obligations *as they arise*, and how far the Funds actually on hand exceed that sum. The results of the Valuation, made on the principles just explained, may be summed up as follows:—

I.—Assets.

1. Amount of Accumulated Funds as *per* the annexed Balance Sheet, consisting of Consols, Loans on Mortgage, Cash in Bank, etc., .	£450,423	19	1
2. Present value of the future *nett* Premiums receivable by the Company under its Life Policies, ...	897,415	17	7
3. Present value of the Guarantees held from other Offices to cover portions of the Company's risks, with the Bonuses attaching thereto,	70,925	16	6
Total Assets, .	£1,418,765	13	2

II.—Liabilities.

1. Claims and other liabilities ascertained, but not payable until after 15th May 1868, . .	£14,189	1	0			
2. Paid-up Capital and Reserve, as at 15th May 1867, less the dividend since paid, . .	65,252	0	5			
3. Present value of £2,392,451, 15s. 7d., the amount of the whole subsisting Policies of the Company, ...	1,234,974	1	2			
4. Present value of the whole vested Bonus additions to these Policies, so far as not already surrendered, ...	51,613	12	6			
5. Present value of Annuities sold and still subsisting, ...	13,561	11	3			
Total Liabilities,				1,379,590	6	4
Nett Surplus or Profit, . . .				£39,175	6	10

Of the above Surplus, the portion which belongs to the *Shareholders* includes (1.) The interest for the year on their Capital and Reserve Funds; (2.) The nett Profits for the year of the Fire Insurance business; and (3.) One-tenth of the nett Profits of the Life Assurance business; and these together amount to £10,417, 3s. 10d. The portion belonging to the *Policyholders*, consisting of nine-tenths of the nett Profits of the Life business, amounts to 28,758, 3s.

IV.—Bonus Declared.

12. The share of the Surplus to be distributed among the Policyholders being £28,758, 3s., will provide for a Bonus Addition to the Policies entitled to participate at the rate of

£1, 10s. per cent. per annum

for the four years since 15th May 1864. The Directors have accordingly much satisfaction in declaring this Bonus.

13. To Policies sharing in the present Division, which may become the subject of claims previous to the next period of Investigation, there will be added a further Bonus at the rate of £1 per cent. per annum for each additional year's existence; and Policies not sharing in the present Division, but becoming entitled to profits before next Investigation, will also, in the event of death, have a Bonus added to them.

V.—Addition to Capital.

14. As regards the Shareholders, taking into account the interest earned during the year on their Capital and Reserve, the nett Profits of the Fire Branch, and their share* (one-tenth) of the nett Profits of the Life Branch, the Reserve is increased to £30,669, 4s. 3d. Following up the practice observed on previous similar occasions, the Directors have now transferred £10,000 from the Reserve to the Paid-up Capital Account, thus adding, without any cash payment, 10s. per share to the sum at the credit of each Shareholder. Each share on which £1 was originally paid,

* More than made up for to the holders of Life Assurance Policies by the share of the expenses borne by the Fire Department.

will now have had added to it out of profits £1, 15s., and will stand at £2, 15s.

The Paid-up Capital will thus be . .	£55,000	0	0
and the Shareholders' Reserve . .	20,669	4	3
Making their Total Funds . . .	£75,669	4	3

The Directors declare a Dividend on the Capital so increased at the rate of 10 per cent. per annum, being one per cent. per annum above the rate declared on the smaller amount of Capital last year.

Balance-Sheet of Scottish National Insurance Company, 15*th May* 1868.

Assets.

Stocks and Shares—						
Consols, ...				£25,000	0	0
Stocks of Scottish Chartered Banks, ...				11,143	13	3
Railway Preference Stocks, ...				22,740	15	11
Scottish National Insurance Stock, ...				2,554	10	0
Market value at 15*th May* 1868, £64,650.				£61,438	19	2
Loans, Mortgages, etc.—						
Secured on Land and other real Securities, ...	£290,217	3	8			
On Government Funds, etc., . .	13,939	7	7			
On existing value of Company's Policies.	19,666	1	4			
To Policy-holders on personal Security,	1,194	13	4			
				325,017	5	11
Government Annuities, ...				491	13	0
Premises in Edinburgh, London, Dublin, etc., . . .				9,904	17	3
Cash in Bank, ...				34,446	4	6
Cash and Drafts at Head Office, ...				54	15	8
Premiums in course of Collection, ...				19,070	3	7
One Month's grace allowed for payment.				£450,423	19	1

Liabilities.

Claims and other Liabilities ascertained, but not payable until after 15th May 1868, ...				£14,189	1	0
Life Assurance Fund—						
Amount ascertained (by detailed valuation) to be the equivalent of the Company's Liabilities at 15th May, 1868, ...	£331,807	10	10			
Ascertained Profits belonging to Policy-holders, appropriated to New Bonus, ...	28,758	3	0			
Together,				360,565	13	10
Shareholders' Capital and Reserve—						
Paid-up Capital, 20,000 Shares at £2, 15s. per Share, . . .	£55,000	0	0			
Reserve, ...	20,669	4	3			
				75,669	4	3
				£450,423	19	1

Examples of Bonus Additions up to 15*th May*, 1868.

Policy issued.	Original Sum Assured.	Vested Bonus Added.	Total Amount now payable at Death.
1844	£1000 0 0	£295 0 0	£1295 0 0
1848	1000 0 0	265 0 0	1265 0 0
1855	1000 0 0	185 0 0	1185 0 0
1860	1000 0 0	125 0 0	1125 0 0

FRIENDS' PROVIDENT INSTITUTION.

Established 1832.

On the 20th of 11th Month, 1867, the Institution completed its Thirty-fifth year;—and the time arrived for making the Sixth periodical investigation, and division of Profits.

The duty of making the necessary calculations was intrusted, as on former occasions, to Charles Ansell, F.R.S., the Consulting Actuary of the Institution. His report as regards Class IX. (Life Assurance) presents the following result:—

The present value of the total Sum Assured, with Bonuses, after deducting the present value of the future Premiums payable in respect thereof, and making proper allowance for the future expenses of management, was	£565,179	18	2
The Balance of Assets standing at the credit of the Class, was	700,154	16	1
Surplus	£134,974	17	11

Of this Surplus the Directors resolved to reserve the Sum of £7,974 17*s.* 11*d.* leaving £127,000 to be divided amongst the Members, in accordance with the Rules.

The relation of the present Division of Profits to those which have preceded it is shown by the accompanying comparative statement.

Date of Division.	Number of Policies in force.	Amount of Policies.	Nett Value of Liability.	Amount of Assets.	Surplus.	Amount Divided.
		£	£	£	£	£
1842	785	550031	60614	84196	23582	18866
1847	1197	825887	117858	159959	42101	37327
1852	1822	1233324	206605	269725	63120	57485
1857	2353	1611166	315073	395951	80878	75000
1862	2801	1914567	436197	533079	96882	91882
1867	3447	2437352*	565179	700154	134975	127000

[* It appears that the Annual Premiums on these Policies amount to £66,967.]

*   *   *   *

The Surpluses exhibited in Class VI. (Endowments) and Class VIII. (Deferred Sums) have enabled the Directors to declare a Bonus to the holders of Policies in each of those Classes, as on former occasions. A small distribution of Profit has also been made in the united Classes IV. and V.

A new feature in the recent Division has been the apportionment of Surplus amongst the Annuitants. In the three principal Annuity Classes, Nos. I., II., and III., the excess of Assets over Liabilities proved to be so considerable as to warrant the Board in devoting a portion of it to the benefit of the Annuitants by permanently increasing the amounts of the several Annuities. Additions varying from 3 to 12 per Cent. of the Annual Sums originally subscribed for were accordingly made; the average increase being about 7 per Cent. The Directors believe this to be the first instance in which an Assurance Company has divided Profits amongst its ordinary Annuitants.

The following Table contains a statement of the position of the various Classes of Assurances, as on the 20th of 11th month, 1867:—

	Nett Liability.	Cash Assets.	Surplus.	Amount Divided.	Balance of Surplus.
	£	£	£	£	£
Class I. Deferred Annuities	14327	17595	3268	1634	1634
II. Do.	14205	15645	1440	720	720
III. Immediate Annuities ..	66977	76678	9701	4850	4851
IV. & V. Endowments	898	1198	300	200	100
VI. Do.	10122	11393	1271	1112	159
VII. Deferred Sums	674	712	38	..	38
VIII. Do.	4450	5628	1178	1031	147
IX. Life Assurance	565083	700058	134975	126830	8145
X. Survivorship Annuities	2253	2274	21	..	21
Totals......	678989	831181	152192	136377	15315

*   *   *   *

The Premiums charged by this Institution are founded upon a Table of Mortality compiled from observations of the Deaths occurring amongst the Members and connections of the Society of Friends; and the Rate of Interest assumed in the calculations is 3 per Cent.

A margin or "loading" of 10 per Cent., to meet these expenses, and to provide against other possible contingencies, is added to the Nett Premiums deduced from the Mortality Table. By the system of Valuation adopted the whole of this "loading" is set aside, and the "Nett" Premiums only are brought into account.

In order to estimate the Sum which should be in the possession of the Office to meet the expected claims on Death as they occur, it is necessary to ascertain what (according to the Table of Mortality and the Rate of Interest adopted) is the present value of the whole Sum Assured with Bonuses, and to deduct therefrom the present value of the Nett Premiums to be received.

*   *   *   *

[SP]ECIMENS OF ADDITIONS TO SUMS ASSURED. CLASS IX.—LIFE ASSURANCE.

[Date of] Issue.	Age of the Assured at Entry.	Original Sum Assured.	Annual Premiums.	Bonus added in 1842.	Bonus added in 1847.	Bonus added in 1852.	Bonus added in 1857.	Bonus added in 1862.	Bonus added in 1867.	Total Sum now Assured.
		£	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
33	48	1000	36 18 4	93 12 0	130 1 0	162 8 0	203 2 0	273 18 0	428 0 0	2291 1 0
33	23	1000	19 14 2	94 17 0	108 0 0	97 19 0	93 1 0	91 15 0	106 8 0	1592 0 0
34	55	1000	48 3 4	95 6 0	160 19 0	217 11 0	282 1 0	387 10 0	604 19 0	2748 6 0
38	37	1000	26 17 6	38 19 0	88 5 0	95 9 0	91 2 0	101 16 0	145 2 0	1560 13 0
43	25	1000	20 12 6	..	59 12 0	91 7 0	86 13 0	86 1 0	97 7 0	1421 0 0
54	54	1000	46 6 8	..	..	..	68 8 0	103 3 0	150 9 0	1322 0 0
54	22	3000	58 7 6	..	..	..	169 15 0	233 14 0	268 5 0	3671 14 0
60	40	1000	29 0 10	..	..	..	..	43 5 0	86 0 0	1129 5 0
60	25	1000	20 12 6	..	..	..	..	39 7 0	84 10 0	1123 17 0

SPECIMENS OF REDUCTIONS OF PREMIUM. CLASS IX.—LIFE ASSURANCE.

Date of Issue.	Age of the Assured at Entry.	Amount of Policy.	Original Annual Premium.	Reduction per annum for Five Years, granted at the Division of 1842.	1847.	1852.	1857.	1862.	1867.	Reduced Premium now payable.	Total Amount of Abatement.
		£	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
833	36	1000	26 4 2	11 3 11	13 10 6	12 18 7	14 15 3	19 7 1	31 9 1	Premium extinguished Annuity of £5. 4s. 11d. payable to the assured	518 12 1
834	50	1000	39 11 8	12 15 2	19 18 9	26 12 7	35 12 6	52 1 8	90 9 10	Premium extinguished Annuity of £50. 18s. 2d. payable to the assured	1187 12 6
834	25	1000	20 11 8	8 10 3	10 16 5	10 4 10	10 3 0	10 12 8	13 12 6	6 19 2	319 18 4
840	37	1000	26 17 6	2 5 2	9 9 3	11 7 11	11 8 7	13 4 3	20 2 6	6 15 0	339 8 4
844	53	1000	44 9 2	..	8 3 11	17 7 9	22 8 8	31 5 5	50 17 6	Premium extinguished Annuity of £6. 8s. 4d. payable to the assured	650 16 3
844	25	1000	20 12 6	..	4 19 0	8 16 9	8 12 1	8 16 9	10 10 4	10 2 2	208 14 7
849	39	1000	28 5 0	..	..	7 6 11	9 19 4	10 8 4	13 0 2	15 4 10	203 13 9
854	60	2000	124 10 0	..	..	..	23 1 4	45 19 5	73 16 7	50 13 5	714 6 8
854	36	2000	52 8 4	..	..	..	12 13 10	17 19 10	21 8 6	30 19 10	260 10 10
854	28	2000	43 16 8	..	..	..	11 5 12	15 14 8	18 6 8	25 10 0	226 15 0
860	44	3000	97 10 0	..	..	..	..	14 17 10	33 7 4	64 2 8	241 5 10
860	23	500	9 18 4	..	..	..	..	1 11 11	3 17 11	6 0 5	27 9 2
865	46	500	17 5 10	..	..	..	..	..	2 16 1	14 9 9	14 0 5
865	24	500	10 2 6	..	..	..	..	..	1 17 11	8 4 7	9 9 7

* * * *

A Statement of Funds and Effects belonging to the Institution on the 20th of 11th Month, 1867.

PROPERTY BELONGING TO THE INSTITUTION.

	£ s. d.
Principal invested on Mortgage Securities............	745846 8 4
Interest accrued thereon	8367 15 8
Loans on Policies	71217 12 11
Interest accrued thereon..........................	1425 7 4
Office Premises at Bradford	3000 0 0
Policy and other Stamps on hand	227 4 6
Overend, Gurney & Co., Limited	4266 10 8
Balance at Bankers' in London.....................	4594 15 1
Do. do. on Security	4408 3 11
Balance in hands of Secretary	115 1 9
Balances due from Agents	54 14 8
	£843523 14 10

DEBTS OWING BY THE INSTITUTION.

	£	s.	d.
Claim Account—Assurances fallen in, but not yet payable	12176	8	7
Balances due to Agents, and Unclaimed Annuities	166	10	8
Balance in favour of the Institution on the 20th of 11th Month, 1867	831180	15	7
	£843523	14	10

CORRESPONDENCE.

AMERICAN TEN YEAR NON-FORFEITURE POLICIES.

To the Editor of the Assurance Magazine.

Sir,—The investigations of Mr. Younger on this class of assurance leave little to be said, but it may be worth while to look at the subject from another point of view.

An assurance for life of 1 is to be effected by annual payments during 10 years, with the option of at any time discontinuing the payments and receiving in exchange a paid-up policy of as many tenths as the number of payments made. Required the payments.

I leave out for the present the restriction usually made, that the discontinuance is not to take place till after the first two years.

The first payment is plainly the single payment which will secure an assurance of $\frac{1}{10}$ for life and a temporary assurance of $\frac{9}{10}$ for one year, and is therefore

$$\frac{1}{10}\cdot\frac{M_x}{D_x}+\frac{9}{10}\cdot\frac{M_x-M_{x+1}}{D_x}$$

or

$$(1)\qquad =\frac{10}{10}\cdot\frac{M_x}{D_x}-\frac{9}{10}\cdot\frac{M_{x+1}}{D_x}.$$

Similarly, the second payment is the single payment which will secure another $\frac{1}{10}$ for life and a temporary assurance of $\frac{8}{10}$ during that year, and we have

$$(2)\qquad =\frac{9}{10}\cdot\frac{M_{x+1}}{D_{x+1}}-\frac{8}{10}\cdot\frac{M_{x+2}}{D_{x+1}}.$$

In the same way,

$$(3)\qquad =\frac{8}{10}\cdot\frac{M_{x+2}}{D_{x+2}}-\frac{7}{10}\cdot\frac{M_{x+3}}{D_{x+2}}$$

....

$$(9)\qquad =\frac{2}{10}\cdot\frac{M_{x+8}}{D_{x+8}}-\frac{1}{10}\cdot\frac{M_{x+9}}{D_{x+8}}$$

$$(10)\qquad =\frac{1}{10}\cdot\frac{M_{x+9}}{D_{x+9}},$$

these values being the same as those obtained by Mr. Younger.

In order to get the *uniform* annual premium which is equivalent to them, we require to know the probabilities of surrender, and as we are ignorant of these, the problem is insoluble, but in practice the Offices can have no difficulty in selecting a rate which shall err on the side of safety.

I subjoin the different payments, commencing at the ages 30, 40, 50, calculated on the New Experience Tables at $4\frac{1}{2}$ percent interest, for an assurance of 1,000.

Age.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
30	34·32	34·27	34·27	34·25	34·24	34·24	34·20	34·10	33·95	33·78
40	43·73	43·76	43·87	43·99	44·11	44·16	44·11	43·94	43·65	43·26
50	58·32	58·46	58·58	58·59	58·46	58·13	57·60	56·86	55·86	54·53

There appears to be only one maximum in each set, the differences becoming large only in the three last payments; so that it may be sufficient to give merely the first, last, and maximum payments for the intermediate years, as follows:—

Age.	25	26	27	28	29	30	31
First payment	30·42	31·19	31·98	32·77	33·56	34·32	35·07
Last „	29·98	30·70	31·44	32·20	32·98	33·78	34·62
Maximum payment	30·54	31·28	32·03	32·79	33·56	34·32	35·13
No. of maximum payment	4	3	3	2	1	1	5
Age.	**32**	**33**	**34**	**35**	**36**	**37**	**38**
First payment	35·90	36·74	37·65	38·67	39·71	40·73	41·72
Last „	35·50	36·40	37·33	38·28	39·24	40·22	41·21
Maximum payment	36·03	36·95	37·89	38·84	39·78	40·75	41·73
No. of maximum payment	5	4	4	3	2	2	2
Age.	**39**	**40**	**41**	**42**	**43**	**44**	**45**
First payment	42·73	43·73	44·80	46·00	47·32	48·77	50·31
Last „	42·22	43·26	44·32	45·41	46·52	47·63	48·76
Maximum payment	42·73	44·16	45·41	46·71	48·02	49·38	50·78
No. of maximum payment	1	6	6	5	5	4	4
Age.	**46**	**47**	**48**	**49**	**50**		
First payment	51·89	53·46	55·06	56·63	58·32		
Last „	49·90	51·05	52·21	53·37	54·53		
Maximum payment	52·21	53·66	55·21	56·85	58·59		
No. of maximum payment	3	3	3	4	4		

As the American Offices (with very few exceptions) employ the Carlisle, or Old Experience, at 4 or $4\frac{1}{2}$ percent, their premiums should be higher than the above, and a comparison will confirm Mr. Younger's conclusion with reference to them.

The restriction as to the non-discontinuance of payments till after two years seems merely intended to allow the effect of selection to recoup the Offices for loss on the first payment by commission and preliminary expenses, and need hardly be considered.

There can be no doubt that this form of assurance is becoming a favourite on this continent, its great recommendation being probably its definiteness as regards the surrender values.

J. B. CHERRIMAN.

University College, Toronto.

ON THE EQUITABLE APPORTIONMENT OF A FUND BETWEEN THE LIFE TENANT AND THE REVERSIONER.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—the words "rough justice" quoted by Mr. Baden at page 284 from my remark at page 280, were used by me with reference, not to the "rateable division" of the fund, but to the division which Mr. Baden advocates. Considerations dissimilar from those which he advanced had led me to feel a decided preference for the latter method, without however my being able to satisfy myself that its applicability could be fully demonstrated.

I cannot regard it as surprising that, in treating upon this subject, so many persons consider the market values of the separate interests to be the point from which to start; but it remains in the next place to be decided upon what principle the life tenant and the reversioner shall share that dormant value or surplus which is only to be realised upon the completion of an arrangement between them for cancelling the existing tenure. An apportionment in the proportion of the market values of the two interests is the only method which I have known to be proposed for dividing such surplus; but I venture to submit that there are two other methods which are just as plausible. It might be alleged that as the consents of the two parties are equally necessary to the contemplated arrangement, the surplus realised by it should be divided equally between them. Or it might, I think, be argued that in addition to the market value of his interest, each party should be credited with the difference between the market value of the interest of the other party and the value of it when computed as a part of the perpetuity; for this is the difference of which each deprives the other until both consent to effect the arrangement.

In illustration of these several methods of dividing the surplus, I subjoin three examples in which the age of the life-tenant is assumed to be respectively 20, 45, and 70; and in each case the net income chosen is £30, assumed to be of the capital value of £850. For estimating the market values, the formulas selected from those illustrated by Mr. Porter are as regards the reversions A 6 per cent Carlisle, and as regards the life-

tenancies $\frac{1}{1-v_5+P}-1$, deducing P from the annual premiums of £1. 14*s.* 7*d.*, £3. 11*s.* 4*d.*, and £11. 7*s.* 0*d.*, respectively, these being approximately the charges made by Companies for non-participating policies of £100 to lives aged 20, 45, and 70 on their last birthdays.

	Example I. Life Tenant aged 20.		Example II. Life Tenant aged 45.		Example III. Life Tenant aged 70.	
	Life Tenant.	Reversioner.	Life Tenant.	Reversioner.	Life Tenant.	Reversioner.
	£	£	£	£	£	£
(A.) Values of the two interests as parts of a perpetuity in possession; being approximately $3\frac{1}{2}$ per cent (Carlisle)	597	253	448	402	207	643
(B.) Market Values of the separate interests	432	136	330	252	156	513
(C.) Balance; or defects in Market Values	165	117	118	150	51	130
(D.) Total dormant value or surplus	282		268		181	
(E.) "Market Values," with the addition to each of a corresponding proportion of the total surplus	646	204	482	368	198	652
(F.) "Market Values," with the addition to each of half of the total surplus	573	277	464	386	246	604
(G.) "Market Values," with the addition to each of the abovementioned defect (C) in the market value of the other....	549	301	480	370	286	564

The incongruity which the results E, F, G, exhibit,—not merely in the difference of their magnitudes in any one example, but more especially in the different order of their magnitudes in the three examples,—is discouraging to the adoption of the market values as a basis of calculation; for I am at a loss to conclude which of these three methods of dividing the surplus is the most plausible. In default of such a conclusion, the disadvantages inflicted upon each party by the other during every year of the joint continuance of the two interests may, I think, be roughly, but not unreasonably, treated as equivalents in the sense of setting them off against each other. If so, the deductions made in computing the market values are to be disregarded on both sides, and there remain only the original values (A) which are indicated for the respective terms by the rate of interest realised upon the fund and by the selected mortality table, and which are advocated by Mr. Baden.

It is observable, too, that any deviation from such a division of the fund causes the assignment to either one party or the other, in respect of

his existing interest, of a larger sum than would be assigned as the value of the corresponding term when calculated as part of a freehold or perpetuity in possession. Any such deviation may therefore be regarded with suspicion in the assumed case of neither party wishing to take advantage of the other.

I am, Sir,

Your very obedient servant,

EDWARD SMYTH.

8, *Mostyn Terrace, North Brixton,*
12*th August*, 1871.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On Insolvency in Life Assurance Companies. By A. H. Bailey, *Actuary of the London Assurance Corporation.*

[Read before the Institute, 27th November 1871.]

THE question,—how insolvent Life Assurance Companies are to be dealt with,—which all who take an interest in assurance affairs have for some considerable time been aware could not much longer be evaded, has during the last three years assumed a pressing importance, and although the subject has lately been discussed in several pamphlets and essays by members of this Institute and others it can hardly be considered exhausted. One of our most distinguished public men, himself one of the ablest lawyers of the day, after a searching and diligent enquiry has come to the conclusion that no better way can be found to settle the affairs of the "Albert," than by distributing among its more than 20,000 assured a sum which, on an average, may be expected to yield about one year's premium for each. It is surely worth consideration whether it is desirable that this decision should be drawn into a precedent, and besides, the subject may interest the members of this Institute for its own sake, because it involves questions concerning estimates of liabilities with regard to which actuaries are by no means agreed, and at which lawyers are in the habit of looking from a different point of view altogether. Certainly we may learn something from the legal profession, possibly they may

pick up a little from us, towards the elucidation of a somewhat intricate matter, and the solution of a by no means easy problem.

The subject is divisible into two parts. (1) When is a Life Assurance Company insolvent? (2) When such insolvency has been proved or is admitted, how should its affairs be arranged?

The first of these considerations involves the question of how the liabilities of a Life Assurance Company ought to be estimated, a question which has frequently occupied the attention of the Institute, and upon which several valuable papers are to be found in its *Journal.* This is not a fitting occasion for a detailed discussion of this subject, but some reference to the principal methods of valuation in use will be useful to elucidate what follows. Until the publication, in Milne's work in 1815, of money values deduced from the Carlisle mortality, no difficulty seems to have been felt. The principle on which a policy was to be valued having once been mastered, the application was easy enough. No table of mortality except the Northampton was then used, no rate of interest for life assurance purposes then thought of other than 3 per cent. And how expenses should be provided for, a question which, although studiously kept in the background, is now more important than those either of mortality or interest, was then disregarded altogether. But with better knowledge premiums from more accurate data were computed, these were lower than the current premiums of the day, and in order to determine the premiums to be charged to the public, additions to the former were made. It is common to say that these additions are made as a provision for expenses. If by this is meant that the additions are governed by the cost of conducting the business, real or estimated, this is a mistake likely to cause confusion. The expenses may or may not be defrayed from this source, but the rate of addition is determined not by the expenditure, but by competition. The additions are arbitrary, subject to this restriction, that the limits of the resulting premiums must be,—on the one hand, the minimum which the assurer will venture to take; on the other, the maximum which the assured can be induced to pay.*

* In the ordinary phraseology a "loading" is put upon the "net premium." Life assurance has not been fortunate in some of its technical terms. The word "loading" is neither elegant nor expressive, but, being in general use and not liable to mislead, it may pass. Not so the expression "net premium," which cannot be allowed. Net is a mercantile term implying the result of a deduction from the gross; the gross never being arrived at by an addition to the net. Thus, net weight is defined to be the true weight of merchandise after allowance has been made for the cask or enclosure; the gross weight being the actual weight of goods and package. The so-called "net premium" of life assurance corresponds more nearly to the prime cost of commerce. Nor is this the only objection. The term has its proper signification in other branches of insurance. In

This new method of constructing premiums occasioned a difference of practice in valuations. Some actuaries, who for distinction shall be called school No. 1, contend that the liability under a policy is to be measured by the difference between the values of the sum assured and of the future premiums payable, computed by tables of the values of reversions and annuities corresponding to the Office premiums. The returns under the Life Assurance Companies Act and the pages of our *Journal* will show that this school does not lack disciples now. To my mind, the objections to the obscurity of the results of this method have never been satisfactorily answered. Others, whom we will call school No. 2, maintain that the loading must first be deducted from the gross premiums, and that then the liability under a policy is to be measured by the difference between the value of the sum assured and of the future pure premiums computed by the rates of mortality and interest employed in the calculation of the premiums. Both schools have this in common—that the valuation must be made to depend on the calculation of the office premiums. In course of time a school No. 3 arose, sometimes but erroneously confounded with No. 2. This school, not concerning itself about the construction of the premium, but taking it as it is, contends that it is essential to determine with as much accuracy as possible what portion of the premium is really required for the risk, and what is the overplus or margin really available for other purposes. And the premium having been divided into these two component parts,—according to this school the measure of the liability under a policy is the value of the sum assured added to the value of the margin less the value of the future gross premiums payable, computed by rates of mortality and interest which will really prevail. The last condition is essential and must be insisted upon; a desire to obtain an accurate table of mortality being sometimes strangely combined with a determination to adopt an inaccurate rate of interest.

Thus, let $P_x =$ Office premium at age x,
$P'_x =$ pure premium do.
$P''_x =$ premium really required for the risk,
so that $P_x = P''_x + m$,

A_x, a_x; A'_x, a'_x; A''_x, a''_x; the values of reversions and annuities corresponding to P_x, P'_x, and P''_x respectively.

marine insurance the net premium is the premium charged in the policy, less the brokerage; this net premium, and never the gross premium, being the amount passed through the books. The expression "pure premium" seems unobjectionable.

Then $V_{x|n} = A_{x+n} - P_x(1 + a_{x+n})$ . . . . (1)

or $V_{x|n} = A'_{x+n} - P'_x(1 + a'_{x+n})$ . . . . (2)

or $V_{x|n} = A''_{x+n} + m(1 + a''_{x+n}) - P_x(1 + a''_{x+n})$ . . (3)

Example. Required the value of a policy for £1000 effected at the age of 35, ten years since, in an Office where the premiums correspond to those of the Carlisle 3 per cent. Table with an addition of 25 per cent., and where the mortality experienced is that of the 17 Offices' Experience, and the rate of interest realised 4 per cent.

$V_{35|10} = 564{\cdot}3 - 27{\cdot}9 \times 14{\cdot}96$ . . . . $= 146{\cdot}9$ (1)

or $V_{35|10} = 508{\cdot}8 - 22{\cdot}3 \times 16{\cdot}86$ . . . . $= 132{\cdot}4$ (2)

or $V_{35|10} = 428{\cdot}6 + 8{\cdot}0 \times 14{\cdot}86 - 27{\cdot}9 \times 14{\cdot}86 = 133{\cdot}6$ (3)

There are others—not a school—for their system, so far as I am aware, has no professors, although it has practitioners, who adopt what has with some truth and humour been called the elastic method of valuation. In that method neither the magnitude nor value of the margin is attempted to be computed, but an *à priori* determination having been made of what the balance should be, the value of the margin is adjusted to conform therewith.

Of life assurance valuations generally I may be allowed to say that in my judgment no complete or sufficient idea of the financial condition of a Company can be formed unless the value of the gross premiums is known. At the same time, I think that the result of a valuation intended for the information of the public should be stated in one item only, *i.e.*, the sum which it is estimated ought to be reserved for the liabilities; also, that while my own adhesion to the principles of school No. 3 has not been shaken by the attacks to whieh they have been lately subjected, I cannot avoid deprecating earnestly the tone and temper in which some of the recent discussions have been conducted, as little calculated to promote the investigation of the truth.

These methods of valuation have all one professed object—the division at the end of any given period of the surplus that may have accrued in that period; a process intended to be periodically repeated in future. They are not altogether applicable when the object is to determine for the ensuing year only, either in consideration of a fixed annual premium what specific sum can be paid if the life fails in the year, or what varying annual premium must be paid to secure a definite sum. These systems, to my mind, embody in their integrity the genuine principles of life assurance;

but the variations of necessity being sometimes in one direction, sometimes in the other, the system itself has proved so unpopular, and been so generally misunderstood, that its promoters have found it expedient to modify their original plan in such a manner as to give greater benefits to those who have been longer assured, thereby, as it seems to me, destroying its theoretical merit.

Again, the circumstances of the case are altered when the object is not to divide profits, but to determine whether or not a Company is solvent. In a vigorous contention in the Court of Chancery as to the solvency of a particular Company, affidavits, embodying valuations made on the principles that have been described, were filed, with the object of proving the Company to be insolvent. On the other hand it was contended that the Company was solvent if the available assets, together with the real present value of the gross premiums, exceeded the real present value of the sums assured. On a dispassionate consideration the conclusion arrived at will probably be that both belligerents were wrong. No equity judge would pronounce a Company to be insolvent, that is unable to meet its present engagements, because it might not be in a position to make future profits. The state of such a Company may be unsatisfactory without necessarily being insolvent. But again, as in these days no Assurance Company can be conducted except at considerable cost, a provision for future expenses is as necessary as for future claims. Undue expenditure, more than any other cause, or than all other causes combined, has been the occasion of insolvency in the past, and is the source of like apprehension in future.

If these principles be sound, it seems to me that in order to solve the question, When is a Life Assurance Company insolvent? the liabilities must be estimated by a modification of the principles of school No. 3. No future profit is now to be thought of, no question of adverse contingencies entertained, and the sole purpose for which the margin of the premium is required is for future expenses. The question then, which, as has been observed, did not arise when the scale of premiums was framed, must, now that competition is at an end, be faced, viz., What will be the actual cost of conducting the business in future? Experience teaches, that if a Company is to continue to compete for new business, it is very difficult to reduce materially its scale of expenditure. And therefore, after weighing what has been urged as to the reduction of cost that *may* be effected, the conclusion to which I have come is, that with this, as with questions of mortality and interest, the

only safe guide is past experience. I think it must be assumed that the future rate of expenditure will be equal to the average rate of the actual expenditure of the past two or three years.

Then if

P = gross annual premiums,

E = all expenses on an average of the last three years,

aP = real present value of future premiums,

A = real present value of sums assured,

C = realised assets,

the Company is solvent, provided

$$C + aP = A + aE$$

Before leaving this part of the subject something must be said about the shareholders' capital and the other assets. In ordinary balance sheets the amount of the paid-up capital properly appears as an item on the debit side; and unless the paid-up capital be intact no dividend to the shareholders ought to be paid. But the case is different when it is a question of solvency. A Company may be able to meet its engagements although its paid-up capital may have been partially or wholly lost, and therefore in considering its solvency this item is rightly omitted. The demand however to include uncalled capital in the assets is to my mind altogether inadmissible. To say nothing of the question of the shareholders' ability to meet calls, it seems sometimes to be forgotten that the very existence of a Life Assurance Company depends upon the accumulation at compound interest of its assets. So that when once it has been ascertained that the funds are deficient, the deficiency will increase from this cause alone, and by no means at a slow pace. Whenever therefore the invested funds are deficient, it seems to be the bounden duty of the Company forthwith to make calls on its shareholders to such an extent as will fill up the void; that this must be done at once, and that the uncalled capital cannot be regarded merely as a weapon in reserve.

Some items of the assets will require attention. A Company in a sound condition has perhaps no better class of investments than loans on its policies. But if the Company be insolvent, so that the sums assured will not be paid in full, the case is altered. This item is then a good asset to the extent only of the aggregate shares in the assets which will be allotted to the policies on which loans have been granted. Any excess over this amount is merely a loan on personal security, and more or less doubtful accordingly. Half-credit premiums and any kind of premiums on credit should

be treated in the same way, except that the excess must be written off altogether, these not being loan transactions for which the borrowers can be sued. Occasionally arrears of half-yearly and quarterly premiums figure as assets. Some modification in the estimate of the liabilities under this class of policies may be allowable; but it can hardly be necessary to discuss whether these so-called arrears are to be treated as well-secured investments.

Having thus endeavoured to ascertain when a Company is insolvent, I now proceed to consider how the affairs of an insolvent Company are to be arranged. And first of all it will be necessary to determine the relative interests of the creditors *inter se* in order to apportion among them (not necessarily distribute) the assets. The creditors may be divided into two classes—(1) The assured who have contracted to pay future premiums, and (2) those who are under no such liability, such as the annuitants and general creditors, including claimants under policies where deaths have arisen or endowments matured. The claims of the former are defined by the 158th section of "The Companies Act, 1862," as being subject to a contingency, and of which "a just estimate is to be made as far as possible of the value." How is this "just estimate" to be arrived at, or in other words, how for this purpose are the policies to be valued? If by the method suggested for ascertaining the solvency of a Company, the values in several instances will be negative. Without going further, this objection seems to me to be fatal. Some positive value must be assigned to every ordinary policy, every one of the assured having sustained some damage by the breach of contract on the part of the Company. And therefore we must come back to some one of the methods 1, 2, or 3 which have already been described for the valuation of a "going concern," to use a now common term. Each policy must be valued separately, the separate interests can then be determined, and the assets apportioned rateably among the creditors. Two courses are then open, and as it seems to me there is practically no other alternative. First, to make a wreck of the concern and distribute the assets as in an ordinary bankruptcy. Second, to continue to receive the premiums from the assured, contracting to pay, instead of the sums assured by the policies, such lesser amounts as the assets and future premiums will suffice to provide. The choice between these two should rest with the creditors. The last is the plan recommended by the actuaries, but somewhat, I think, to their surprise it has been received with little favour either by the assured or by the Judges to whom these questions have been referred. Up to

the present time the assured have evinced no disposition to continue the payment of the same premiums for reduced benefits. The effect on their minds of such a proposition seems to have been a disgust with, and distrust of, life assurance altogether. *Chat échaudé craint le feu.* The Judges seem unwilling to interfere with the general rights of creditors to a distribution of assets which have accrued by the stoppage. They seem to find the 22nd clause of the Life Assurance Companies Act unworkable, because it provides that the reduction of the contracts is to be a substitute for winding up, instead of a mode of winding up; and more than all, they are evidently sceptical both as to the safety and expediency of schemes of reconstruction generally. No progress will be made unless it can be shown that the actuaries' proposal is both safe in itself, and conducive to the interests of the assured.

Now the measure of the policyholder's interest in the insolvent Company is the value of his policy. A distribution of the assets will give him merely a dividend on this value, a small sum even under favourable circumstances, whereas he probably expects to receive a dividend on the amount which he has paid. But it may fairly be presumed that the object for which the assurances were effected was to provide sums at death; surely then their object is better served by consenting to a reduction, even a material reduction, of the amounts assured, than by receiving small doles in present money, which doubtless in many instances would be uselessly frittered away. In his judgment in the "Albert" case, Lord Cairns dismissed reconstruction schemes as unsafe, and was further of opinion that one of the objections to any such schemes was, that the immediate creditors and annuitants would thereby obtain an undue advantage over the assured. While it is impossible to avoid being struck with the pains taken, the patience displayed, and the grasp of mind with which an unfamiliar and somewhat intricate subject was seized by the arbitration of the "Albert" throughout the protracted proceedings, I hope I may be allowed, with all deference, to say, that in these two points Lord Cairns was, I think, under a misapprehension. The supposed value of the goodwill having been probably urged, Lord Cairns enquired of 29 of the principal Companies whether they would work a reconstruction scheme, and was answered in the negative. He thereupon expressed himself as follows:—"If the Companies or any one of "them had accepted the proposition to take over the insurances "for their own benefit, I should have had that proof of the safeness "of it as a commercial speculation. On the other hand, I am very

"sorry to say I can but look on the refusal of the Companies to "undertake a reinsurance for their own benefit as the strongest "practical proof to my mind that the scheme as a commercial "undertaking would not be a safe one, and would not be one "which I could properly or safely recommend to the policyholders "of this Company to adopt. The result therefore is, that I must "put aside altogether any idea of reconstruction or reconstitution." The Companies must, I think, have been surprised at the inference drawn from their refusal. Their real reason, it can hardly be doubted, was the conviction that the goodwill of an insolvent Company is of no value; on the contrary, the payment of reduced sums assured would be a proceeding likely to involve them in considerable ill-will. And the agents, whose future services are one of the chief inducements for an ordinary transfer, must be dismissed, unless the interests of the policyholders are to be further jeopardised, and the vicious system which has occasioned the insolvency to be further continued.

Lord Cairns's other objection must, I think, have arisen from the manner in which the proposal was submitted to him. The actuaries consulted did not suggest that it was necessary to ascertain what dividend the insolvent Company could pay, that the annuitants and immediate creditors were first to be paid their claims in cash, and that then the remainder of the assets should be handed over to the reconstructed Company. The plan which they did propose—which I firmly believe to be practicable and desirable as a general mode of arranging the affairs of an insolvent Life Office—was some such as this.

A scale of premiums to be framed as low as can safely be adopted; it would be convenient, but not essential, to adopt the non-participating scale of the Company. Such of the policyholders as agree to concur in the scheme to be at once assured for such amounts as the premiums they pay will secure, according to this scale, at their present ages, additions to be made to these amounts in the manner afterwards provided. In this way payment could be partially resumed very soon after the stoppage, and the inconvenience of not being assured at all mitigated. The liquidation of the Company, always a slow process, might then proceed independently, dividends being declared from time to time as the assets are realised, but no payment made to one particular class of creditors in preference to another. The direct creditors (including annuitants) would receive their dividends in cash, while to the assured would be assigned reversionary sums equivalent to their

shares of the dividend, the aggregate amount of the latter being handed over to the managing body of the reconstructed Company, either in cash or in *approved* securities. To the policyholders who decline to continue the payment of their premiums it was proposed to give dividends in the form of paid-up policies, refusing anything of the nature of surrender values. But seeing how unpopular this refusal would be, and feeling the force of the argument, that the terms of the contract must not be pressed upon one party at a time when it has been broken by the default of the other party, I believe that the dissentient policyholders might receive their dividends in cash along with the direct creditors. Of course no reconstruction would be practicable without the concurrence of a considerable body of the assured. Periodical valuations would be necessary (they might be annual if desired), the problem to be solved being what sums a given fund and a given amount of future annual premiums will assure. I do not think the members of this Institute will regard this problem as insoluble, or incapable of being worked out in practice.

Lastly, the affairs of the insolvent Company should be under the control of the assured, and be managed by persons of their appointment. Whatever another Company can do for them, they can do better, because more economically, for themselves. Not seeking for further business they will be under no temptation to spend money in cultivating connexions, and will not fear to incur the odium of stopping the payment of commission and otherwise reducing expenditure. They should put their own shoulders to the wheel, and not call upon any Hercules to help them to drag their waggon out of the mire.

The following account of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

Mr. T. B. SPRAGUE fully agreed with Mr. Bailey that it is impossible to form a correct view of the financial position of a Life Insurance Company, unless the value of the gross premiums is ascertained; and he further thought that in any report intended for the information of the public the value of the gross premiums should be stated, and also the value of the loading or of the amount thrown off as a reserve. He was of opinion that Mr. Bailey's objection to the use of the phrase "net premiums" is well grounded; that the words are certainly ambiguous, and therefore open to objection. But he did not admire the phrase Mr. Bailey proposes instead —that of "pure premium." For the opposite of pure is impure, and what can an impure premium be understood to mean? He thought a better phrase would be the "risk premium."

The historical sketch which Mr. Bailey had given of the methods of valuing the liabilities of Life Offices was not only interesting, but also very important for anybody who wishes to understand the present methods adopted by Offices in England. The study of the past is important in almost all subjects, and to anybody who wishes to understand the methods of valuation now used in England, and the methods of dividing profits, nothing would afford so much light as a study of what has been done by prosperous Companies of good credit in the past. The enumeration of the schools given by Mr. Bailey was, however, incomplete. If he had considered a little further, if he had devoted some hours' study to the various returns filed with the Board of Trade he might have found a great many other schools besides those three which he has selected.

Mr. Bailey very properly argued that in order to determine as to the solvency of a Life Office we must use the most correct data that we can find. We must use the rate of interest that is most likely to be realized with safety in the future, and we must use a table of mortality which expresses as nearly as can be judged the mortality we may expect will prevail. There can be no doubt, also, that the gross premiums must be valued, and a margin thrown off. Mr. Bailey made a remark which seemed to be a little out of place, as to a determination which sometimes prevails to use a wrong rate of interest with a true rate of mortality. Now, of course, in judging of the solvency of an Office, any preference for a low or safe rate of interest would be quite out of place. But in administering the affairs of a successful and prosperous Insurance Company, he (Mr. Sprague) thought the manager may very wisely, properly, and defensibly choose to make his valuation at a lower rate of interest than he knows he will realize. If any defence were required for that, it would be sufficient to say that the premiums are generally calculated at a lower rate of interest than it is known will be realized in the future, and any Company that uses premiums calculated at 3 per-cent does not act consistently in making its valuation at 4 per-cent. In order to be consistent, it should calculate its premiums at 4 per-cent.

Mr. Bailey had proposed two most important questions. First, When is a Life Assurance Company insolvent? There was only one objection to Mr. Bailey's method of answering that question. Mr. Bailey says we must throw off from the value of the gross annual premiums the value of the expenses which have been incurred in past years. But he (Mr. Sprague) thought that in valuing to see whether a Company is insolvent, we must bear in mind that the expenses can be reduced considerably when we cease to seek for new business, and we must throw off the value of that reduced expenditure, and not of the expenditure which prevailed while the Company was getting new business. Then, again, Mr. Bailey had said nothing about the necessity of examining whether the method of valuation has brought out any negative values of policies. Of course, the policies having negative values must be discarded, and considered as of no value. Mr. Bailey well sums up his remarks under this head by saying that if a Company is found to be in such a position that there is no prospect of getting any future profits, still that is no reason for declaring itself insolvent. The inability to give profit is a wholly different thing from insolvency. Then, in estimating the solvency, it is quite right to omit from the balance sheet the paid-up capital. That is all pledged to the policyholders as part

of their security, and if the Company can show realized assets sufficient, under correct assumptions, to meet its liabilities, no matter whether a part of those assets is shareholders' capital or not, the Company is solvent. He fully agreed with Mr. Bailey, that unpaid capital is a different thing. Until the capital is paid up it is not right to take credit for it. But Mr. Bailey had further said that unless the capital were intact, no dividend ought to be paid. He, on the contrary, saw no objection to the shareholders receiving dividend on so much of their capital as remained—half a dividend, if half the capital were lost.

The second question is—A Company being found to be insolvent, what is to be done with it? This question has had to be considered by lawyers in two well-known cases recently. The lawyers have naturally consulted the actuaries, and it appears from what Mr. Bailey says, that in the long run the actuaries and the lawyers have taken different views of the course to be pursued. The actuaries have unanimously recommended a reduction of the sums assured, the same premiums being paid, but the lawyers have agreed at last that that is impracticable. The lawyers say, "You must wind up this Company; you must ascertain the contributories and the creditors; you must realize the assets and divide them amongst the creditors in proportion to their claims." In approaching a question of this sort, the lawyers are hampered by Acts of Parliament and by precedents; they have to consider what is the law, what the Legislature has ordered as to the means of determining and regulating the affairs of insolvent Companies generally. But the actuaries are able to take a much higher view; they need not consider what is the law, but what is most consistent with real and substantial justice to all parties. In fact, we may sum it up by saying that the courts of this country are not courts of justice but courts of law. In former years, when the equity courts originated in this country, the first idea was, no doubt, that they should be courts of justice, freed from the strict and rigid rules of law, to decide between man and man, according to the principles of eternal justice. That there is no doubt they did for a long time, but now they have formed a new series of legal maxims; and the judges of the equity courts are no longer allowed to appeal to principles of eternal justice, but they are tied to maxims and precedents in the same way as common law lawyers are bound to appeal to the principles of the common law; and what we want is a new system which shall allow this freedom again to lawyers to appeal to eternal justice. And that has been done in late years to a not inconsiderable extent, by passing the special acts for the arrangement of the affairs of the London, Chatham, and Dover Railway and the affairs of the Albert Insurance Company. In each of these cases an arbitrater was appointed, substantially, it may be said, to deal with them according to his ideas of real and true justice, no matter what the law was, and his decision is final and without appeal.

Well, having come to this point, that the assets are to be divided in proportion to the claims of the creditors, the next point is to estimate the value of each man's claim. Mr. Bailey says that we must make such a valuation that every holder of an ordinary whole term policy shall be a creditor, inasmuch as every such policyholder has lost something; but he (Mr. Sprague) dissented from that view. When we want to ascertain whether a Company is insolvent we value gross premiums, throwing off a certain amount for expenses, and that brings out a certain deficiency.

He thought the obviously correct method of proceeding was to ascertain what portion of that deficiency is chargeable to each separate policy consistently with the principles of that valuation. Mr. Bailey says, on the contrary, we must have two valuations; we have had one valuation to see whether the Company is insolvent, and having done that, we must have another, on totally different principles, to estimate the claims of the several policyholders. The affairs of a Life Company are very different from those of the Companies which Parliament had principally in view when it passed the Acts relating to the winding up of Companies. In particular, in the case of a large Company, the ordinary creditors, the policyholders, are numbered by thousands, and according to the ordinary process of winding up, every creditor has to make out his claim and send it in. It was out of the question for 10,000 or 12,000 policyholders singly to put a value upon their policies, and send it in. The officials of the Company have all the materials for making a valuation, and when once the principle is laid down, it is in every way better that they and not the policyholders should make the valuation.

Lord Cairns has decided in the case of the Albert that every man's policy is to be valued, and he is to have a dividend upon that value. But it is impossible to value with anything like justice the various claims of all the policyholders. The policies on the lives of those who are in health may be easily valued; but out of such a large number, there are many who are in a bad state of health, and it is impossible to put anything like a fair value upon their chance of longevity, so as to value their policies. Therefore, this method is highly unjust as regards the policyholders who are in bad health and will shortly die; and as regards the annuitants in bad health, it is too favourable. This difference between people in good and bad health is thus entirely disregarded by Lord Cairns's decision, which to that extent is extremely unjust. The only correct way to value the policies of individuals separately is to wait and see how long each man will live, and then you can ascertain what loss he has sustained. If he lives so long that he would only have paid one more premium, and then dies, you must deduct the value of that premium from the value of the sum assured; and similarly for two or any other given number of premiums. That principle affords a simple method of bridging over the interval between the date of stoppage of an insolvent Company and the date of reconstruction of a new Company. All persons who die within that interval should be taken as immediate creditors, discounting their sums assured, and the payments they would have made if the Company had not stopped. But the more correct, and indeed the only proper way to estimate interests of this sort is to estimate them in a lump; take them as we value the policies of an Assurance Company, and in that way we can do justice and fix a proper value on each man's interest. As regards surrender values, there is the difficulty which Mr. Bailey mentions, but it is very easily got over. Supposing that, according to contract or custom, the policyholders in an insolvent Company had a claim to a certain amount of surrender value. If the Company can pay 4*s.* in the pound, give them that 4*s.* in the pound on the old surrender value, which is quite a different thing from the value of their policy as creditors of the Company. He concluded by expressing a hope that the discussion of this important practical question would be attended with some practical results.

Mr. W. P. Pattison felt that there was a widespread distrust among the public, and feared it would extend to the profession itself. The sooner, therefore, the circumstances which have brought about the failure of Offices whose transactions were of great magnitude, were fully considered, the more desirable would it be for those who belong to the profession of actuary. The European and the Albert Offices did not fail until they were forced to do so. They could not go on paying their claims, and were thus forced to closed their doors. It was exceedingly important that this should not be the case in future. When an Office has passed a certain point, let its transactions be brought to a close. Some principles should be at once laid down by which the insolvency of an Office can be distinctively declared.

No doubt excessive expenditure is the primary cause of insolvency in a Life Office; but if the valuation which immediately follows is made upon true principles, the tendency of the expenditure is at once brought to light, and if it continues its high rate of expenditure, its future insolvency is made clear. Now, that has been got over in some cases by entirely erroneous principles of valuation. In the first place, the gross premiums, less an insufficient percentage, and then, without any deduction whatever for expenses, have been valued, and wrong tables of mortality and wrong rates of interest have been used. The question of valuing by gross premiums is far more important than it is sometimes considered, as not only are the future expenses not provided for, but a very large number of policies are actually converted into assets, which are relied upon to meet claims upon the old policies. He thought, therefore, there is no doubt that to a very large extent the wide-spread ruin which has been brought about by the insolvency of Offices has been due either to dishonesty or to incompetency in gentlemen who claim to be actuaries.

He thought it was quite time that the actual mortality which Offices experience should be exhibited side by side with the mortality assumed in the valuation. A great many Offices assume the Carlisle Table of Mortality, when that table is not in the least applicable to them. The difference in the mortality experienced by different classes of Offices is very large indeed. Some Offices experience a mortality at least 20 per-cent below that experienced by a large number of others. Even if the Carlisle is a true table to measure the mortality experienced by an Office during the whole currency of its risks, it is inadmissible after the effect of selection has passed. During the first five years after selection the rate of mortality is considerably lower than that of the Carlisle Table, though it is much above that table after the first five years. With regard to the rate of interest, two or three Offices have assumed 4 per-cent in their valuation, though they have been actually realizing only 3 per-cent. Mr. Sprague had referred to the rate of interest, and justified, upon the ground of consistency, the assumption of 3 per-cent. He (Mr. Pattison) thought it could be put upon a higher ground, namely, that of equity. If policy-holders pay a higher rate of premium than necessary, in order to participate in the profits, he did not think we should charge them a premium based upon 3 per-cent and then value the whole of the policies at a higher rate of interest. In the same way equity allows that the old policyholders should be required to provide in the valuation for the expenses of managing their business, and that the new business should not in the least be relied upon for the carrying on of an Office.

He thought that all policyholders should rank as creditors. In testing whether an Office is solvent or insolvent you may fairly deduct 10 per-cent from the gross premiums in the valuation, but that is before the contract is broken. If the Office has sufficient funds in hand, valuing with a deduction of 10 per-cent from the gross premiums, due allowance being made for the policies converted into an asset, the Office must be pronounced not insolvent; but when the Office has not got that fund, the contract is broken, and the policyholder has then a right to say, "I consider I am a creditor, having paid so much money."

Mr. C. Walford wished to say a few words upon the legal aspects of the case. The special Acts referred to by Mr. Sprague gave the arbitrator power to deal equitably and not legally with the questions which are involved in the failure of an Office; but the arbitrator could not go beyond the general statute law of the land; and if any large class of policyholders chose to rely upon their rights under the statute law, nothing the arbitrator could do would affect those rights in any way whatever. Therefore, it was useless for the arbitrator to recommend any scheme which gave to one or more sections of policyholders rights which they could not have got under the general law, because it would have been upset by those who dissented from such recommendation. It was in that view that Lord Cairns gave the decision as to the impossibility of reconstruction.

Mr. Bryant (of New York) said that in America an insolvent Company is dealt with in a very different manner from the English. The procedures are to a certain extent fixed. There is no controversy there amongst the legal gentlemen as to whether a Company is solvent or insolvent, for that question is settled by an official valuation, made by an authorized Superintendent appointed by the Government. The standards of valuation are, to be sure, fixed without much regard to the facts. In some States 4 per-cent interest, and in others $4\frac{1}{2}$ per-cent is adopted; and as certain reserves have to be made on each policy and kept intact, it becomes a question of inquiry whether the net assets in hand are large enough to supply those reserves. If a Company can show that its assets are sufficient to meet the policy liabilities thus computed, it has sufficient assets, and is solvent; but if the reserve is not sufficiently large to meet, by the State standards of valuation, those engagements, it is technically insolvent, whether commercially so or not. In England, on the contrary, account was taken not merely of the rate of interest that may be realized, or the table of mortality used, but of the actual premiums of the company, and an opinion formed according to the facts, and not by an arbitrary assumption. In case of insolvency, it appeared a proper inquiry—How came the Company in its present state—what brought about the insolvency? Is there not something at work here which will be always at work, and which may defeat any plan for carrying out the Company's engagements? How has the Company transacted its business? Have extra risks been assumed without extra premium? Has a poor class of lives been taken? These are questions which ought to be investigated, as well for the purpose of guidance in any scheme of reconstruction or transfer, as for the purpose of bringing home to those to whom belongs the charge of negligence, or whatever else it may be that has caused the insolvency.

Mr. E. Galsworthy thought if the questions suggested by Mr. Bryant had been put to the Companies which have failed it might possibly have

prevented the disasters which have taken place. He doubted whether the Albert failed because it could not pay its way. He thought it stopt payment because it could not get the funds it had in another part of the world. If the European had stopt three years ago, the deficiency as compared with that now estimated would have been comparatively small.

He agreed with Mr. Sprague in thinking that Mr. Bailey is not right in throwing off so much as 20 per-cent from the gross premiums. Mr. Bailey found that the Albert charges amounted to 23 per-cent, but if he had looked further into the past of the Albert he would have found that the expenses were about 5 per-cent of the renewal premiums, and just so much more as the new business cost; and, practically, as the new business rose and fell, so the expenses rose and fell. The moment they diminished their expenses the new business diminished, and *vice versâ*. By stopping the new business there would be a continuous expense of 5 per-cent upon the renewal premiums, and, of course, something more for necessary establishment charges; but these two together would never have come to anything like 20 per cent. Of course we must agree that the insolvency of a Life Assurance Company is a very different thing from its inability to pay a bonus; but, unfortunately, the moment it is known that an Office is unable to pay a bonus, then, practically, the insolvency seems to begin, because the new business diminishes. That appeared to be the great difficulty. He did not think the surrender values of policies which have been taken out on diseased lives can be determined upon the same principles as those which have been taken out on ordinary lives. A man at age 20 takes out a policy, and pays a premium equal to that charged for age 40; another does the same, and within five or ten years one of them gets worse and dies, and the other, not having died, goes to the Office, and says, "I not only expect a surrender value, but I expect a very large one, because I paid you a very large premium." But the money paid by him has gone to pay for the one who died; and until the lives which have been accepted as diseased have been examined again when they come to surrender their policies, he could not see how any just surrender value can be given.

Mr. H. HARBEN referred to the case of a man whose premium was originally high from bad health, but whose health had since improved, and thought the improvement of health should be taken into consideration, and he should be admitted as a claimant for a less sum than otherwise would be the case. He had heard of persons feigning a disease or malady; and thought if any allowance were made in calculating the policy value on account of supposed bad health, persons might be tempted to feign disease, and thus prove for a large sum upon the estate, and defraud the insolvent Insurance Company. That appeared to be a practical difficulty, and he thought that Lord Cairns's method, however hard it may be in certain cases, provides a solution of the difficulty, and over an average of 20,000 lives deals fair justice. As regards section 22 of the Life Assurance Companies Act, the lawyers object to what they believe to be an unfortunate word in the clause itself. A deed of annuity is a contract, but if a claim on a policy has fallen in, it is no longer a contract, but an accrued debt. He believed that if that clause were properly worded, and a general interpretation clause were put into the Amendment Act to the effect that the

term "contract" should include all policies, whether current or accrued, and all annuities, the lawyers would soon be able to come to some decision.

With reference to Lord Cairns's decision not to reconstruct the Albert, Lord Cairns's circular to the Companies upon the subject did not say, "Will you take the Albert Office from the date of its failure, and guarantee all its policies as on that date, minus those who elect to prove upon the assets?" but it said, "Will you take so many policies, and for every policy I agree to pay you so much?" That means that all those who choose to select a good Company and prove upon the new Company were to go to them, and all the good lives were to go anywhere else, and proving upon the Albert estate, were not to be paid for. That being so, the selection would have been dead against the Office. That, he thought, was a most unwise circular. But if a circular had been sent round asking whether the Company would take the policies as on that date, guaranteeing them all, the Company to have all the assets, many of the Offices would probably have been glad to meet the proposal.

Mr. B. Newbatt was inclined, on the principle that prevention is better than cure, to think that the plan existing in America, which stops an Office before it has done any real mischief, or, practically, any mischief at all, is better than Mr. Bailey's, or indeed any scheme which only attempts to patch up a Company that has utterly and entirely broken down. Mr. Bailey's scheme deals only with what we may call legal insolvency. But there is an insolvency of another kind, which may perhaps be fitly called moral insolvency, where an Office, though legally solvent, yet finds itself in a position wholly unable to carry out the contracts into which it originally entered. Part of the contract entered into by an Office which issues policies on the participating scale of premium, is to give something—it may not be very much—in addition to the bare sum assured named in the policy; something corresponding at least to the excess of premium which is paid on a participating policy—that is to say, such a sum should be forthcoming in the aggregate and on the average as the with-profit premiums, if treated as non-profit premiums, would have secured to the assured in the first instance, had they elected to take non-profit policies. The absence in any Office of ability to do this is a form of moral insolvency to which, as well as to other forms of a deeper shade, the American system addresses itself; and the question of dealing with such a case here can hardly fail, sooner or later, to be forced on our attention. He quite agreed with Mr. Pattison that the question of the rate of interest to be used in the valuations is not one of expediency, as Mr. Sprague put it, but is as much a point of principle as the question of what loading shall be set aside for future expenses and contingencies. In this country this question of interest has never been raised at all in this form. It had only gone to this point—Is the rate used in the valuation a reasonable rate? Is it a rate which the Office is justified in assuming that for a long series of years it will be able to make on its assets? That question being determined in favour of the Office, the whole matter appears to have been regarded as settled. But in America the principle has been urged that "the reserve ought in all cases to follow the premium." He alluded to Mr. Peet, a representative and champion of a class of Offices which have sprung up in America, whose object is to insure at the lowest possible rate of premium. Mr. Peet's Office bases its premiums upon 6 per-cent

interest, and he says, "It will be as monstrous to call upon us, dealing with a class of insured who are content to take a state of things which 6 per-cent premiums and a 6 per-cent reserve will give them, to value upon a 4 per-cent basis, as it is improper for an Office that bases its calculations upon 4 per-cent to assume 6 per-cent in its valuation." He (Mr. Newbatt) thought that a valuation in this country also should in all cases follow the premium; and that it is as unfair, whilst charging premiums formed upon a 3 per-cent basis, to give only the security of a 4 per-cent reserve, as it is to include in the valuation, for the purpose of obtaining a divisible surplus, any portion of the loading which has been put on to the premiums for the purpose of meeting the expenses. In both cases alike, though it may not be in the same degree, the assured is deprived of part of the security for which he has paid. He thought that actuaries should show somewhat more of unity of opinion both in principle and practice, and somewhat more of *esprit de corps* than they have been content to exhibit hitherto.

Mr. A. Baden agreed with Mr. Bailey with regard to the valuation by the rate per-cent on which the tables are constructed. It seems a little pedantic to say that because we, the profession, have constructed our premiums upon a 3 per-cent basis, we should always use that 3 per-cent basis in the valuation. It may or may not be true that we have constructed our tables upon a 3 per-cent basis, but what does it matter if we take a 4 per-cent valuation. The real point is, whether the margin reserved is proportionately enlarged. If we have put a certain margin on a 3 per-cent premium, and only use that proportion in making a 4 per-cent valuation, unquestionably we are wrong; but if we get a margin proportionate to the 4 per-cent valuation, it does not seem to be much of a practical question whether we use 3 or 4 per-cent in our valuation. But there is one good reason which justifies the use of 3 per-cent valuation, which cannot be considered the true rate of interest, namely, that it is the means of making a reserve for a "rainy day," which is sure to come when the rate of mortality increases as the influence of selection wears out on our books. If, instead of valuing by one table for all classes of risks, we valued the risks on our books for five years by one table, and for more than five years by another table showing a higher rate of mortality, then there would be no need for such an expedient. As it is, the mode of adopting 3 per-cent for 4 per-cent is a rough-and-ready, tho, on the whole, a fair means, for providing for that deficit which would otherwise arise when the advantages of selection have worn out amongst the risks on our books.

The President was sorry to say, the subject of Mr. Bailey's paper was one of the greatest interest and importance at the present time. He could not but admit that, both as a member of a scientific profession and an Englishman, he felt humiliated by the circumstances which have taken place during the last three or four years. It was now nearly half a century since he first became connected with the business of life assurance. At the commencement of that period the insolvency of a Life Assurance Company, which is now a common subject of discussion, was hardly ever mentioned or thought of. It was very well understood that Offices might be established, and failing to get sufficient business to make it worth while to continue, would wind up their affairs; but any notion of the collapse of Offices of such standing, as regards the amount of business transacted, as those which have

failed, never seemed to enter into the mind of any one connected with life assurance business. Professor de Morgan has said that there is no enterprise of any kind that admits of being managed with so much safety and stability as a well-conducted Life Assurance Company. And those events which we all so much lament and deplore have been the result of mismanagement of the most painful, of the most discreditable, kind,—the result of gross folly, gross presumption, and gross self-sufficiency. In every other profession—in law, in medicine, or any profession involving scientific knowledge,—a man is supposed to require special education, but everybody in the world seems to think he understands life assurance, and the consequence is that in any general conversation you hear more nonsense talked about life assurance in an hour than about any other subject whatever. He would relate what took place on one occasion at a meeting of an Office in which he is a shareholder, one of the oldest and most eminent Offices in the country. There was a great debate about a quarter of a century ago as to whether a larger bonus could not be given. A committee of five was appointed, amongst whom was a celebrated calculater; and they reported that to give a larger bonus would be fatal to the Office. They had the opinion of five of the oldest and most eminent actuaries of the day—Professor De Morgan, Mr. Griffith Davies, Mr. Ansell, and two others, and they employed the late Mr. David Jones to make the valuation, —and these gentlemen all gave their opinion in direct opposition to the proposal of increasing the bonus; and one of the committee got up in his hearing at the meeting, and said it was a most extraordinary and singular thing that these five actuaries should all entertain this opinion, and not only they but other actuaries entertained it also, and "he could not beat it out of their heads!" That remark was not received with any astonishment by the meeting, it did not create surprise, but if it had been a question in medicine, anyone expressing such an opinion would have been driven out of the room. He must confess that he did not see any chance of winding up satisfactorily an insolvent Life Assurance Company. An actuary might lay down a good scheme,—but what is the essence of life assurance? It is confidence, and the moment you lose that, you lose the cohesion, the combination, the welding force, and the whole thing tumbles to pieces. No Insurance Company ought to become insolvent, particularly a Company with a subscribed capital; because, if it was properly managed, before it could get into a state of insolvency, it could, by the assistance of its subscribed capital, transfer its risks to another Office that would undertake to work out the contracts. That is the only way in which the business of a Life Office can be satisfactorily wound up.

Mr. Bailey in reply stated, that he never supposed that he or anybody else could possibly bring about a satisfactory way of arranging the affairs of an insolvent Life Assurance or any other Company. But he thought it worth while for actuaries to endeavour to make, what in familiar phrase is called, "the best of a bad job," and a bad job it will be at the best. He thought the better way to get out of that bad job was by dividing the assets.

With regard to the rate of interest, he thought when a man goes to effect an insurance upon his life he does not care one straw what the rate of interest is, or how the premium is arrived at. The first rate other than the Northampton was deduced from the Carlisle Table with 4 per-cent interest,

and 40 per-cent was put upon that; subsequently the rate had been reduced to 3 per-cent. The rates of premium are the results of competition and nothing else. But when an Office comes to make a valuation for any purpose whatever, the question is not what rate of interest the premiums have been computed upon, but what rate will be realized in the future. We want to know for our purpose what sum we must lay aside—not what is safe but what is true, and the only way in which we can arrive at what will be really required is to use data which you think are as nearly true as possible. The sum of money to be divided is another question altogether.

As to paying commission, if an Office goes on it must pay commission. You can only stop the payment of commission by the winding up process; and if an Office is to continue, there was, he thought, abundant evidence to show that it cannot materially reduce its rate of expenditure. Therefore, he repeated that the rate of expenditure which has been incurred in the last three or four years is a fair test for a going Office, but not for one that is winding up.

On the Laws of Sickness and Invalidism; and their relation to the Law of Mortality. By W. M. MAKEHAM, *Fellow of the Institute of Actuaries.*

[Read before the Institute, 18th December 1871.]

IN a paper read before this Institute many years ago, by Mr. S. Brown, occur the following judicious remarks relative to Statistics of Mortality and Sickness:—

"It has frequently been remarked," says Mr. Brown, "that "however varied and uncertain may be the occurrence of the events "to which the life of a single individual is exposed, the average "return in a large mass of persons is so regular as to be predicted "with confidence within very small limits of error. On the "uniform happening of these events under similar circumstances, "or the discovery of the laws by which they are governed, the "actuary depends for the application of the theories of probabilities "to the many useful purposes for which they may be rendered "available. The principal difficulty which he has to encounter is "the indefinite character of the event itself. In endeavouring to "ascertain the law of mortality at any period of time, or in any "particular country, the event is certain and definite whenever it "does occur. The attention is only required to the difference of "the circumstances which cause it—the age of the party, the "nature of the disease, or the locality within certain boundaries of "which a greater or less intensity of disease may prevail. But a "much greater difficulty arises when the event itself which is under "observation is of an uncertain character; such for instance as the "average duration or effects of sickness in a large number of cases.

"That there are peculiar diseases, the origin of which may be "distinctly traced, which run through a certain regular course, "and in which the event to the sufferer may depend upon his "passing safely thro' a known crisis, may be admitted. When we "come however to apply the definition of sickness to the question "whether the member of a sickness club may be incapacitated for "pursuing his daily avocations, we shall find it in the great majority "depending not so much on the natural laws of sickness as on "the varying opinions of medical men as to what constitutes the "degree of sickness which entitles to relief, or the tenacity and "firmness of mind of the patient himself. Some men will readily "yield to the first depressing sensations of illness; whilst others "will toil on, though suffering both in mind and body."*

Notwithstanding this inherent vagueness in the phenomenons designated by the technical expression of "Sickness,"—which in all probability accounts in a great measure for the enormous difference in the results of the experience of different Societies,—it will generally be found that in each different collection of facts the sickness experienced, like the mortality, bears a definite relation to the age. Nor is there any thing in this contrary to what might have been predicted from our knowledge of the laws of statistical science. If no physiological changes were produced by time in the organisation of the human body, so that the natural predisposition to sickness remained always the same, we should certainly expect to find, in a large collection of facts, the total amount of experienced sickness distributed pretty equally over all ages,—simply, from the absence of any cause or reason why one age should have a larger share than another. In other words, the sickness experienced at each age would oscillate (within limits more or less wide) about a certain constant quantity—such constant quantity being the average of the whole. But this *hypothetical* case differs in no essential respect from the *actual*. Instead of fluctuating about a constant quantity, the *actual* experience fluctuates about a quantity which varies with the age, and which may be expressed by a function (more or less complicated) of the age. The determination of this variable quantity is no doubt a much more difficult operation than simply striking an average. Nevertheless, we may be quite certain that the relation in question exists,—and the discovery of the form of the function by which it is expressed is the object of an investigation of the mathematical law of sickness.

* "On the Uniform Action of the Human Will, as exhibited by its Mean Result in Social Statistics." By Samuel Brown, Esq. *Assurance Magazine*, vol. ii page 341.

As Sterne says, "An illustration is no argument;—neither "is the wiping a glass clean a syllogism;—but we may all see the "better for it." We have an interesting instance of the principle referred to in the foregoing remarks in the following verifications of the celebrated Petersburg Problem given by Professor De Morgan

Throw at which Head first occurred.	Results of Experiment.				Theory.
	1.	2.	3.	4.	
1	1061	1048	1017	1039	1024
2	494	507	547	480	512
3	232	248	235	267	256
4	137	99	118	126	128
5	56	71	72	67	64
6	29	38	32	33	32
7	25	17	10	19	16
8	8	9	9	10	8
9	6	5	3	3	4
10	..	3	2	4	2
11	..	1	1	..	1
12	..	..	1	..	
13	..	..	..	..	
14	..	1	..	..	1
15	..	..	..	..	
16	..	1	1	..	
	2048	2048	2048	2048	2048

in vol. x p. 251 of the *Journal of the Institute,*—the results of which may be compared to a series of observations of the yearly decrements in a body consisting of 2048 individuals subject to a constant force of mortality. A coin is thrown until head appears. Let this succession of throws, be it H, or TH, or TTH, and so on, be called a *set.* The Table shows the results observed by four different persons who each threw 2048 sets. Now let us suppose that the true law of the progression in this case is, *a priori,* unknown, and that the facts observed are given as the data for calculating the probabilities of a fifth trial. A profound mathematician and philosopher, like Demoivre or Gompertz, would probably be satisfied that the true series is a geometrical progression, the ratio of which is $\frac{1}{2}$,—and would know that this assumption would afford a better basis for calculating the probabilities of a fifth trial than any one of the four series, or than the mean result of the whole; while others again, and probably the great majority, would feel more confidence in keeping strictly (as they would say) to the *facts,* instead of relying on a *mere* hypothesis. I see symptoms in Dr. Thiele's able contribution on the law of mortality in the last number of the *Journal* that this matter will in future have to be

dealt with by a more scientific mode of treatment than it has hitherto met with in this country.*

Another illustration of this important matter may be borrowed from the doctrine of Political Economy,—In defining the term "Natural Value," Mr. J. S. Mill describes it as "the point about "which the value oscillates and to which it always tends to return; "the centre value towards which, as Adam Smith expresses it, the "market value of a thing is constantly gravitating; and any devi-"ation from which is only a temporary irregularity . . . . On "an average of years sufficient to enable the oscillations on one "side of the central line to be compensated by those on the other "the market value agrees with the natural value; but it very seldom "coincides exactly with it at any particular time. The sea every-"where tends to a level; but it never is at an exact level; its "surface is always ruffled by waves, and often agitated by storms. "It is enough that no point at least in the open sea is permanently "higher than another. Each place is alternately elevated and "depressed, but the ocean preserves its level."

I cannot conclude these preliminary observations without referring to Mr. Woolhouse's admirable "New Method of adjusting Mortality Tables"—the utility of which in investigations of this nature can scarcely be over-estimated. Those who have devoted much attention to these matters must have realized the difficulty of deducing trustworthy conclusions from the rough data obtained from observation. The method of grouping together a certain number of terms will be found to give very different results by varying the mode of grouping. It was for this reason that in my paper "On the Principles to be observed in the construction of Mortality Tables," I expressed a preference for the late Mr. Finlaison's method of adjusting his observations on annuitant life, —a method which, altho' devoid of any scientific basis, yet had the merit of avoiding the disadvantage referred to. What Mr. Finlaison attempted to accomplish by a sort of "rule of thumb" has been thoroughly and scientifically attained by Mr. Woolhouse's ingenious contrivance; and we shall now be able to trace the true results of any series of observations with a degree of accuracy hitherto unattainable. We shall now be able to measure the exact force of those disturbing elements which present themselves more or less in all such observations, and from a comparison of different results to deduce satisfactory conclusions respecting them. As I do not

* It is too often forgotten, at least by English actuaries, that all Tables of Mortality are necessarily "hypothetical" when applied to determine the future mortality in a constantly changing body of individuals brought together like the members of an Assurance Society.

propose to do more on this occasion than break ground as it were in my subject, I shall not have in this paper to avail myself of Mr. Woolhouse's method, but I have thought it proper not to omit to record here my high appreciation of its merits.

In Mr. A. G. Finlaison's First Report on the Experience of Friendly Societies we have a voluminous mass of observations on sickness and mortality, from which I have deduced the particulars contained in the following Table. They comprise all the information given in the Table on page 2 of Mr. F.'s Report, but arranged in the form which the doctrine of Life Contingencies has rendered familiar to us. The contents of the several columns will be understood from the following explanation of the notation used:—

L_x = Number living on the usual form of Mortality Table.

S_x = Number sick of the L_x living at age x.

$H_x = L_x - S_x$. This will represent the number of "healthy" persons if S_x include sickness of all kinds.*

I_x = Number invalided during the year between the ages x and $x+1$.

[*Note.*—Throughout this paper the terms "invalided" and "invalidism" are used to denote the act of declaring on the sick list.]

D_x = Yearly decrements of Column L_x.

D'_x = " " H_x.

$R_x = I_x - D'_x$ = number of recoveries during the year if S_x include all kinds of sickness, or if the deaths take place only among the body denoted by S_x.

Thus the ordinary mortality table is split up (so to speak) into two sections, distinguished respectively by the symbols H_x and S_x, so that L_x being the number living at age x, we have (for all values of x) $L_x = H_x + S_x$. The symbol S_x denotes the number actually on the sick list *at the time of completing their xth year of age,*—while H_x represents the number *not* on the sick list at that time. Again, the column headed I_x contains the number of those who, *during the year following the completion of age x,* pass from column H_x into column S_x, while R_x on the other hand is intended to show how many pass during the same period from column S_x back again into column H_x. The columns H_x and S_x, in short, may be considered as two distinct mortality tables, between which (that is between the individuals composing which) two streams of inter-migration are continually in action; while I_x and R_x respectively show the number of migrations which annually take place.

* S_x is the number of persons sick at the exact age x, of whom some die in the year, and others recover, their places being taken by others of the healthy, some of whom, again, die, and so on, in the course of the $(x+1)$th year.

x	L_x	H_x	S_x	I_x	R_x	D_x	D'_x
20	6253·5	6134·2	119·3				
21	6201·8	6084·4	117·4	1496·7	1451·0	45.8	45·7
22	6156·0	6038·7	117·3	1454·9	1409·4	47·6	45·5
23	6108·4	5993·3	115·1	1410·2	1364·8	45·4	45·4
24	6063·0	5947·9	115·1	1380·4	1340·2	42·5	40·2
25	6020·5	5907·7	112·8	1339·2	1289·8	42·4	40·4
26	5978·1	5867·3	110·8	1289·0	1248·3	40·2	40·7
27	5937·9	5825·6	111·3	1255·3	1211·2	44·0	44·1
28	5893·9	5781·5	111·4	1252·0	1205·9	44·2	46·1
29	5849·7	5736·3	113·4	1221·3	1176·0	45·4	45·2
30	5804·3	5691·2	113·1	1199·3	1144·2	49·4	55·1
31	5754·9	5646·0	108·9	1182·2	1140·6	44·7	41·6
32	5710·2	5604·4	105·8	1144·0	1106·2	39·6	37·8
33	5670·6	5566·6	104·1	1136·0	1088·0	47·4	48·0
34	5623·2	5518·6	104·6	1144·2	1094·7	46·3	49·5
35	5576·9	5469·1	107·8	1152·2	1101·6	47·3	50·6
36	5529·6	5418·5	111·1	1139·4	1094·0	43·2	45·5
37	5486·4	5373·1	113·3	1130·2	1078·4	48·4	51·8
38	5438·0	5321·3	116·7	1140·3	1085·3	52·8	55·0
39	5385 2	5266·3	118·9	1125·2	1071·6	53·8	53·6
40	5331·4	5112·7	118·7	1116·5	1055·8	61·0	60·7
41	5270·4	5152·0	118·4	1323·5	1067·2	53·2	56·3
42	5217·2	5095·7	121·5	1099·0	1040·1	54·0	58·9
43	5162·9	5036·8	126·1	1102·3	1043·9	58·3	58·4
44	5104·6	4978·4	126·2	1085·6	1026·9	60·3	58·7
45	5044·3	4919·7	124·6	1071·4	1007·7	60·7	63·7
46	4983·6	4856·0	127·6	1093·1	1025·3	60·7	67·9
47	4922·9	4788·2	134·7	1090·1	1021·1	65·9	69·0
48	4857·0	4719·1	137·9	1090·3	1019·9	64·9	70·4
49	4792·1	4648·7	143·4	1101·3	1027·0	70·4	74·3
50	4721·7	4574·4	147·3	1072·3	999·2	70·1	73·1
51	4651·6	4501·3	150·3	1084·6	1007·3	70·3	77·3
52	4581·3	4424·0	157·3	1061·2	979·0	80·2	82·2
53	4501·1	4341·8	159·3	1068·3	990·8	73·9	77·5
54	4427·2	4264·3	162·9	1083·8	1000·4	82·0	83·4
55	4345·2	4180·9	164·3	1061·9	976·1	85·9	85 8
56	4259·3	4095·1	164·2	1033·2	924·8	101·2	108·4
57	4158·1	3986·7	171·4	1032·6	924·5	99·8	108·2
58	4058·3	3878·5	179·8	1025·7	911·3	110·9	114·4
59	3947·4	3764·1	183·3	1012·3	910·2	94·3	102·1
60	3853·1	3662·0	191·1	1021·6	901·0	106·3	120·6
61	3746·8	3541·4	205·4	994·5	893·9	87·8	100·6
62	3659·0	3440·8	218·2	978·8	864·5	103·7	114·3
63	3555·3	3326·5	228·8	932·5	810·8	116·1	121·7
64	3439·2	3204·8	234·4	930·4	811·5	112·5	118·9
65	3326·7	3085·9	240·8	940·1	808·1	114·6	132·0
66	3212·1	2953·9	258·2	950·4	794·7	142·9	155·7
67	3069·2	2798·2	271·0	913·0	766·5	133·2	146·5
68	2936·0	2651·7	284·3	831·8	687·2	134·4	144·5
69	2801·6	2507·1	294·5	847·4	679·9	159·1	167·5
70	2642·5	2339·6	302·9	802·5	624·0	145·5	178·5
71	2497·0	2161·1	335·9	755·2	572·2	155 9	183·0
72	2341·1	1978·1	363·0	754·2	601·2	148·2	153·0
73	2192·9	1825·1	367·8	675·8	545·5	137.0	130·3
74	2055·9	1694·8	361·1	637·6	489·5	150·7	148·1
75	1905·2	1546·7	358·5	586·4	441·7	148·9	144·6
76	1756·3	1402·0	354·3	545·2	397·3	163·2	148·0
77	1593·1	1254·0	339·1	505·7	368 6	133·5	137·1
78	1459·6	1116·9	342·7	453·4	281·2	174·0	172·2
79	1285·6	944·8	340·8	366·6	258·7	129·0	107·8
80	1156·6	836·9	319·7	345·3	212·3	156·6	133·1
81	1000·0	703·9	296·1				

The average duration or "expectation" of sickness during the year to an individual alive at the commencement of the year will be

$$\frac{1}{L_x}\int_x^{x+1} S_x dx = \frac{S_{x+\frac{1}{2}}}{L_x} \text{ (nearly).*}$$

Calling this σ_x, we have $L_x . \sigma_x = S_{x+\frac{1}{2}}$. Mr. F. gives the value of σ_x for each age, expressed in weeks, from which I have computed the values of $S_{x+\frac{1}{2}}$ and thence have deduced the values of S_x by taking it $=\frac{1}{2}(S_{x-\frac{1}{2}}+S_{x+\frac{1}{2}})$. The values of H_x are then found from the equation $H_x + S_x = L_x$.

Mr. Finlaison has a column showing the "number of persons returned as sick in the year at each age." These will consist of (1) the number who begin the year on the sick list, and (2) the number invalided in the course of the year. Consequently, dividing by the "number of persons returned as liable to sickness" we get the ratio $\frac{S_x+I_x}{L_x}$. Hence, equating and solving for I_x we have the process by means of which the last-mentioned function is calculated. Finally, the column R_x is deduced from the equation $H_{x+1} = H_x - I_x + R_x$.

The last equation is true only upon the assumption that the deaths at each age occur exclusively among the lives comprised in S_x,—in other words, that S_x includes all sickness which terminates fatally, however limited or however extended may be its duration. In Mr. Finlaison's Tables this, I believe, is by no means the case. Hence, the numbers contained in the Column R_x cannot be correct, and they are given merely to complete the scheme which I propose should be adopted for setting forth the elementary facts relating to the theory of Sickness and Invalidism.

The Column S_x affords the means of computing the values of allowances during sickness upon precisely the same principles as ordinary life annuities—the "continuous" system being substituted for the more usual plan of annual intervals. It is, however, more especially with reference to the investigation of the law of sickness, to which subject I now proceed, that this mode of treatment has been adopted.

It has, I believe, been noticed by more than one writer that the average yearly sickness at each age exhibits in many observations a strong tendency to a constant ratio to the annual deaths,—and, I believe, the premiums for sickness allowances are sometimes calculated upon the hypothesis of such constant ratio. The law in

* For the reason already stated I have not thought it necessary to resort to any very elaborate processes for determining these values.

question, if true, would lead to a very curious result which, I think, has never, hitherto, been pointed out,—viz., that the total expectation of sickness is a constant quantity at all ages. For the total expectation in question is

$$\frac{L_x\sigma_x + L_{x+1}\sigma_{x+1} + \ldots\ldots}{L_x};$$

and putting $L_x\sigma_x = mD_x$ (D_x denoting the yearly decrement of life) where m is a constant expressing the constant ratio supposed to obtain, the series becomes,

$$m\frac{D_x + D_{x+1} + D_{x+2} + \ldots\ldots}{L_x} = m.$$

This result of course shows that theoretically the hypothesis cannot be true,—for necessarily the expectation of sickness can never exceed the expectation of life, and the latter quantity diminishes without limit as the age increases.* Nevertheless, it must follow that in the observations which exhibit a tendency to the supposed law, the total expectation of sickness during a great portion of life is nearly constant.

If we examine the series D'_x, which consists of the yearly decrements of the series H_x, we shall find that, like the decrements of life, after diminishing for a few terms it then forms an increasing series with a progressive rate of increase until, somewhere about the age of 70, it attains a maximum; after which it rapidly becomes insignificant. From this it is probable that the form of function which expresses the series L_x will, with a suitable modification of the constants, be found also to represent the series H_x. In other words, that the decrements of life and the decrements of health follow the same law.

To put this supposition to the test, I will take the five terms of H_x corresponding to ages 20, 35, 50, 65, and 80. The second differences of the logarithms of these five terms should (if the hypothesis be true) show some approach to a geometrical progression.

x	$\log H_x$	Δ_x	Δ^2_x
20	3·78776		
		− 4984	
35	3·73792		− 2775
		− 7759	
50	3·66033		− 9336
		− 17095	
65	3·48938		− 39576
		− 56671	
80	2·92267		

* A similar objection applies to Gompertz's hypothesis of sickness, viz., $\log \sigma_x = a + b.c^x$ (See *Journal*, vol. xvi p. 342). After a certain extreme age the average number of weeks' sickness in the year will exceed the number of weeks in the year.

Adding and subtracting alternately (in $\log H_x$) the quantity p determined from the equation

$$4p = \frac{(\Delta^2_{35})^2 - \Delta^2_{20} \times \Delta^2_{50}}{\Delta^2_{20} + 2\Delta^2_{35} + \Delta^2_{50}}$$

we get a pure geometrical series in the second order of differences;

x	$\log H_x$	Δ_x	Δ^2_x	$\log(-\Delta^2_x)$	$\Delta(\log - \Delta^2_x)$
20	3·78869				
		− 5170			
35	3·73699		− 2403	3·3808	
		− 7573			+·6063
50	3·66126		− 9708	3·9871	
		− 17281			+·6062
65	3·48845		− 39204	4·5933	
		− 56485			
80	2·92360				

Let us now see to what extent the original figures have been modified in the process:

x	H_x		Difference.	Yearly Decrement.
	Original.	Modified.		
20	6134	6147	+13	50
35	5469	5457	−12	51
50	4574	4584	+10	73
65	3086	3079	− 7	132
80	837	839	+ 2	133

It is a disadvantage of the method employed that the greatest differences occur at that part of the table where the decrements are smallest. By adopting a different method much more satisfactory results might be obtained. Seeing however that the greatest alteration scarcely exceeds one-fourth of the corresponding yearly decrement, I do not think it necessary to attempt a closer adjustment, considering the nature of the materials in hand.

It appears then that the connection between the laws of sickness and mortality is a very simple and obvious one. We have now to investigate the series denoted by I_x which represents the number invalided at each year of age. The following is the form of the preparatory table required for calculations connected with this branch of the subject:—

x	K_x	I_x	x	K_x	I_x	x	K_x	I_x
21	6265·3	149·7	41	3786·3	112·4	61	1644·8	99·5
22	6106·6	145·5	42	3673·9	109·9	62	1545·3	97·9
23	5961·1	141·0	43	3564·0	110·2	63	1447·4	93·2
24	5820·1	138·1	44	3453·8	108·6	64	1354·2	93·0
25	5682·0	133·0	45	3345·2	107·2	65	1261·2	94·0
26	5549·0	128·9	46	3238·0	109·3	66	1167·2	95·0
27	5420·1	125·6	47	3128·7	109·0	67	1072·2	91·3
28	5294·5	125·2	48	3019·7	109·0	68	980·9	83·2
29	5169·3	122·1	49	2910·7	110·1	69	897·7	84·7
30	5047·2	120·0	50	2800·6	107·2	70	813·0	80·3
31	4927·2	118·2	51	2693·4	108·5	71	732·7	75·5
32	4809·0	114·4	52	2584·9	106·1	72	657·2	75·4
33	4694·6	113·6	53	2478·8	106·8	73	581·8	67·6
34	4581·0	114·4	54	2372·0	108·4	74	514·2	63·8
35	4466·6	115·2	55	2263·6	106·2	75	450·4	58·6
36	4351·4	113·9	56	2157·4	103·3	76	391·8	54·5
37	4237·5	113·0	57	2054·1	103·3	77	337·3	50·6
38	4124·5	114·0	58	1950·8	102·6	78	286·7	45·3
39	4010·5	112·5	59	1848·2	101·2	79	241·4	36·7
40	3898·0	111·7	60	1747·0	102·2	80	104·7	34·5
41	3786·3		61	1644·8		81	170·2	

The column K_x denotes the sum of I_x from age x to the end of life. Hence $K_x - K_{x+n}$ gives the number of invalidisms between age x and age $x+n$, and dividing by L_x we get the average number to each person living, which might be called the expectation of invalidism during the next n years. If we construct a column of the values of $K_x v^x$ (which corresponds with the ordinary D column of the Annuity Table) we have the means of computing the value of an allowance of £1 for each invalidism to which a given individual may be subject.

The series I_x does not at first sight exhibit any striking similarity to the series representing the decrements of life,—nevertheless, we shall have reason to conclude that the two series really follow the same law;—that is to say, that they are each the differences of a function of the form $ab^{q^x}.s^x$. I have accordingly first calculated the value of K_{81} (or the sum of I_x from 81 upwards) by assuming the law in question, and now proceed to show that the complete series thus obtained is of such a character as to justify the assumption.

$\log K_{21}=3·79632$		
	-12472	
$\log K_{33}=3·67160$		-2246
	-14718	
$\log K_{45}=3·52442$		-6462
	-21180	
$\log K_{57}=3·31262$		-14769
	-35949	
$\log K_{69}=3·95313$		-36268
	-72217	
$\log K_{81}=2·23096$		

Modifying the series according to the method explained in my

paper "On the Law of Mortality" (*Journal*, vol. xiii p. 356) we get the following result:

$$\begin{array}{llllll}
\log K_{21}=3{\cdot}79544 & & & & & \\
 & -12295 & & & & \\
\log K_{33}=3{\cdot}67249 & & -2569 & & \log=3{\cdot}40976 & \\
 & -14864 & & & & +{\cdot}38270 \\
\log K_{45}=3{\cdot}52385 & & -6201 & & \log=3{\cdot}79246 & \\
 & -21065 & & & & +{\cdot}38270 \\
\log K_{57}=3{\cdot}31320 & & -14968 & & \log=4{\cdot}17516 & \\
 & -36033 & & & & +{\cdot}38272 \\
\log K_{69}=2{\cdot}95287 & & -36131 & & \log=4{\cdot}55788 & \\
 & -72164 & & & & \\
\log K_{81}=2{\cdot}23123 & & & & &
\end{array}$$

To show to what extent it has been necessary to alter the original figures in order to obtain the required result, I give the following comparison:

x	$K_x - K_{x+12}$	
	Original.	Modified.
21	1562	1540
33	1349	1363
45	1291	1284
57	1156	1160
69	727	727

That is to say, if the terms of the first series (showing the number of invalidisms actually observed during intervals of 12 years each) be modified to the extent shown by the second series,—by which the latter is brought into conformity with the required law,—and if the series be then completed according to the same law, we shall obtain by summation a series K_x, which is a function of the form $abq^x s^x$.

The number of observations included in Mr. Finlaison's enquiry amounts to 800,000,—*i.e.*, to two-thirds of the number of observations in the recent investigation of the "Experience" Mortality (H^M). I therefore rely with some degree of confidence upon the conclusion which, I think, is fairly deducible from the preceding examination of Mr. Finlaison's results, viz., that the computation of allowances for sickness and invalidism may be reduced to the same form as the calculation of life annuities and reversionary sums, and that a single table of the well-known integral $\int_v^\infty e^{-v}v^{n-1}dv$ will suffice for all practical purposes to furnish us with the whole of the elementary values required in the application of actuarial science to the business of Life Assurance and Friendly Societies.

In conclusion, I have to state that not only may the law repre-

sented by the function $ab^{q^x}s^x$ be extended to the whole period of life, as I have already shown in a paper published in the last number of the *Journal of the Institute*, but it may be modified so as to adapt itself to accidental departures from the law, and thus follow the results of observation with any degree of approximation that may be thought necessary. And this too without surrendering those peculiar facilities in calculation which form the distinguishing property of the function, and of no other function that can possibly be devised. As before stated, however, I confine myself on this occasion to an examination of the broad features of my subject, leaving these and other details for another opportunity.

Life Insurance in the Time of Queen Elizabeth.

To the Editor of the "Journal of the Institute of Actuaries."

Sir,—Mr. J. O. Halliwell, the well-known antiquary, communicated to me that in the course of some researches at the British Museum he had discovered a manuscript having reference to some legal proceedings taken in respect of a life policy granted in the days of Queen Elizabeth.

I have had a copy made of the manuscript and now send it to you, thinking that it may be of sufficient interest for insertion in the *Journal.*

Yours truly,
ROBERT TUCKER.

Lombard Street,
December 1871.

British Museum. MSS. Lansdowne, No. 170, *fo.* 123.
(*Cæsar Collections. Admiralty, &c.*)

An order given by m^{r} Docter Dale and m^{r} Docter Cæsar, Judges of the Admiraltie the viijth daye of March in the xxvijth yere of the raigne of o^{r} souaigne Ladye Elizabeth by the grace of god Queene of England ffraunce and Ireland Defender of the faith, &c.

Whereas the ll^{s} of her ma^{tes} most honõrable privy Counsell haue com̄itted vnto vs Valentine Dale and Julius Cæsar Docters of lawe the hearing and ordering of a matter in variance betwen Richard Martin Citizen and Alderman of London complainant on thone side, And John Barker, Leonard Holidaye, William Browne, John Castelin & Anthony Marlo^{r}, Henry Cletherow, Edmond Hogan, John Stokes, Henry Colthirst and Nicholas Style, John Newman,

Symon Lawrenc and Oliu͛ Stile, Parnell Towerson widowe executrix of William Towcrson deceased, William Becher and Rob̃rt Brooke defendantes on thother side, for and concerning an afsurance made by the said defendantes vpon the life of William Gybbons deceased; vpon long debating of the said matter and deliberat hearing of the learned Counsell of bothe the said ꝑties, It appeared that the said defendantes did by theire writing subscribed w^th theire owne handes bearing date the 18 daie of June in the yeare of our Lord god 1583 make a certeine contract of an assurance w^th the said Complainant named amongest m͛chantes a policy, the tenor whereof ensueth. In the name of god amen. Be it knowne vnto all men by theise p̃sentes that Richard Martin Citizen and Alderman of London doth make afsurance and causeth himself to be afsured vpon the naturall life of William Gybbons Citizen and salter of London, for and during the space of xij monethes next ensuinge after the vnderwriting hearof by the assurers heareafter subscribed fullie to be complete and ended. The w^ch assurance wee the ꝑsons heareafter named m͛chantes of this Citie of London for and in consideraco͞n of certeine currant money of England by vs received at the subscribing hereof, of the said Richard Martin after the rate of viij^li sterling ꝑ cent (whereof we acknowledg ourselves and everie of vs by these p̃sentes trulie satisfied & paid) do take vpon vs to beare. And we do assuere by thiese p̃sentes that the said William Gybbons (by what addico͞n so eu͛ he is or shalbe named or called) shall by Gods grace contynue in this his naturall lief for & during the space of xij monethes next ensuing after the vnderwriting heareof by everie of vs the assurers, or in default thereof everie of vs to satisfye content & paie or cause to be satisfied contented and payd vnto the said Richard Martin his executors administrators or afsignes, all such seu͛all sum̃es of money as we the afsurers shall hereafter seu͛ally subscribe, promising and binding vs eche one for his owne part, our heiers executors & administrators by these p̃sentes, That if it hapen (as god defend) the said William Gibbons to dye or decase out of this p̃sent world by any wayes or meanes whatsoevere before the full end of the said xij monethes be expired, that then we our heiers, executors or assignes w^thin two monethes next after true intimaco͞n thereof be to vs our heires executors or administrators lawfullye given, shall well and truly content and pay or cause to be contented and paied vnto the said Richard Martin his executors administrators or assigns all such

sum̃e and sum̃es of money as by vs th'assurers shalbe heareafter seũally subscribed wthout any further delaye: It is to be vnderstanded that this p̃sent writing is and shall bee of asmuch force, strength and effect, as the best and most surest pollicy or writing of assurance w^{ch} hath bene eũ heretofore vsed to be made upon̄ the life of any ꝑson in Lumbard Street, or nowe wthin the Roiall Exchange in London. And so the afsurers be contented and doe ꝓmise and binde themselues and everie of them theire heiers executors and administrators by these p̃sentes to th'afsured his executors admĩnistrators and afsigns for the true ꝑformance of the p̃mifses according to the vse and custome of the said street or Royall exchange: And in testimony of the truth the afsurers have hearevnto seũally subscribed theire names and sum̃s of money afsured. God send the said William Gibbons helth and long lief. yeven in the office of afsurance wthin the Royall Exchange aforesaid the xviijth day of June 1583. Accordinge to w^{ch} polycy, John Barker hath bound himselfe to paie 50^{li}. Leonard Holydaye to paie 25^{li} Willm̃ Browne to paie 25^{li} John Castelin and Anthony Marlor to paie 25^{li} Henry Clitherowe to paie xxv^{li}, Edmund Hogan to paie 33^{li} 6^{s}. 8^{d}. John Stokes to paie 33^{li} 6^{s}. 8^{d}. Henry Colthirst and Nich̃as Stile to paie 25^{li}. John Newmā to paie 25^{li} Symon Lawrence and Oliuer Stile to paie 25^{li} the said executrix of Wilm̃ Towerson deceased to paie 33^{li} 6^{s} 8^{d}. Wilm̃ Becher to paie 25^{li} and Robert Brooke to paie 33^{li} 6^{s} 8^{d}: And that the said assurers in consideracõn of the said afsurance did receave of the said Comp^{t} diu͛s som̃es of money after the rate of viij vpon the hundred. And that the said Wilm̃ Gibbons did come to his death vpon the xxixth day of May next after the making of the said contract and policy, whereof intimacõn was given by the officers appointed therevnto to eche of the said afsurers seũally for the paiment of the said seũall som̃es w^{ch} eũy one of them had bound them selfes vnder his hand to afsuere to the said Alderman Martin in the said polleey and was to paie wthin two monethes after intimacõn so given, the w^{ch} intimacõn was given as followeth, to John Barker the 20 of August 1584 at noone exchange, to Leonard Holydaye the 16 of August 1584 at noone exchange, to William Browne the 17 of August 1584 at noone exchange, to John Casteline and Anthony Marler the xij of August 1584 at night exchange, to Henry Clitherowe the 11 of August 1584 at noone exchange, to Edmond Hogan the 31 of October 1584 at noone exchange, to John Stoks the 11 of

August 1584 at noone exchange, to Henry Colthirst and Nicholas Stile the 8 of August 1584 at 2 of the clocke in the afternoone, to Symon Lawrence and Oliũ Style the 22 of August 1584 at noone exchange, to Parnel th'executrix of Willm Towerson deceased the 7 of September 1584, by Willm Stone her servant, to William Becher the 13 of August 1584 at noone exchange, to Robrt Brooke the 11 of August 1584 at noone exchange. The whiche defendantes haue refused to paie the said somes of money and haue alleadged that William Gybbons did live full 12 monethes accompting 28 daies to euy of the same monethes whereby there should be nothing due vnto the said Compt by vertue of the said contract. And although by the lawes of the Realme in cõtractes made for matters done and happened wthin this Realme betwene man and man being not mchantes nor made after the mãner or vsage of mchantes the moneth is taken to be accompted after the rate of 28 daies for the moneth. Yet notwthstanding forasmuch as by the genall lawes vsed amongest all Christian nations and by comon vsage it is vndoubtedly taken that the moneth is to be accompted after the rate of 30 daies vnto the moneth. And forasmuch also that aswell diũs lerned in the Civill lawe as also the Lo. maior of London and the Aldermen his brethren and suck likewise as are specially deputed and appointed by vertue of an order made by the L. maior of the Citie of London and th'aldermen, and ratified by aucthority of her mates Counsell Comissions for the hearing and ending of all matters of assurance and diũs others as well notaries as mchantes Englishmen and strangers of the best and most skilfull sort haue declared theire opinions that by the contract aforesaid made according to the custome and vsage of Lumbard street and the Riall exchange, the moneth is to be accompted according vnto the course of the Kalender and not after 28 daies to the moneth. And forasmuch also as Richard Candeler the Clerke of the assurances by whome the said contract was drawne and penned did affirme vpon his othe that the meaning of the said contract was that the assurers should be bound for a whole yeare. And further also the said complainant hath offered him selfe to take his othe vpon the holie evangelist, that his meaning was at the tyme of the said contract to accompt the said 12 monethes for one whole yere and offereth the like othe vnto the said defendantes, wch they of their part would not accept; a thinge in determinacõn of Lawe declared to be *manifestæ turpitudinis, nec velle*

iurare, nec iuramentum deferre. And forasmuch also as the said defendantes doe take these two monethes, wch they haue given vnto them by the said policye for the paiment of the said seuall somes wch they haue assured, to be accompted after the course of the Kalender, and not after 28 daies to the moneth. And forasmuch as also John Stokes and Henry Clitherow haue in an acquitance of bill of receipt written wth theire owne handes bearing date the 6 of March 1583 conserned and vnderstood twelve monethes mentioned in an instriment of afsurance or pollicy vpon the life of the said William Gibbons bearing date the 24 of february 1583, for one whole full and complet yeare. It is therefore the daie and yeare abouesaid ordered by vs the said Valentine Dale & Julius Cæsar, Docters of Lawe, That the said defendantes and euy of them shall before the feast of Thañunciation of the Virgin Mary nex ensuing, paie vnto the said complainant the seuall somes of money specified in the said seuall subscriptions vnto the said pollicy together wth the consideracōn for the forbearing of the same since the time of the said intimacōn according to the vsuall rate accustomed amongst mchantes.

This order being sett downe by the Judges of the Admiraltie was confirmed and ratified by the lls and others of her mates most honorable privye counsell the xiiijth of March 1587 at Grenwich, and by them comaunded to be entered into the Register of Counsell to remayne of Record to the benefitt of the pties therein mentioned, and to be hereafter kept & observed in like cases of assurance.

pnt₃

THERLE OF SHREWSBURY, L: STEWARD,
THERLE OF BEDFORD, L: CHAMBERLAIN,
L: HUNSDON,
Mr. THRĒR,
Mr. COMPTROLLER,
Sr HENRY SIDNEY,
Mr. VICECHAMBERLAINE,
Mr. SECRITARY,

} *of the Privy Counsell.*

JOH: WILKES, *one of ye Clerks of ye Counsell.*

CORRESPONDENCE.

ON THE ADJUSTMENT OF MORTALITY TABLES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The important problem of adjusting Mortality Tables has lately been discussed by Dr. Kanner, and I hope the following statement of his theory will not be devoid of interest to the readers of this periodical. The publication of this speculation was the last work of Dr. Kanner, who suddenly died of small-pox in the beginning of this month at Berlin, in the midst of his assiduous life.

Dr. Kanner introduced his theme by the following problem (published in No. 16 of the *Annalen des gesammten Versicherungswesens*). Assuming that it has been observed that out of 994 persons of a given age 4 have died in the course of a year, and that out of 3012 persons one year older, 72 have died in the course of a year: further, that we have reason to presume that the chance of dying in a year at the higher age exceeds that at the lower, by ·01: what value must be attributed to the chance of dying in a year at the younger age in order to make the mortality really observed at the two ages the most probable? Or, in other words, what is the most probable mortality for each of the two ages, to be deduced from the observations, under the condition that the probability of dying in a year is ·01 greater at the higher age than at the younger?

Let x_0 be the probability of dying in a year at the younger age, then $x_0+{\cdot}01$ is the same probability at the higher age; and according to well known rules of the theory of probabilities, the probability of the hypothesis that the mortality of the younger age is x_0 becomes proportional to

$$\frac{(1-x_0)^{990}x_0^{4}\{1-(x_0+{\cdot}01)\}^{2940}(x_0+{\cdot}01)^{72}}{\int_0^1(1-x)^{990}x^4dx\int_0^1(1-x)^{2940}x^{72}dx}.$$

This becomes a maximum when the numerator is a maximum; and then we have

$$\frac{d}{dx_0}\{(1-x_0)^{990}x_0^{4}({\cdot}99-x_0)^{2940}(x_0+{\cdot}01)^{72}\}=0$$

or,

$$-\frac{990}{1-x_0}+\frac{4}{x_0}-\frac{2940}{{\cdot}99-x_0}+\frac{72}{{\cdot}01+x_0}=0,$$

whence we get

$$4006x_0^3-4032x_0^2+35{\cdot}9594x_0+{\cdot}0396=0.$$

The only real root of this equation of the third degree is $x_0={\cdot}01$;

therefore the rates of mortality,

·01 for the younger age,
·02 for the higher age,

are the sought most probable rates corresponding to the two observations, under the condition that the rate at the higher age exceeds that at the younger by ·01.

In Number 17 of the *Annalen* Dr. Kanner gives the development of

the theory in general, after having pointed out that the absence of a clear perception of the problem had induced mathematicians almost to consider it insoluble. They left the adjustment of a Mortality Table to esthetic taste. What abstract thought was not able to perform, the eye and the feeling were to decide, by the aid of a graphic exhibition of the numbers!

Dr. Kanner conceives the problem in the following way:

Given the numbers of observed persons of different ages and the numbers of them who die in the course of a year;

Required the most probable mortality for each age.

If there are given no other conditions, which the mortalities of the different ages are to satisfy, an *unadjusted* Mortality Table is wanted. If, on the contrary, the rates of mortality at the different ages are to depend on each other and satisfy certain conditions, expressed by equations, the rates are made subject to a law, arbitrary, but answering to experience; and an *adjusted* Table of Mortality is to be computed.

The first of these problems may be considered as a special case of the second.

Let $$m_0, m_1, m_2, \ldots . m_\mu,$$

denote the numbers of persons dying in a year, at $\mu+1$ ages,

$$n_0, n_1, n_2, \ldots . n_\mu,$$

the numbers of persons of the same ages who survive the year, these numbers having been found by careful observation.

Let $$x_0, x_1, x_2, \ldots . x_\mu,$$

denote the rates of mortality at the different ages. It is required to find their most probable values.

Evidently the solution of the problem requires that the product

$$x_0^{m_0}(1-x_0)^{n_0}x_1^{m_1}(1-x_1)^{n_1} \ldots . x_\mu^{m_\mu}(1-x_\mu)^{n_\mu}=\mathrm{V}$$

must be a maximum. This condition is satisfied by putting its partial differential coefficients equal to 0.

Thus
$$\frac{d\mathrm{V}}{dx_0}=0,\ \frac{d\mathrm{V}}{dx_1}=0, \ldots . \frac{d\mathrm{V}}{dx_\mu}=0.$$

In this way, each age being considered separately and independently of the other ages, the most probable rate of mortality at each age is found to be equal to the ratio of the number of persons dying in a year to the number living at the beginning of the year. The observation gave us nothing but the proportion of the persons dying out of a certain number living; and therefore it allows no other conclusion but what is the most probable hypothesis of mortality for each different age. The observation does not include any connection or relationship between the single mortalities $x_0, x_1, \ldots . x_\mu$, and if the *observation* is to be the sole basis of our conclusions, we are obliged to consider the rates of mortality $x_0, x_1, \ldots . x_\mu$, as independent of each other. By solving the above equations,

$$\frac{d\mathrm{V}}{dx_0}=0,\ \frac{d\mathrm{V}}{dx_1}=0, \ldots . \frac{d\mathrm{V}}{dx_\mu}=0,$$

we find

$$x_0 = \frac{m_0}{m_0+n_0},\ x_1 = \frac{m_1}{m_1+n_1}, \ldots\ldots x_\mu = \frac{m_\mu}{m_\mu+n_\mu}.$$

The observation enables us also to state the probability of each of these hypothesises, as well as the probability of any other hypothesis which we may like to make; but there is not the least support for any adjustment, if we intend to proceed on the basis of the observation exclusively. Every adjustment is opposed to the result of observation, and the problem of adjustment cannot be introduced at all until we have established, either tacitly, or explicitly, relations between the unknown rates $x_0, x_1, \ldots\ldots x_\mu$, —conditions which these values are to satisfy, and with which the values $\frac{m_0}{m_0+n_0}, \frac{m_1}{m_1+n_1}, \ldots\ldots \frac{m_\mu}{m_\mu+n_\mu}$, cannot be reconciled. Only then can we raise the question of adjustment, the question of what values we shall select for $x_0, x_1, \ldots\ldots x_\mu$, so as to satisfy the relations in question and at the same time make V a maximum; for this latter condition is the basis which must be maintained, if we wish to find out the most probable hypothesis to be derived from our observations.

If the values which link $x_0, x_1, \ldots\ldots x_\mu$, together are substituted in V, one or more quantities are to be determined in such a way that V shall be a maximum, and thus the problem of adjustment will be solved.

The most perfect way to express the relations between $x_0, x_1, \ldots\ldots x_\mu$ consists in considering x as a function of the age. Thus,

if $$x = f(t, a, b, c \ldots\ldots)$$

where t is the age, and $a, b, c, \ldots\ldots$ are constants to be determined, this may be called the mathematical law of mortality.

If $t_0, t_1, \ldots\ldots t_\mu$ are the ages to which the observations refer, we have to determine $a, b, c, \ldots\ldots$ in the expression

$$\{f(t_0, a, b, c, \ldots\ldots)\}^{m_0}.\{1-f(t_0, a, b, c, \ldots\ldots)\}^{n_0}\ldots\ldots\ldots$$
$$\ldots\ldots\{f(t_\mu, a, b, c, \ldots\ldots)\}^{m_\mu}\{1-f(t_\mu, a, b, c, \ldots\ldots)\}^{n_\mu} = \mathrm{V}$$

so that V shall be a maximum.

For brevity, write

$$\mathrm{V} = f_0^{m_0}(1-f_0)^{n_0} f_1^{m_1}(1-f_1)^{n_1} \ldots\ldots f_\mu^{m_\mu}(1-f_\mu)^{n_\mu}$$

Then forming the partial differential coefficients with respect to $a, b, c, \ldots\ldots$ and putting each $=0$, we have

$$\left.\begin{array}{l}\frac{m_0}{f_0}\cdot\frac{df_0}{da} - \frac{n_0}{1-f_0}\cdot\frac{df_0}{da} + \frac{m_1}{f_1}\cdot\frac{df_1}{da} - \frac{n_1}{1-f_1}\cdot\frac{df_1}{da} + \ldots\ldots - \frac{n_\mu}{1-f_\mu}\cdot\frac{df_\mu}{da} = 0 \\ \frac{m_0}{f_0}\cdot\frac{df_0}{db} - \frac{n_0}{1-f_0}\cdot\frac{df_0}{db} + \frac{m_1}{f_1}\cdot\frac{df_1}{db} - \frac{n_1}{1-f_1}\cdot\frac{df_1}{db} + \ldots\ldots - \frac{n_\mu}{1-f_\mu}\cdot\frac{df_\mu}{db} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{array}\right\} (a)$$

—as many equations as there are constants to be determined.

It is not to be denied that the practical application of this theory offers still a great many difficulties, that the form of the last equations is in general so very complicated that we must almost despair of solving them exactly. But these points, at all events, are of secondary importance. It

may be of importance to find out the way for practical solution of the problem too, but the theory of the question itself had first to be investigated, or else the way which leads to the solution would not have been discernible. In establishing a sound theory, we must never suffer ourselves to contemplate the difficulties that may arise afterwards, when the theory is to be applied for practical purposes; for such a course would darken the accuracy of our conclusions. I see Dr. Kanner's principal merit in his clear proof, that the question of adjustment depends only on the state of relationship which is presumed to exist between the rates of mortality at the different ages, and that the condition that the product V or $x_0^{m_0}(1-x_0)^{n_0} \times x_1^{m_1}(1-x_1)^{n_1} \dots x_\mu^{m_\mu}(1-x_\mu)^{n_\mu}$ is to be a maximum, must be made the basis of our conclusions and computations.

Pursuing his examinations, Dr. Kanner mentions the ineffective efforts which have been made to solve the question of adjusting a mortality table by introducing the method of least squares, a method applicable only where the probability of a deviation from the correct value is the same for a positive deviation as for a negative one of the same magnitude, which is not the case in our question.

The general term of the equations (a) is

$$\frac{m}{f}\cdot\frac{df}{da}-\frac{n}{1-f}\cdot\frac{df}{da},$$

which may be put in the form

$$-\frac{m+n}{2f(1-f)}\,2\left(f-\frac{m}{n+m}\right)\frac{df}{da}.$$

The incorrect application of the method of least squares, in assuming $\frac{(m+n)^3}{2mn}$ to be the weight of the observations, leads us to the form

$$\frac{m+n}{2\cdot\frac{m}{m+n}\cdot\frac{n}{m+n}}\cdot 2\left(f-\frac{m}{n+m}\right)\frac{df}{da}.$$

Although the two forms resemble each other, their difference is remarkable. If the weight of the observation could be made equal to $\frac{m+n}{2f(1-f)}$, the forms would become identical; but I doubt very much whether there exists more than a mere resemblance, and whether the method of least squares can be correctly applied to the problem of adjustment. As Dr. Kanner remarks, the weight of the observation, in this form, being dependent on the correct values to be found, cannot be determined before the solution of the problem is given.

I am, Sir,

Yours faithfully,

Hamburg, 30*th Nov.* 1871. WILHELM LAZARUS.

*** We regret greatly to hear of the death of Dr. Kanner, who has done much to promote sound views in the theory of life contingencies; and whose last production, explained in the foregoing letter, is, altho difficult of practical application, a most valuable contribution to the theory. We consider his premature death a heavy loss to the actuarial profession.—Ed. *J. I. A.*

TABLES DEDUCED FROM THE NEW EXPERIENCE TABLE H^{MF} (ADJUSTED).

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having had, by the kind assistance of Mr. James Stark, junr., some monetary tables computed from the Table H^{MF} of the "*Mortality Experience of Life Assurance Companies collected by the Institute of Actuaries*," I beg to place them at your disposal for publication in the *Journal.*

They are calculated at 3 and 4 per cent. from the table of mortality adjusted by Mr. Woolhouse on Makeham's modification of Gompertz's theory, the formation of which table he has fully explained in the *Journal*, vol. xv pp. 404–408; but I have used the logarithms throughout to five places only, considering them amply sufficient for ordinary official purposes.

Mr. Makeham's formula expressing the law of mortality is

$$\log l_x = \log k - x \log a + q^x \log g$$

and the constants deduced from the original observations in Table H^{MF} are

$$\log k = 5{\cdot}04119$$
$$\log a = {\cdot}00286$$
$$\log g = {\cdot}00041$$
$$\log q = {\cdot}04$$

The following will show the close approximation to the original facts of the number living at the decennial ages by the adjusted table, and also a comparison of the values of life annuities by the New Experience, by the Experience of 17 Offices, and by the Carlisle Table.

Age.	Table H^{MF} Number Living.		Value of Single Life Annuity 3 per Cent.		
	Original Facts.	Adjusted Table.	By New Experience.	By Experience of 17 Offices.	By Carlisle Table.
20	9554	9581	21·956	21·797	21·694
30	8904	8890	19·912	19·754	10·556
40	8128	8137	17·254	17·123	17·143
50	7183	7198	14·004	13·820	14·303
60	5847	5843	10·368	10·188	10·491
70	3805	3822	6·784	6·685	7·123
80	1411	1454	3·789	3·799	4·365
90	159	142	1·729	1·516	2·499

I remain, Sir,
Yours obediently,

SAMUEL BROWN.

Guardian Assurance Office,
11 *Lombard Street,* 31*st Aug.* 1871.

TABLE OF MORTALITY H^{MF} (adjusted), *showing the Number Living, the Number Dying, and the chance of dying in a Year, for every Age from* 10 *to* 100.

Age x	l_x	d_x	$1-p_x$	Age x	l_x	d_x	$1-p_x$
10	102698	698	·00679	56	64534	1430	·02216
11	102000	695	·00682	57	63104	1493	·02366
12	101305	693	·00684	58	61611	1557	·02528
13	100612	690	·00686	59	60054	1627	·02710
14	99922	688	·00688	60	58427	1696	·02902
15	99234	688	·00693	61	56731	1769	·03119
16	98546	685	·00695	62	54962	1842	·03350
17	97861	685	·00700	63	53120	1916	·03608
18	97176	684	·00704	64	51204	1992	·03890
19	96492	684	·00709	65	49212	2063	·04192
20	95808	684	·00714	66	47149	2136	·04529
21	95124	683	·00718	67	45013	2203	·04894
22	94441	685	·00725	68	42810	2266	·05293
23	93756	686	·00732	69	40544	2322	·05729
24	93070	687	·00739	70	38222	2372	·06205
25	92383	691	·00748	71	35850	2410	·06722
26	91692	692	·00755	72	33440	2437	·07289
27	91000	697	·00766	73	31003	2451	·07904
28	90303	701	·00775	74	28552	2448	·08576
29	89602	704	·00787	75	26104	2429	·09304
30	88898	712	·00800	76	23675	2391	·10098
31	88186	716	·00812	77	21284	2333	·10961
32	87470	726	·00830	78	18951	2254	·11897
33	86744	732	·00844	79	16697	2156	·12910
34	86012	741	·00862	80	14541	2037	·14010
35	85271	753	·00883	81	12504	1901	·15199
36	84518	765	·00905	82	10603	1747	·16484
37	83753	779	·00930	83	8856	1583	·17872
38	82974	793	·00956	84	7273	1408	·19364
39	82181	808	·00983	85	5865	1230	·20972
40	81373	828	·01017	86	4635	1053	·22703
41	80545	845	·01049	87	3582	879	·24541
42	79700	868	·01090	88	2703	717	·26522
43	78832	890	·01129	89	1986	568	·28628
44	77942	917	·01177	90	1418	438	·30871
45	77025	943	·01224	91	980	326	·33247
46	76082	975	·01281	92	654	234	·35758
47	75107	1007	·01340	93	420	161	·38406
48	74100	1042	·01406	94	259	107	·41181
49	73058	1080	·01479	95	152	67	·44081
50	71978	1121	·01558	96	85	40	·47097
51	70857	1164	·01642	97	45	23	·50217
52	69693	1212	·01739	98	22	12	·53427
53	68481	1261	·01841	99	10	5	·56714
54	67220	1315	·01958	100	5	5	1·
55	65905	1371	·02080				

Table H^{MF} (adjusted). *Commutation Table, showing also the Value of the Life Annuity a_x with Interest at 3 per-cent, for every Year of Age from* 10 *to* 100.

Age x	D_x	N_x	M_x	a_x
10	76417·	1793457·	21952·	23·469
11	73687·	1719770·	21448·	23·339
12	71053·	1648717·	20961·	23·204
13	68513·	1580204·	20489·	23·064
14	66060·	1514144·	20033·	22·921
15	63694·	1450450·	19591·	22·772
16	61410·	1389040·	19162·	22·619
17	59208·	1329832·	18748·	22·461
18	57081·	1272751·	18346·	22·297
19	55028·	1217723·	17956·	22·129
20	53047·	1164676·	17577·	21·956
21	51134·	1113542·	17209·	21·777
22	49288·	1064254·	16853·	21·593
23	47505·	1016749·	16506·	21·403
24	45785·	970964·	16169·	21·208
25	44122·	926842·	15841·	21·006
26	42517·	884325·	15521·	20·799
27	40967·	843358·	15209·	20·586
28	39469·	803889·	14904·	20·368
29	38022·	765867·	14607·	20·142
30	36624·	729243·	14317·	19·911
31	35274·	693969·	14032·	19·674
32	33968·	660001·	13754·	19·430
33	32705·	627296·	13480·	19·181
34	31484·	595812·	13212·	18·924
35	30304·	565508·	12949·	18·661
36	29162·	536346·	12689·	18·392
37	28056·	508290·	12433·	18·118
38	26986·	481304·	12180·	17·836
39	25949·	455355·	11930·	17·548
40	24945·	430410·	11682·	17·254
41	23972·	406438·	11436·	16·954
42	23030·	383408·	11192·	16·648
43	22116·	361292·	10948·	16·337
44	21229·	340063·	10706·	16·019
45	20368·	319695·	10464·0	15·696
46	19533·	300162·	10221·9	15·367
47	18721·	281441·	9978·9	15·034
48	17932·	263509·	9735·2	14·695
49	17165·	246344·	9490·4	14·351
50	16419·	229925·	9244·0	14·004
51	15692·	214233·	8995·7	13·652
52	14985·	199248·	8745·4	13·297
53	14296·	184952·	8492·4	12·938
54	13624·	171328·	8236·8	12·576
55	12968·	158360·	7978·0	12·212
56	12328·	146032·	7716·1	11·845
57	11704·	134328·	7450·9	11·474
58	11094·	123234·	7182·1	11·108
59	10499·	112735·	6909·9	10·738
60	9917·0	102818·	6633·7	10·368
61	9348·7	93469·	6354·2	9·998
62	8793·3	84676·	6071·2	9·630
63	8251·1	76425·	5785·1	9·263
64	7721·8	68703·	5496·1	8·897
65	7205·3	61498·	5204·4	8·535
66	6702·1	54796·	4911·1	8·176
67	6212·3	48584·	4616·3	7·821

3 *per-cent Commutation and Annuity Tables*—(continued).

Age x	D_x	N_x	M_x	a_x
68	5736·1	42848·	4321·1	7·470
69	5274·2	37574·	4026·3	7·124
70	4827·3	32747·	3733·0	6·784
71	4395·9	28351·	3442·1	6·449
72	3981·0	24370·	3155·2	6·122
73	3583·3	20787·	2873·5	5·801
74	3204·0	17583·	2598·5	5·488
75	2843·9	14739·	2331·8	5·183
76	2504·1	12235·	2074·9	4·886
77	2185·7	10049·	1829·4	4·598
78	1889·5	8159·9	1596·8	4·319
79	1616·2	6543·7	1378·6	4·049
80	1366·5	5177·2	1176·0	3·789
81	1140·9	4036·3	990·10	3·538
82	939·29	3097·0	821·70	3·297
83	761·61	2335·4	671·45	3·066
84	607·27	1728·1	539·27	2·846
85	475·42	1252·7	425·13	2·635
86	364·77	887·89	328·32	2·434
87	273·74	614·15	247·86	2·244
88	200·54	413·61	182·65	2·062
89	143·07	270·54	131·01	1·891
90	99·136	171·40	91·29	1·729
91	66·535	104·86	61·55	1·576
92	43·121	61·743	40·06	1·432
93	26·895	34·848	25·078	1·296
94	16·083	18·765	15·078	1·167
95	9·1846	9·5803	8·624	1·043
96	4·9864	4·5939	4·700	·921
97	2·5611	2·0328	2·426	·794
98	1·2379	·7949	1·156	·642
99	·5597	·2352	·513	·420
100	·2352	.. ..	·253	·000

TABLE H^{MF} (adjusted). *Values of Annuities on Two Joint Lives of Equal Ages, at 3 and at 4 per-cent Interest.*

Ages.		At 3 per-cent Interest.	At 4 per-cent Interest.	Ages.		At 3 per-cent Interest.	At 4 per-cent Interest.
10	10	19·861	16·924	55	55	8·971	8·349
11	11	19·738	16·843	56	56	8·637	8·056
12	12	19·610	16·758	57	57	8·304	7·763
13	13	19·477	16·669	58	58	7·972	7·469
14	14	19·340	16·576	59	59	7·643	7·176
15	15	19·153	16·479	60	60	7·317	6·885
16	16	19·050	16·378	61	61	6·994	6·595
17	17	18·897	16·273	62	62	6·675	6·307
18	18	18·739	16·163	63	63	6·360	6·021
19	19	18·576	16·049	64	64	6·050	5·740
20	20	18·408	15·930	65	65	5·746	5·463
21	21	18·233	15·806	66	66	5·448	5·189
22	22	18·053	15·677	67	67	5·156	4·921
23	23	17·867	15·544	68	68	4·872	4·658
24	24	17·675	15·404	69	69	4·594	4·401
25	25	17·478	15·260	70	70	4·325	4·150
26	26	17·275	15·111	71	71	4·063	3·906
27	27	17·065	14·954	72	72	3·810	3·669
28	28	16·849	14·794	73	73	3·566	3·440

Joint Life Annuities, 3 and 4 per-cent—(continued).

Ages.	At 3 per-cent Interest.	At 4 per-cent Interest.	Ages.	At 3 per-cent Interest.	At 4 per-cent Interest.
29 29	16·626	14·627	74 74	3·330	3·218
30 30	16·398	14·454	75 75	3·104	3·003
31 31	16·163	14·276	76 76	2·886	2·797
32 32	15·922	14·091	77 77	2·678	2·599
33 33	15·675	13·901	78 78	2·479	2·410
34 34	15·422	13·704	79 79	2·290	2·229
35 35	15·162	13·501	80 80	2·110	2·056
36 36	14·896	13·292	81 81	1·939	1·892
37 37	14·624	13·078	82 82	1·777	1·736
38 38	14·348	12·858	83 83	1·624	1·588
39 39	14·065	12·631	84 84	1·480	1·449
40 40	13·776	12·399	85 85	1·345	1·318
41 41	13·482	12·161	86 86	1·218	1·194
42 42	13·182	11·917	87 87	1·099	1·079
43 43	12·879	11·668	88 88	·988	·971
44 44	12·570	11·414	89 89	·885	·870
45 45	12·257	11·154	90 90	·789	·776
46 46	11·940	10·890	91 91	·700	·690
47 47	11·619	10·622	92 92	·618	·609
48 48	11·295	10·349	93 93	·543	·535
49 49	10·968	10·072	94 94	·474	·468
50 50	10·639	9·792	95 95	·411	·406
51 51	10·308	9·508	96 96	·354	·349
52 52	9·975	9·221	97 97	·301	·297
53 53	9·640	8·933	98 98	·249	·246
54 54	9·306	8·642	99 99	·182	·180

TABLE H^{MF} (adjusted). *Commutation Table, showing also the Value of the Life Annuity a_x with Interest at 4 per-cent, for every Year of Age from* 10 *to* 100.

Age x	D_x	N_x	M_x	a_x
10	69379·	1354883·	14599·9	19·529
11	66257·	1288626·	14146·5	19·449
12	63275·	1225351·	13712·4	19·366
13	60425·	1164926·	13296·2	19·279
14	57702·	1107224·	12897·7	19·189
15	55101·	1052123·	12515·7	19·095
16	52615·	999508·	12148·4	18·997
17	50239·	949269·	11796·7	18·895
18	47969·	901300·	11458·6	18·789
19	45799·	855501·	11133·9	18·679
20	43725·	811776·	10821·7	18·566
21	41744·	770032·	10521·5	18·447
22	39850·	730182.	10233·3	18·323
23	38039·	692143·	9955·4	18·196
24	36309·	655834·	9687·8	18·063
25	34655·	621179·	9430·1	17·925
26	33072·	588107·	9180·9	17·783
27	31560·	556547·	8940·9	17·634
28	30114·	526433·	8708·5	17·481
29	28731·	497702·	8483·7	17·323
30	27409·	470293·	8266·6	17·159
31	26144·	444149·	8055·5	16·989
32	24934·	419215·	7851·4	16·813
33	23776·	395439·	7652·4	16·632
34	22669·	372770·	7459·5	16·444
35	21609·	351161·	7271·7	16·251
36	20594·	330567·	7088·2	16·051
37	19623·	310944·	6909·0	15·846

4 *per-cent Commutation and Annuity Tables*—(continued).

Age x	D_x	N_x	M_x	a_x
38	18693·	292251·	6733·5	15·635
39	17802·	274449·	6561·7	15·417
40	16949·	257500·	6393·4	15·193
41	16131·	241369·	6227·6	14·963
42	15348·	226021·	6064·9	14·726
43	14597·	211424·	5904·2	14·484
44	13877·	197547·	5745·7	14·235
45	13187·	184360·	5588·7	13·981
46	12524·	171836·	5433·5	13·720
47	11888·	159948·	5279·2	13·455
48	11278·	148670·	5125·9	13·183
49	10692·	137978·	4973·4	12·905
50	10128·	127850·	4821·4	12·623
51	9586·9	118263·	4669·7	12·336
52	9066·9	109196·	4518·3	12·043
53	8566·4	100630·	4366·7	11·747
54	8085·4	92544·2	4215·0	11·446
55	7622·4	84921·8	4062·9	11·141
56	7176·6	77745·2	3910·4	10·833
57	6747·8	70997·4	3757·5	10·522
58	6334·8	64662·6	3604·0	10·208
59	5937·0	58725·6	3450·1	9·891
60	5554·1	53171·5	3295·4	9·573
61	5185·5	47986·0	3140·4	9·254
62	4830·5	43155·5	2984·9	8·934
63	4489·1	38666·4	2829·2	8·613
64	4160·7	34505·7	2673·5	8·293
65	3845·0	30660·7	2517·9	7·974
66	3542·2	27118·5	2362·9	7·656
67	3251·7	23866·8	2208·6	7·340
68	2973·6	20893·2	2055·6	7·026
69	2707·9	18185·3	1904·3	6·716
70	2454·6	15730·7	1755·2	6·409
71	2213·7	13517·0	1608·7	6·106
72	1985·5	11531·5	1465·6	5·808
73	1770·0	9761·5	1326·5	5·515
74	1567·4	8194·1	1191·9	5·228
75	1377·8	6816·3	1062·7	4·947
76	1201·6	5614·7	939·45	4·673
77	1038·7	4576·0	822·77	4·406
78	889·28	3686·7	713·30	4·146
79	753·36	2933·3	611·60	3·894
80	630·86	2302·4	518·06	3·650
81	521·62	1780·8	433·08	3·414
82	425·32	1355·5	356·83	3·187
83	341·55	1013·9	289·45	2·969
84	269·72	744·16	230·74	2·759
85	209·13	535·03	180·53	2·558
86	158·91	376·12	138·36	2·367
87	118·11	258·01	103·64	2·185
88	85·696	172·31	75·77	2·011
89	60·545	111·76	53·91	1·846
90	41·551	70·210	37·26	1·690
91	27·619	42·591	24·920	1·542
92	17·727	24·864	16·086	1·403
93	10·950	13·914	9·989	1·271
94	6·4856	7·4283	5·955	1·145
95	3·6680	3·7603	3·377	1·025
96	1·9722	1·7881	1·825	·907
97	1·0033	·7848	·934	·782
98	·4802	·3046	·441	·634
99	·2151	·0895	·194	·416
100	·0895	.. ..	·095	

INSTITUTE OF ACTUARIES.

[The Council of the Institute, with the object of assisting the junior members to pass the examinations required for the certificate of competency, resolved to appoint a Tutor for the current year who should form a class for the instruction of candidates for the Second Year's Examination, and who should as part of such course of instruction give three lectures open to all members of the Institute. Mr. W. Sutton, B.A., was accordingly appointed Tutor for the present year, and we believe that his lectures, altho' primarily intended for students, will be found of sufficient interest for our readers generally, to justify our printing them in full, which we have therefore done.—Ed. *J. I. A.*]

Lecture I.

(*Delivered* 2 *October* 1871.)

Gentlemen,—I need say but very little as to the purpose for which we are here to night, as you are all acquainted with the steps taken by the Council with the object of assisting the junior members of the Institute in their endeavours to obtain its certificate of competency. The few remarks I am about to make will constitute the first of the three lectures, open to all members of the Institute, which the Council have decided shall accompany, or rather form a part of, the course of class instruction in the subjects of the Second Year's Examination. Those subjects constitute a very important, I might say the most important, portion of the various branches of professional knowledge with which members of the Institute must have a thoro' and exact knowledge, before they can expect to obtain a certificate of competency as an actuary. It has been often pointed out by gentlemen who take a prominent part in the proceedings of the Institute, and whose opinion is of very great weight in such matters, that a sound knowledge of the subjects of the Second Year's Examination, and dexterity in the practical application of that knowledge, are absolutely indispensable to any degree of success in the profession of an actuary; and, for myself, I am convinced that the time and study spent in their attainment will well repay you.

On looking at the syllabus it will be seen that those subjects naturally fall into three divisions; and I do not think that the arrangement of the syllabus can be advantageously improved upon.

Taking, then, the syllabus as our guide, I propose to divide the course of class instruction into three parts in the following manner:—

Part I.	Theory of Logarithms.
	Elements of the Theory of Probabilities.
	Compound Interest.
Part II.	Tables of Mortality.
	Construction of Auxiliary Tables.
Part III.	Annuities and Assurances on Lives.
	,, ,, Survivorships.
	Miscellaneous Questions.

As to the time to be devoted to the subjects in each part, that will of course to some extent depend upon circumstances; but I think we may say that it will be approximately proportional to the number of questions set in each group of subjects. At the same time this is to be understood as being only a rough guess as to the time we shall find it necessary to devote to each group, and as in no way indicating their relative importance as subjects of study. Our great care must be that however little we do, that little shall be done thoro'ly and well; for I am sure by long experience that nothing is so disheartening to a student as to have to retrace his steps, and I would therefore much rather that we should proceed very surely if somewhat slowly at the first onset. It is possible, however, that some of you from previous reading may be able to proceed faster than the others. Should such be the case, I propose to make two divisions of the class, each division reading the different subjects simultaneously, but the higher division going deeper into each subject than the lower. There is an additional *a priori* argument in favour of this arrangement in that some of you may not contemplate going in for the examination in December next. However, these are matters of detail which we can easily settle hereafter. There is only one thing I wish to impress upon you, and that is, immediately you begin to feel that you are getting out of your depth, stop. Do not go on in the expectation that you will understand all right in time; but stop and go no further, or as little as possible, until you are sure that your footing is firm. I believe that much of the difficulty students find in reading for the Second Year's Examination arises in this way. Unfortunately there is no good text-book for students, with well selected examples such as most of the mathematical text-books of repute have, and by means of which the student can at any time easily test for himself

whether his progress is sound or not; and so, frequently, a considerable time elapses before he finds that he has not properly grasped what he has been reading, and the task of turning back becomes then all the more disheartening.

Considering then the first group of subjects contained in what we will call Part I it is to a certain extent a matter of indifference in what order we take them; for you will see, as we advance, how necessary all of them are to the various developments of the theory of life insurance, which is more particularly the business of an actuary. At the same time there are many questions which come before him with which money contingencies dependent upon life have nothing to do. For instance, the rates of contribution of members of Land and Building Societies, and the very extensive class of financial transactions, which may be briefly described as that of loans repayable by instalments. Of this latter kind are the loans made by corporations of towns on the security of the rates, and the loans under the Lands Improvement Act in consideration of an annual rent charge for a term of years. The practice, too, is extending to Manufacturing Companies, such as Wagon Companies, and the like, which sell a great part of their produce under this kind of arrangement. As an instance, take the case of the owner of a small coal mine who needs trucks to transport the produce by rail, but being unable to pay for them in the ordinary way, agrees to pay by instalments extending over a term of, say, 5 or 7 years. In fact, I think we may say that the practice is growing more and more into use, and sooner or later will become general. In all of these transactions, as a rule, the rate of interest to be charged is known beforehand, so that the calculations involved are not at all of a difficult character. There are other cases of loans repayable by instalments, however, that are not so simple. I allude to that class of loan transaction which Mr. Gray has discussed in a paper published in vol. xiv of the *Journal of the Institute of Actuaries*, p. 91, where the method of repayment is of a complicated character, and the determination of the rate of interest becomes a difficult problem. I will read to you the statement of one of the examples treated by him, which I take as an instance.

Example 1. A loan of £10,000,000 is contracted at 3 percent; and the debt is represented by 10,000 bonds, nominally of £1,000 each. These are to be paid by lot with a premium of 25 percent, as follows:—94 the first year, 102 the second, 110 the third, and so on, increasing by 8 each year until the 40th, by which time the whole 10,000 bonds will have been paid off. It is required to

determine the rate percent that the loan costs the borrower, and also the rate that will be realized by the holders of the bonds. As Mr. Gray remarks, these two rates will be the same where the whole of the loan is held by a single individual, but this is by no means necessarily the case where there is more than one bondholder, as is usual in this class of transaction.

Lieut.-Col. Oakes has published a book of Loan and Debenture Tables, in which is tabulated the average rate of interest paid and realized, according to different set of circumstances, in transactions of this nature. Compound interest, too, is, as I need scarcely remind you, the means for the determination of fines for the renewal of leases, and the valuation of all kinds of landed property.

In Parliament there is every now and then introduced by the Chancellor of the Exchequer for the time being a scheme for reducing the National Debt, and it must be a source of pleasure to an actuary to be able to understand clearly the merits of such schemes, and if occasion requires it to explain those schemes to others.

Now, in questions of the character here enumerated the doctrine of Compound Interest is the basis on which everything depends, and all of them need a sound theoretical knowledge of that doctrine to enable you to give satisfactory answers when such questions come before you. With regard to the text-book on the doctrine of Compound Interest, you will find the greater portion of what is necessary to be read in Jones's Annuities. There are however several important points which that book does not treat of, or treats only very cursorily. First of all, there is little or no mention made of the important distinction between the nominal and real rate of interest, where interest is payable more than once a year. Secondly, he does not discuss the two methods of calculating the interest for a fractional term. These two methods are as follows:—If i be the interest on 1 for 1 year, then one method gives $(1+i)^{\frac{1}{2}}-1$ as the interest for half a year, and the other method $\frac{i}{2}$. Now, as many of you are aware, this has been a subject of keen controversy among mathematicians and actuaries, and is certainly one to which your attention ought to be drawn. D'Alembert, Demoivre, and Francis Baily advocated the first method, and Mr. Milne and the late Mr. Peter Hardy advocated the latter. I only mention these names as being familiar to all of you. You will find a clear statement of both sides of the question in Mr. Peter Hardy's

Doctrine of Compound Interest. There is, again, the question of instantaneous interest, which, together with the corresponding instantaneous mortality, forms the basis of Mr. Woolhouse's Theory of Continuous Annuities and Assurances, of which I shall have more to say in a future lecture. At the same time, I cannot here refrain from remarking, in passing, how important an addition this is to the theory of life contingencies, and how incumbent it is upon you, as the actuaries of the future, to become acquainted with it. Once more, Jones's book does not distinguish in annuities certain between the two parts of which each instalment is composed, the one part going towards repayment of the original capital, and the other being a term's interest on that portion of the capital still outstanding. For this point you may refer to Mr. Peter Gray's letter on the component parts of a terminable annuity, in vol. xi of the *Journal*, p. 172. As a matter of course, too, no mention is made in Jones of the very instructive as well as interesting class of questions so lucidly treated by the late Mr. Peter Hardy in the 1st vol. of the *Assurance Magazine* in a paper "On the Values of Annuities, which are to pay certain "given Rates of Interest on the Purchase Money during the "whole term of their continuance, and to replace their Original "Values, on their expiration, at certain *other* given Rates." Singularly enough, a paper touching on this very point was read before the Institute of Surveyors last year, this fact showing clearly that the question has a real practical value, and as such one which all actuaries should be familiar with.

I think then that it will be sufficient for most purposes to take Jones as our text-book, using the late Mr. Hardy's little book and Baily's standard work on the Doctrine of Interest as our books of reference. We shall first take the elementary propositions where interest and instalment are payable yearly, and then go on to the more complicated cases where interest and instalment are payable so many times a year, not necessarily the same in number, and lastly consider questions of a miscellaneous character. To any of you fond of studying the historical side of a question, I may safely recommend our President's papers published in the *Journal of the Institute* some years ago (see vol. vi p. 301, vii p. 311, viii p. 68, ix p. 61), which contain in a short space a very complete historical account of the practice of charging interest.

The next subject in our Part I is the Elements of the Theory of Probabilities, and by this is not meant the Elements of the

Theory in its entirety, but only of a very small portion of it. A perfect acquaintance with the theory as it at present stands requires a thoro' knowledge of the highest mathematics and a very considerable period of study. As the theory by which errors of observation are corrected, it is of immense service in astronomy, and is even directly applied in such a matter as the adjustment of a ship's compasses. In daily life, too, its principles are acted upon more or less by people to whom even the very word probability is almost unknown.

As a very simple instance of the manner in which the theory of probabilities is actually brought to bear on life insurance matters we will take the case of a man who is entitled to receive £100 a year hence, his present age being 40. He wishes to receive a sum down in lieu of this—what should that sum be? Taking the Carlisle mortality table we find that out of 5075 persons alive at the age 40, 5009 only are alive at the end of a year. The fraction $\frac{5009}{5075}$ is called the probability of a person, age 40, living a year. If this person were entitled to £100 at the end of a year, whether alive or dead, then at 5 percent interest the present value of this £100 would be £95. 4*s*. 9*d*.; and taking into consideration the fact that there is only the probability $\frac{5009}{5075}$ of his living a year, $\frac{5009}{5075}$ths of £95. 4*s*. 9*d*. is the sum to which under the circumstances he is fairly entitled, and this is called in mathematical language the value of his expectation. I take this case, an extremely simple one, as it affords us an opportunity of seeing how the two things, interest and probabilities, are involved in all money contingencies depending upon human life. At the same time we see how the higher branches of the theory of probabilities may be applied. In the first place, you will naturally ask, why take the Carlisle table instead of others equally to be relied upon? In the next place, on what grounds is it assumed because in the Carlisle experience 5009 were alive at 41 out of 5075 alive at 40, that such will be the experience in other bodies of persons? To such questions as these for a satisfactory answer recourse must be had to the theory of probabilities.

The portion of the theory of probabilities which is actually needed for the ordinary business of an actuary is so inconsiderable, that a German writer, Dr. Bremiker, has remarked that "Just as "the arithmetic mean has always been regarded as the most "probable result before the method of least squares was known, "so in the same way has the value of an insurance been calculated "according to the arithmetic mean without regard to its error, "inasmuch as that value was assumed to be the average of all the

"payments to be expected according to the mortality table. This "cannot strictly speaking be called an application of the calculus "of probabilities; and if nevertheless we find both in old and new "text books on life assurance a chapter on the calculus of proba- "bilities, this might just as well have been left out, since in "reality no application is made of it." (See Mr. Sprague's translation of Dr. Bremiker's paper in the current volume of the *Journal*, p. 218.)

The above example will serve to explain what Dr. Bremiker means in the passage just quoted. Suppose that 5075 persons, age 40, are desirous of equally contributing to form a fund, out of which at the end of a year those who are still living shall receive £100 each. What should be the amount of each person's contribution? Now, assuming that the mortality during the year will be exactly in accordance with that of the Carlisle Mortality Table, 5009 persons will be alive at the end of a year, and the present value of the £100 to be paid to each of them is £95. 4*s*. 9*d*. Consequently, 5009 × £95. 4*s*. 9*d*. is the amount to which the fund ought to be made up, and as there are 5075 contributing members, each's contribution is clearly $\frac{5009}{5075}$ × £95. 4*s*. 9*d*., the result obtained above.

You will obtain a good idea of those points in the theory of life insurance to which the theory of probabilities is applicable from the following extract from the late Mr. Galloway's article on Probabilities in the Encyclopædia Britannica. "* * * Although "English writers * * * have almost without exception "confined themselves to the explanation of the methods of com- "puting annuity tables and of determining from them the values "of sums depending on life contingencies, the aid which this "branch of economy derives from the general theory of probabili- "ties, is by no means confined to the consideration of such "elementary questions. The number of observations necessary "to inspire confidence in the tables, the extent to which risks may "be safely undertaken, the comparative weights of different sets "of observations, and the probable limits of departure from the "average results of previous observations in a given number of "future instances, are all questions of the utmost importance, "which come within the scope of the calculus (*i.e.* of probabilities), "and cannot, in fact, be justly appreciated by any other means." On the general value of the theory of probabilities Mr. Samuel Brown remarks in his historical sketch of the theory which appeared in vol. vi p. 134 of the *Journal*—"The application of the

" theory is still only in its infancy in this country. Assurance " on pure theoretical principles is almost entirely confined to the " contingencies of human life; whilst the insurance of property " against whole or partial loss from any cause, and legal, moral, " social and political questions, open up fields of research, and for " application of the science." Vol. vi p. 147.

However, one of the practical results of the higher part of the theory, the method of least squares, has recently been made use of in the graduation of mortality tables by Professor Oppermann and Dr. T. N. Thiele of Copenhagen, as you will see in an essay by the latter, a translation of which by Mr. Sprague appears in the *Journal*, vol. xvi p. 313; and almost the first proposition you will learn will show you that the probability of living a year as ordinarily calculated from a mortality table is only approximately true. But as I have already stated, in preparing for the Second Year's Examination you are not required to go very deeply into the subject—all that is expected of you is a knowledge of the elementary propositions in direct and inverse probabilities, and for this the chapter on probabilities in Todhunter's Algebra, or the short treatise attached to Jones's Annuities, and written by Sir J. Lubbock and Mr. Bethune, will amply suffice. As the latter is not easily obtained I propose taking the former as the text-book in this subject. To those of you who may feel disposed to carry your reading further, I would recommend the late Professor De Morgan's Theory of Probabilities, or Mr. Galloway's book, already referred to, as containing all or nearly all of the Theory as at present developed. As instructive examples of the application of the theory to life insurance questions, I would refer you to Sir J. Lubbock's papers in the *Journal* some years ago (vol. v p. 197 and p. 277), to Dr. Bremiker's paper on Risk already quoted, to Mr. Sprague's paper (vol. xiii p. 20) on the same subject, and to the translation of Dr. Kanner's paper (vol. xiv p. 439), also on Risk. Another application of this theory which will assist you in obtaining clear ideas of the subject, is Sir J. Lubbock's paper on the Clearing House, published in vol. ix p. 141, of the *Journal*. M. Quetelet's Letters on the Theory of Probabilities contain a very admirable exposition of the application of the theory to a variety of statistical and scientific subjects, and are written in addition in a style well adapted for students. Sir John Herschel's review of this book, reprinted in the *Journal* (vol xv p. 179), is also well worthy your attention, forming as it were an exhaustive summary of M. Quetelet's book, and at the same time elucidating

and illustrating it in a masterly manner. Then you will find in the late Professor De Morgan's Essay on Probabilities a complete account in popular language of all that has been actually done in the theory, illustrated throughout by instances taken from the theory of life insurance. Mr. Whitworth of St. John's Coll. Cambridge, has written a small book entitled "Choice and Chance," which is more elementary than the books above mentioned, and in addition contains some excellent examples for practice.

For myself, I am sure that the time and study spent upon the theory of probabilities will tend to enlarge your views of the theory and practice of life insurance, and give you a power to grasp the general bearings of any actuarial question put before you attainable by no other means.

The last subject for consideration to-night is the Theory of Logarithms. Logarithms are the actuary's principal instrument in making his calculations, particularly where these calculations are of an extensive character. In the Matriculation Examination of the Institute you were required to show proficiency in actual working with logarithms, but it is thought that the competent actuary should know something about logarithms themselves as well as be expert in using them; and accordingly you are required to show a fair knowledge of what logarithms are and how their values are calculated and tabulated. There are two methods by means of which logarithms may be treated, (1) by the doctrine of ratios, and (2) by deductions drawn from the properties of the exponents of powers. With regard to the first of these methods we shall have nothing to do. This certainly was the way in which Kepler, Halley, Simson, and others, treated the subject, but the ideas on which this doctrine of ratios depends are so very difficult to grasp and of so very intricate a nature, that this method has gradually grown into disuse, and the second method only is now used. Starting, then, with the definition of a logarithm, viz., that if $N=a^x$, then x is called the logarithm of N to the base a, we shall take this exponential a^x, and expand it into series by means of which we can obtain other series expressing x, the logarithm of N, in terms of N, from which, given N, x can be immediately calculated. A difficulty, however, will have to be got over; because the first series obtained will be found not to be convergent except in a particular case, and it will therefore be necessary to transform this series into others which will be convergent in all cases. Other points, too, will come before us, which we shall have to notice. The series obtained will only be the series expressing

the value of log N when the base is ϵ, that used in the Napierian logarithm, and this will have to be allowed for, and so on. In practice, a variety of ingenious expedients are made use of to enable the calculations to be easily made; and then, too, when the values *are* obtained, other ingenious expedients are applied in the necessary process of verification. But the actual calculation of tables of logarithms is a subject into which we need not go far; at the same time we must go far enough to enable you to take up Mr. Peter Gray's little book on the formation of logarithms and anti-logarithms, and read it from the beginning to the end without any difficulty. The study of that book will serve to give you some idea of the immense patience, ingenuity, and calculating skill required for the formation of an ordinary table of logarithms. As an additional reason why a knowledge of the theory of logarithms is necessary to you, I may mention that when reading some of the higher investigations in the theory of life insurance you will meet with some of the series mentioned above, more particularly those where the Napierian base ϵ is used. For instance, the series for ϵ_x and $\log_\epsilon(1+x)$ are constantly made use of, and familiarity with them will be found indispensable. As text-books in this subject I propose to take Mr. Todhunter's Algebra and Plane Trigonometry, referring from time to time to Mr. Gray's little book alluded to above.

Having now referred to each of the subjects forming Part I of our course, I wish to make a few remarks of a more general nature. In the first place, you will find that a perfect familiarity with the subjects of the Matriculation Examination is absolutely necessary to any progress whatever in the subjects of our course; I mean by this, that those of you who feel that their acquaintance with the elementary mathematics of the Matriculation Examination is at all rusty or incomplete, must make a point of looking over those subjects again, and endeavour to become as familiar with them as possible. In the next place, I wish most distinctly to impress upon you the advisability of not only acquiring familiarity with the mathematics of the Matriculation Examination, but as far as in you lies extending it to the higher branches. A careful perusal of the *Journal of the Institute* from its foundation up to the present time will show you conclusively how the higher mathematics are becoming more and more applied in the developments of the theory of life insurance. Mr. Sprague's series of papers on Annuities, Mr. Woolhouse's Theory of Continuous Annuities and Assurances already referred to, Mr. Makeham's

papers on the Law of Mortality and other subjects, all require for their understanding a knowledge of the Differential and Integral Calculus, and of the Theory of Finite Differences. It may be said that for practical purposes it is sufficient to be able to apply the results of such investigation, but this I cannot admit. Before applying any of these results, you ought to be able to follow the demonstrations upon which they depend step by step, and satisfy yourselves that they are correct. Of course I do not mean that it is indispensable you should have at your command the acute analytical skill and mathematical facility requisite to make such investigations. This would be fixing far too high a standard. But I do think that papers of the kind mentioned above ought not to be a sealed book to you, and that you should do your best to enable yourselves to follow closely the processes of reasoning by which the results of these investigations are obtained.

Lecture II.

(*Delivered* 10 *November* 1871.)

In the first lecture we considered those preliminary subjects which it is necessary the actuarial student should be well grounded in before he can profitably proceed to what we may call the technical branches of his profession. We have during the last month, therefore, gone over the principal points connected with those preliminary subjects, and the time has now come when we must go a step further in our course, and endeavour to obtain a first sight, as it were, of how the knowledge already acquired is to be made use of hereafter. In accordance, then, with the plan laid down in my first lecture, I propose to-night to consider the next two subjects in the Syllabus of the Second Year's Examination, viz.:—

Tables of Mortality, and
Construction of Auxiliary Tables.

The late Mr. Babbage begins his introduction to his book on Life Assurance thus:—" Nothing is more proverbially uncertain " than the duration of human life, when the maxim is applied to an " individual; yet there are few things less subject to fluctuation " than the average duration of a multitude of individuals.

" The number of deaths happening amongst persons of our " own acquaintance, is frequently very different in different years; " and it is not an uncommon event that this number shall be " double, treble, or even many times larger in one year than in the

" next succeeding. If we consider larger societies of individuals, " as the inhabitants of a village or small town, the number of " deaths is more uniform; and in still larger bodies, as among the " inhabitants of a kingdom, the uniformity is such, that the excess " of deaths in any year above the average number seldom exceeds " a small fractional part of the whole.

" This fluctuation may be confined within still smaller limits if " we exclude from the enumeration those portions of the population " which are most exposed to the casual effects of disease or want, " during seasons when the supply of food is scanty; and if there " existed any data by which we might separate the effects arising " from deaths among the young and the poor, there are good " grounds for presuming that the variation from the mean would be " yet very greatly reduced."

The substance of these remarks is the key to the entire theory of life assurance, and their accuracy is proved more conclusively every day by experience. Throughout the whole range of statistics nothing is better established than this fact, that altho' for any particular individual of a certain age, no matter what, any prediction as to the exact time he will live is valueless because of the possibility of very considerable error, yet if we have 1000 such individuals we can within comparatively narrow limits tell beforehand how many of them will die within a certain number of years from the present time, and near enough for practical purposes we can calculate how many of them will die in each particular year of age; and it is in this latter way that mortality tables have become of important use in the affairs of life. Any particular individual of the 1000 may be well to-day and dead to-morrow, or he may be hale and hearty at 90, but he cannot tell; and he therefore insures himself against this risk of uncertainty by sinking his individuality, so to speak, and consenting to become a mere unit of the 1000. And here Insurance Companies come in and take upon themselves this risk attaching to each individual of the 1000; because they know that what is a tremendous risk in the case of an individual becomes when there are a large number of such a safe venture, and they can calculate with accuracy sufficient for all practical purposes, what amount of contribution they should receive from each individual of the 1000 in order that the fund formed from these contributions should always be in a solvent state. They take a mortality table formed from the observations of persons as nearly similar as practicable in circumstances and position to the 1000 they are about to insure, and from this mortality table they calculate by

a method we shall soon become acquainted with, what each of the persons who enters should pay either in a single payment, or by instalments, so that the funds of the Company may be enabled to pay each person on his death a certain sum: and in doing so the Company feels confident that it is not undertaking what would at first sight appear to be a very risky kind of business. They argue thus—All experience shows that in this matter of death, proverbially uncertain as it is to the individual, yet that where you have a number of persons of a certain age sufficiently large to give a fair average, nothing is more certain than that the experience of the past will repeat itself in the future, or so nearly repeat itself as that the deviation is, practically speaking, of little consequence, and can be easily allowed for. Of course, the method actually adopted in practice is slightly different from this. There are various kinds of fluctuation or uncertainty against which even an Insurance Company must insure itself, such as the possibility of the number of lives at different ages being too small to yield an average approximating to that of the mortality table, the varying amount of the sums at risk at the different ages, the chance of epidemics, and so forth; and for these risks the Company makes an addition to the calculated cost of insurance sufficient in its judgment to cover them all. However, with these refinements we have for our present purpose nothing to do. Our attention must be directed to the different methods adopted for obtaining these aforesaid mortality tables, and to consider briefly the respective merits and demerits of each method.

The *first method* of forming a mortality table is to take from the burial registers of the particular town, district or country under observation, particulars of the number who die at each age, the entries in the registers extending over a series of years, and from these results to calculate backwards, as it were, the number who must have been living at the beginning of each year of age. Dr. Price, in the construction of the Northampton Table, took the numbers living at the age 0, *i.e.*, the number of births, from the registers of baptism, and we shall see hereafter how far this course affected the accuracy of his table. This method of construction is now looked upon, generally speaking, as of little value, and may be considered practically to have been discarded. To give accurate results, it assumes a state of things which may be said in practice never to exist, namely, that the number of births is exactly equal to the total number of deaths in the year, and that the population is neither increased by immigration nor decreased

by emigration. For further information as to this method of constructing mortality tables I refer you to Mr. Farren's paper in the *Assurance Magazine*, vol. i p. 40, which contains a very interesting account of the manner in which the first mortality table—that called the Breslau Table—published in the year 1693, was constructed by the illustrious Dr. Halley; also, to a short paper by M. Quetelet, containing a remarkably clear explanation of this method of construction, of which a translation is given by Mr. Brown in the *Journal*, vol. iv p. 27. You will also find some interesting remarks by Dr. Farr on this method of construction in the 8th and 12th Annual Reports of the Registrar-General. There is also a very lucid account of this method in the *Penny Cyclopædia*, under the head *Law of Mortality*.

The *second method* is to compare the numbers living at each year of age, as obtained by actual enumeration, with the deaths that occur, as obtained from the registers, extending over a certain period of time adjacent to that when the enumeration was made. Thus, for instance, if an examination were made at the beginning of any year of the number living at each year of age and again at the end of the year, and the deaths that occurred during that year at each year of age were obtained from the registers, the facts thus obtained would enable us to form a mortality table of considerable accuracy, in the case where the numbers living at the different ages are of some magnitude. If the numbers observed are not very large, this method would have to be carried over a series of years to give results which may be relied upon.

Another method, more valuable perhaps than that just described, is that adopted in the case of Insurance Companies, Annuity Societies, and the like, where the age at the time of coming under observation is accurately known, and each member's career can be traced from the time of coming under observation until his death or discontinuance, or the end of the period over which the observations extend. When we come to compare the two latter methods we see that each of them has its own particular merits and demerits. In the former of the two we get very satisfactorily indeed the rate of mortality prevailing at a particular time, in a particular town, or district; but the time over which our observations extend may be a favourable one, or not. The trade of the place may be in a prosperous state, or the reverse, its sanitary condition may be at that particular time good or bad, there may be a striking absence of epidemical diseases, or not, and so on. Whatever the particular set of circumstances may

have been during the period over which the observations extended, we must bear in mind that the mortality table thus derived is only based upon the facts deduced from the experience of a particular town or district, during a particular period of time. We cannot say with anything like positiveness that this mortality table will be safe for general use. Even for that town or district itself, we do not get the rate of mortality prevailing among particular classes of that community, which, as we know, is a very important matter. As an extreme instance of what I mean, suppose that according to the second method, as above described, a mortality table has been constructed for the town of Sheffield. Now the mortality among the Sheffield grinders is well known to be much greater than that of the community at large; and as the inhabitants of Sheffield are mostly grinders or belong to similar unhealthy callings, this fact must have a material influence upon the rate of mortality as found to prevail there. It would be manifestly incorrect to consider that the mortality among the wealthy manufacturers and well-to-do tradesmen of the place would at all correspond with the rate as given by the mortality table. Then, again, there is the question of the mortality prevailing among males and females respectively, which in one well known case—that of the Carlisle Table—have been treated as identical. These and many other similar points have to be considered in determining the relative value of the tables formed after this method.

If, again, we take the mortality tables formed after the third method, we shall see similarly certain advantages and disadvantages. In these cases, as the observations extend over a considerable period of time, the presence or absence of epidemics, and the other conditions of a temporary character, such as the state of trade, and the sanitary condition, do not materially affect the general result; but we still lack information upon several points. We obtain no information, generally speaking, with regard to the mortality prevailing among different classes of the community. In most Insurance Companies the social position of the assured varies from that of princes of the blood and peers to that of domestic servants and labourers, so that altho' it may be said generally that the lives under observation are those of the well-to-do classes, yet that is about all we can say. Of course, this objection does not apply with so much force in the case of Government Annuitants and certain Friendly Societies, where the lives are generally those of persons of pretty much the same social position and calling. At the same time we have now lost all that is

valuable in the way of ascertaining the mortality of particular districts, as from the nature of the case the business of most Insurance Companies embraces an area considerably larger than that of the United Kingdom.

In connection with the rate of mortality found to prevail among assured lives, I ought to make mention of the important question of the effect of selection. Two causes tend to render the mortality experience of Insurance Companies different from that of the community at large, one being that at the time of coming under observation the lives are known to be healthy, whereas in the community at large the experience includes not only the healthy, but diseased and weakly lives, and even those on the point of death. The other cause is that the assured has the option of discontinuing or surrendering his policy at any time, and since as a rule he will not surrender or discontinue unless in good health, and as it is pretty certain his policy will be kept up if he is in bad health, it is quite possible that after a certain number of years from entering under observation the proportion of good lives to bad may be less than in the community at large. Altho' this question of the effect of selection is one of such considerable interest and importance to actuaries, its complete solution is still wanting. You will find in Mr. Higham's paper in the 1st volume of the *Assurance Magazine*, p. 179, and in Mr. Sprague's recent paper on the subject, vol. xv p. 328, all that has been hitherto effected towards its solution.

The above remarks will tend to suggest to you the various points for consideration in the construction of mortality tables generally, and in fixing upon the particular mortality table to be employed in calculating the tables of premiums of an Insurance Company, or in other investigations which come before an actuary.

So far, however, we have only got the rough results of observation. As already pointed out, these results will certainly show more or less irregularity, and it becomes necessary for insurance purposes to carefully eliminate such of these irregularities as are, to the best of our knowledge, accidental and brought about, perhaps, by the untrustworthiness of the facts or the paucity of the numbers under observation. In other words, what we have got at present are not the mortality tables actually employed in practice, but are only the basis on which those mortality tables are founded. They have still to undergo what is called the process of graduation, before they can be used to any extent by the actuary. This process of graduation is one of the most delicate and at the

same time one of the most important, requiring all the mathematical skill that can be brought to bear upon it, and also the greatest care in the exercise of that skill, lest the distinction between the irregularities of the rough results and what I may term their peculiarities is lost sight of. As an instance, I refer you to a letter by Mr. A. H. Bailey, in the 14th volume of the *Journal*, p. 247, in which that gentleman points out how a certain peculiarity in the rough results of observation for the period of early manhood, is made to disappear in the graduated tables that have been formed from those observations.

You will observe that I have hitherto used the term rate of mortality when speaking of mortality tables. The rate of mortality at any age, as you are aware, means in its ordinary acceptation the ratio which the number who die before completing the next year of age bears to the number who enter upon that year of age; or, in symbols,

$$\text{Rate of mortality at age } x = \frac{l_x - l_{x+1}}{l_x} = -\frac{\Delta l_x}{l_x}$$

Let us now suppose that in a certain fractional part of a year immediately after passing the age x, say δx, a certain number of deaths have occurred, $l_x - l_{x+\delta x} = -\delta l_x$, what will be the rate of mortality at age x deduced from this? Now we have the proportion

$$\begin{aligned}\delta x : 1 \text{ year} &:: \text{No. of deaths in interval } \delta x : \text{No. of deaths in 1 year}\\ &:: l_x - l_{x+\delta x} \qquad : \text{No. of deaths in 1 year}\\ &:: \frac{l_x - l_{x+\delta x}}{l_x} \qquad : \frac{\text{No. of deaths in 1 year}}{l_x}\\ &:: -\frac{\delta l_x}{l_x} \qquad : \text{rate of mortality required}\end{aligned}$$

$$\therefore \text{ rate of mortality at age } x = -\frac{1}{l_x} \cdot \frac{\delta l_x}{\delta x};$$

and if we suppose this portion of a year, δx, to be indefinitely small, we get an expression for the rate of mortality which is known as the *force of mortality* at the age x. This "force of mortality" will, of course, vary at each instant in proceeding from the age x to the age $x+1$, and the average value of this quantity for all ages between x and $x+1$ is what we have defined above as the rate of mortality at the age x. It will, perhaps, tend to clear your ideas on this point if we take an illustration.

Suppose a man has to walk a mile in a certain time, say 15 minutes. Now, it is quite clear that he may walk at very different rates during that time, and yet at its expiration have just completed the mile. He may begin very slowly, and gradually quicken his

speed, or he may continue at one uniform speed all through, or he may now accelerate his speed and at another time lessen it. In all these various ways he may walk, and yet arrive at the end of the mile in exactly 15 minutes from the time of starting; and we should say that the average rate at which he has walked the mile is 4 miles an hour, and this is clearly the average of the different rates at which he has walked during the 15 minutes. The average rate of 4 miles an hour corresponds to the rate of mortality at the age x, the rate at which he started walking corresponds to the force of mortality at the age x, and the rate at which he was walking at any particular instant during the 15 minutes corresponds to the force of mortality at a certain age between x and $x+1$.

Thus, to take a numerical example from the Carlisle table, the force of mortality at age 40 is ·0125 and at age 41 is ·0135, and for intermediate ages slowly increasing from one value to the other, whereas the rate of mortality at age 40 is ·0130. This explanation will possibly enable you, in a rough sort of way, to comprehend the meaning of the term *force of mortality;* but you will see that a perfect understanding of it requires some knowledge of the Differential Calculus. You will remember that in the course of our last month's study we have had before us the question of an instantaneous rate of interest, called by Mr. Woolhouse the force of discount; and here we have an instantaneous rate of mortality, called the force of mortality, and you can now form some notion of how the two may be combined in the theory of life insurance, giving us the values of what have been called by Mr. Woolhouse Continuous Annuities and Assurances, *i.e.,* instead of the values of annuities payable yearly, and assurances payable at the end of the year in which death occurs, the values of annuities payable momently, and of assurances payable at the instant when death occurs, and it is in these latter forms that the formulas of our theory become amenable to the processes of the higher mathematics. To those of you desirous of becoming acquainted with this force of mortality and its uses, I cannot do better than refer you to Mr. Woolhouse's and Mr. Makeham's papers in the *Journal,* where you will find full and lucid explanations of the term, and very interesting examples of its application.

Before leaving the question of the construction of mortality tables there is one point about which I think it will be well to say a few words. You see in connection with most mortality tables another column giving what is called the expectation of life at each age. Now, altho' as a matter of theory the meaning of

the expectation of life at any age is a perfectly simple matter, yet from various causes this phrase has been the source of endless confusion. What does the column above alluded to, with the heading "expectation of life," really represent? If put into the algebraical form $\frac{1}{l_x}\{\frac{1}{2}d_x+1\frac{1}{2}d_{x+1}+2\frac{1}{2}d_{x+2}+\ldots\}$, we see that it denotes the average number of years which the l_x persons of the mortality table live thro'; and if put into the equivalent form

$$\frac{1}{l_x}\{\tfrac{1}{2}(d_x+d_{x+1}+d_{x+2}+\ldots)+l_{x+1}+l_{x+2}+l_{x+3}+\ldots\}$$
$$=\frac{1}{l_x}\left\{\frac{l_x}{2}+l_{x+1}+l_{x+2}+\ldots\right\}$$
$$=\tfrac{1}{2}+p_{x.1}+p_{x.2}+\ldots$$
$$=\tfrac{1}{2}\text{ year}+p_{x.1}\times 1\text{ year}+p_{x.2}\times 1\text{ year}+\ldots$$

we see that it gives us the sum of the values of the mathematical expectation of a man aged x living each year to the end of life. The first confusion has been to consider this expectation of life as the number of years which it is probable that a person of the age x will live—in other words, that if the expectation of life at age x is m years, then it is just an even chance that an individual of present age x will live m years! Of course you will see at once that there is no connection whatever between the expectation of life, or average duration of life at any age, and the term of years of which the probability that an individual will attain to is $=\frac{1}{2}$. This latter quantity is the *vie probable* of the French writers, and when translated into English as the *probable lifetime* has been mistaken for the expectation of life.

Another misuse of the term expectation of life arises thus:—Suppose that the average duration of human existence in England in the year 1840 was found to be 34 years, and in 1860 to be 32 years—in other words, that the expectation of life at the age 0, or birth, had decreased 2 years during that interval of 20 years. What conclusion can we draw from this result as to the relative longevity of the inhabitants of the country at the two epochs? Clearly none whatever, unless we know the proportion between the numbers living at each age at the two epochs. And yet over and over again results of this kind are quoted to show how certain countries are declining in material prosperity and physical vigour, or the reverse.

Another blunder into which this phrase expectation of life has led, and I am afraid still leads many persons, has to do with the application of mortality tables to pecuniary matters. Few

persons except actuaries can see why an annuity on a man's life is not the same thing as an annuity certain for a term of years equal to his expectation of life as given by the mortality table employed; and altho' this mistake has been repeatedly corrected, it is still ever and again misleading some one. To you, however, to whom the fact that there *is* a difference between the two benefits is well known, I need not say more than refer you to Mr. Sprague's mathematical demonstration of the question, published in the *Journal*, vol. x p. 52, and xiii p. 381.

Having thus considered the different methods of construction of mortality tables and various other points connected therewith, I have only a very short time at my disposal for the consideration of other tables formed from these mortality tables, and which are called, in the Institute Syllabus, Auxiliary Tables. This phrase is rather vague, and embraces not only the tables deduced directly from a mortality table but tables of the kind commonly known as Commutation Tables. It is difficult to make the method of construction and use of these tables intelligible to you except by actual examples, which would take up too much time just now, and therefore I do not propose to go into details on the present occasion. Very much of one kind and another has been written about these commutation tables, as to who was really their first inventor, whether he only invented a part of them or the whole, and so forth; but with these questions we have nothing to do. A more important question for us is the principle of notation to be adopted in the use of them; and here I may say that we shall make use of that given in Mr. Gray's *Tables and Formulæ*, p. 118, and which is that generally employed. It is a great pity that needless cause of confusion should have been introduced by different writers adopting different methods of notation; because in such a matter as the use of tables everything depends upon uniformity of habit; and a person long accustomed to the use of, say, the ordinary method, would scarcely ever be free from the liability of mistakes when using, for instance, Dr. Farr's method of arrangement. Professor De Morgan's papers "On the Calculation of Life Contingencies," reprinted in the *Journal*, vol. xii p. 328, and xiii p. 129, contain a very complete exposition of the use of commutation tables, and Mr. Gray's papers on the same subject in vol. x pp. 84, 169, 220, give in a clear and lucid manner an elementary account of such tables, which you will find amply sufficient for practical purposes.

Lecture III.

(*Delivered* 1 *December* 1871.)

In Parts I and II of our course we have been engaged in laying the foundation of our work, and having completed that we are now about to rear the superstructure. In Part I we saw how the Doctrine of Compound Interest, and the Theory of Probabilities must be combined if we wish to obtain the values of certain pecuniary expectations, where the chances involved can be ascertained or are given. In Part II we were occupied with the consideration of the different methods by which the values of those chances can be ascertained when they are chances depending upon human life; a mortality table in its ordinary form containing all the datums required to enable us to calculate the chances of any particular case. And, now, in Part III we are about to consider how to estimate the values of these aforesaid pecuniary expectations, when the chances involved are such as can be deduced from a mortality table.

It will scarcely be necessary for me to dwell upon the importance of this subject. A moment's consideration will suggest to you almost numberless instances where the pecuniary interests involved depend in some way, or other, upon the contingencies of human life. Mr. Baily, in the preface to his *Doctrine of Annuities and Assurances* gives a very interesting summary of the various kinds of real and personal property in which the system of tenure depends in some way or other upon the duration of human life. A close examination of these will enable us to make a broad distinction between the different kinds, and to classify them under two heads—the first, containing all those depending upon the *continuance* of a certain *status;* the second, all those depending upon the *failure* of a certain *status.* Speaking generally, the first class may be known by the name of *annuities,* and the second by that of *assurances* or *reversions.* Of course, in practice, these two classes become more or less intermingled, and what is a reversion or assurance to-day, may to-morrow become an annuity and *vice versâ.*

The proper method of deducing the values of ordinary annuities and assurances, altho' to us apparently very simple, and almost self-evident, has been the growth of more than one hundred years. Dr. Halley made, in England at any rate, the first attempt to compute the values of life annuities; but as he only calculated them for every fifth year of life, and at one rate of interest, his results were not directly of very great utility. The next attempt, following after Dr. Halley, was Demoivre's *Annuities on Lives,* calculated on the author's well known hypothesis of equal and

uniform decrements throughout life. Mr. Simpson's *Doctrine of Annuities and Reversions,* published in 1742, was an improvement in some respects upon Demoivre, as instead of calculating the values of annuities from an hypothetical law of mortality, he calculated them from the "real observations of life." With regard to his values of annuities Mr. Baily observes, "His tables being deduced "from the rate of mortality in London only, are found not to be "sufficiently adapted for general use; and his Rules being deduced "partly from the hypothesis of M. De Moivre, and partly from "real observations, had been ascertained not to be sufficiently "correct."—(See *Preface* to Mr. Baily's work referred to above.) James Dodson soon afterwards (1753–1755), in the last two volumes of his *Mathematical Repository,* made improvements in the analytical reasoning by which to investigate the values of various pecuniary interests whether annuities or reversions, but his numerical results, being founded on Demoivre's hypothesis, are of little value. In fact, it was not until Dr. Price published the fourth edition of his work on *Reversionary Payments,* in 1783, that we can be said to have been possessed of annuity tables constructed in an accurate manner. Even these tables are now looked upon with distrust, as they are founded upon the Northampton Mortality Table, which it has been shown was based on incorrect assumptions. Mr. Morgan, of the Equitable Life Office, and a contemporary of Dr. Price, in his *Doctrine of Annuities and Assurances,* published in 1779, made many important extensions of the theory of annuities and survivorships, and also in his subsequent papers published in the *Philosophical Transactions.* Mr. Baily followed Mr. Morgan, and in his *Doctrine of Annuities and Assurances,* besides pointing out various points where Mr. Morgan had erred, laid down the proper methods to be employed, these being much the same in substance as those still employed. Since the publication of Mr. Baily's book, only two works of any distinctive importance have appeared in England—Mr. Milne's treatise *On the Valuation of Life Annuities and Assurances,* and Mr. Griffith Davies's work on *Life Annuities;* but it must not be inferred that no progress has been made in the development of the theory of life annuities and assurances. I need only recall to your minds the writings of the late Mr. Gompertz and of the late Professor de Morgan, of Mr. Woolhouse, Mr. Sprague, Mr. Makeham, Mr. Jellicoe, the late Sir John Lubbock, the late Mr. Orchard, Mr. Gray, Dr. Farr, to remind you of the important advances we have made since Mr. Baily's time. And it must be remembered, too, that progress has been made in two ways. We have not

only got better constructed mortality tables, so that the numerical results of our theory are more correct, but that theory itself has been improved.

Having now gone historically over the ground, we will proceed in some detail to consider the various kinds of annuities and assurances as they are treated at the present time. Let us take, first, the value of an annuity or an assurance where the status involved is only that of one life of a given age. In the usual case, where the payments of the annuity are made at the end of each year, and where the rates of mortality and interest are annual, such an annuity is simply the sum of a series of terms of the form $p_{x,n}v^n$, where n has successively the integral values 1 to $\omega-x-1$, ω being the limiting age of the mortality table employed. In other words, the value of the annuity is the same as the sum of the values of the mathematical expectations of receiving 1 at the end of every year to the end of life. Another way of looking at this series is, as Mr. Sprague first pointed out, to consider it as an annuity certain of $\omega-1$ payments, the first payment being $p_{x,1}$, the second $p_{x,2}$, and so on, the last being $p_{x,\omega-x-1}$, and the sum of all the payments being equal to the curtate expectation of life. Similarly, the value of an assurance payable on x's death is the sum of a series of terms of the form $q_{x,n}v^n$, for integral values of n from $n=1$ to $n=\omega-x$. This, in the same way as the annuity, may be looked upon as an annuity certain of ω payments, the first being $q_{x,1}$, the second $q_{x,2}$, and so on, the last being $q_{x,\omega-x}$, the sum of all the payments together making 1. Another way of putting this value of an annuity on the life of x is this:—Let there be l_x persons of the age x, of whom l_{x+1} are alive at the end of one year, l_{x+2} at the end of two years, and so on. It is required to ascertain what the amount of contribution of each of the l_x persons should be to enable the fund thus formed to pay every one of them who is alive at the end of each year a sum of 1. This is the same thing as solving the simple equation

$$l_x\mathrm{X}=l_{x+1}v+l_{x+2}v^2+\ \ldots\ +l_{\omega-1}v^{\omega-x-1}$$

which gives $\mathrm{X}=\dfrac{l_{x+1}}{l_x}v+\dfrac{l_{x+2}}{l_x}v^2+\ \ldots\ +\dfrac{l_{\omega-1}}{l_x}v^{\omega-x-1}$;

and this in our previous notation becomes

$$\mathrm{X}=\sum_1^{\omega-x}p_{x,n}v^n, \text{ the values of } n \text{ being integral,}$$

if we assume that $\dfrac{l_{x+1}}{l_x}=p_{x,1}$, $\dfrac{l_{x+2}}{l_x}=p_{x,2}$, &c. This assumption, however, is not theoretically true, altho' it is the assumption made

in practice. As appears from the preceding example, the values of an annuity, as ordinarily calculated, is simply the average value of the payments made at the end of each year to such of the l_x persons as, according to the mortality table employed, are alive at the end of each year. For a detailed consideration of this point, I refer you to Sir J. Lubbock's paper "On the Calculation of Annuities," reprinted in the *Journal of the Institute*, vol. v p. 197. Similar remarks, of course, apply to assurances.

Now, we must here call attention to one important point. From the manner in which we have put the expressions for an annuity and an assurance, it will be seen that in the annuity no payment is made at the end of the year in which the life x has died, nor, in the assurance, is the sum assured payable until the end of the year in which death occurs. I need hardly remind you that neither of these conditions obtains in practice. In the annuity of practice, a proportionate part is paid for the year in which death occurs, and in the assurance payment is generally made at a given short interval after death. But we shall come to this point again, further on.

The method just laid down is sufficient to enable us to find the value of an annuity or assurance, not only on a single life, but on the joint continuance or failure of any number of joint lives, the only difference being that instead of $p_{x,n}$ and $q_{x,n}$ we should have to put the corresponding values $p_{xyz\ldots,n}$, $q_{xyz\ldots,n}$. In all these cases, it must be remembered that the value of the annuity or assurance is simply the average value of the payments made, assuming that the mortality table employed accurately represents the number alive at the completion of each year of age. The actual calculation of tables of the values of annuities and assurances, altho' a comparatively easy matter when there is only one life, becomes very laborious when two or more joint lives are involved. For an account of the different methods of calculation I refer you to Mr. Gray's *Tables and Formulæ.*

Hitherto I have only alluded to the simple case where the annuity is entered upon at once, and for the remainder of life, and the assurance is for the whole of life, and the value of each benefit is estimated in terms of a single payment down. But, as you are aware, the annuity and the assurance may be either deferred, temporary, or intercepted, and paid for not by one payment down, but in a variety of ways. Arranging the two series thus,

$$\begin{aligned} a_x = {} & p_{x,1}v + p_{x,2}v^2 + \ldots\ldots + p_{x,t}v^t \\ & + p_{x,t+1}v^{t+1} + p_{x,t+2}v^{t+2} + \ldots\ldots + p_{x,t+m}v^{t+m} \\ & + \ldots\ldots \end{aligned}$$

$$\begin{aligned}A_x = &\, q_{x,1}v + q_{x,2}v^2 + \dots + q_{x,t}v^t \\ &+ q_{x,t+1}v^{t+1} + q_{x,t+2}v^{t+2} + \dots + q_{x,t+m}v^{t+m} \\ &+ \dots\end{aligned}$$

we see that the annuity and assurance may be thus considered:—

Annuity for life $=$ Temporary Annuity for t years + Annuity deferred t years

$=$ Temporary Annuity for t years + Intercepted Annuity for m years after t years + Annuity deferred $t+m$ years

Assurance for life $=$ Temporary Assurance for t years + Assurance deferred t years

$=$ Temporary Assurance for t years + Intercepted Assurance for m years after t years + Assurance deferred $t+m$ years.

In the usual case of practice, where the consideration for an assurance or for a deferred annuity is payable not in one payment down, but by instalments extending over life or a series of years, the method of procedure is this: the purchaser of the deferred annuity or of an assurance undertakes to pay the Company an annuity for a term or for life, and the amount of this annuity is simply a question of proportion, the insured be it remembered having the option of breaking the contract at any time.

There is another class of benefits which are a little more complicated in their nature than those we have just considered, viz., Endowment Assurances. Here the assured receives two benefits—one being a certain sum payable if he live a certain number of years, the other being a certain sum payable if he do not live that number of years. Thus, for instance, an endowment assurance of 1 payable at 60 or previous death, on a life now 40, is nothing but an endowment payable at 60, together with a temporary assurance for 20 years, on a life now 40. Now, if instead of 60 we take the limiting age of the mortality table ω, we see that an ordinary whole life assurance may be described as an endowment assurance payable at ω or previous death. Hence, we should naturally expect that the formulas for endowment assurances are the same in form as those for whole life assurances, and they are so as a matter of fact. As an important illustration of the use of this property, Mr. Sprague has shown in vol. viii p. 111, of the *Journal*, that we are thus enabled to use Orchard's *Assurance Premiums* in the calculation of endowment assurance premiums. In connection, too, with endowments and endowment assurances, it may not be out of place to call your attention to a matter of

detail in the calculation of the premiums. As you are aware in these cases it is not an unusual event for an endowment policy to contain a provision that if death occur the premiums paid shall be returned. Mr. Ambrose Smith in vol. viii of the *Journal*, p. 116, gave the correct expression for the net premium in this case, and since then Mr. Gray and Mr. Hardy, in the Solutions of the Second Year's Examination Questions of 1868, vol. xv p. 242, have again called attention to the point involved, which indeed occurs in a somewhat different form in ordinary business. We know perfectly well that if you put on, say, 5 percent to the cash price of an article to obtain the credit price, and then take 5 percent off the credit price for cash, you are a loser by the transaction. So in endowments with returnable premiums in case of death, if you only charge the insurer for a temporary insurance of the *net* premiums paid, and in case of death pay his executors the *gross* premiums, you are clearly a loser by that part of the transaction.

Next in order to annuities and assurances on joint lives we may place Annuities and Assurances on Successive Lives. In the case of annuities on successive lives at the decease of the lives in possession, other lives of given age are nominated; and similarly in the case of assurances. This part of the theory of life insurance forms an important feature in many institutions of the country. The tenure of property by ecclesiastical corporations, and those numerous estates known by the name of copyhold estates are instances of annuities and assurances on successive lives, and the freehold of an advowson may in the same way be considered as an annuity on successive lives. These questions have been very ably discussed in the pages of the *Journal* by Mr. Peter Gray, vol. ii pp. 1 and 271, and by the late Mr. Weddle, vol. xiii p. 221. You will in those gentlemen's papers find all that is really essential to a sound understanding of the manner in which such questions are to be treated.

The next set of questions in our subject relate to Annuities and Assurances on the Last Survivor of a certain number of lives. These theoretically possess no intrinsic difficulty. The one great difficulty in practice is, that when we have obtained a formula for the benefit in question, and that formula involves annuities on three or more joint lives, we have, as a rule, only approximate means of obtaining the values of those annuities, since the labour required to calculate even a three joint life annuity table is prodigious. As a very instructive instance of how this difficulty may be got over in practice, I need only refer you to the last number of the *Journal*, where the value of an annuity on the last

survivor of five lives is calculated by means of a valuable formula given by Mr. Woolhouse in vol. xi p. 321 of the *Journal.*

We now come to one of the most complicated class of questions with which an actuary has to deal—Survivorship Annuities and Assurances, where the benefit depends upon the deaths of the given annuitants, or of the assured lives taking place in some assigned order relative to other lives. Even with only three lives involved, the calculations required are of considerable difficulty and complexity. Mr. Milne and Mr. Baily have investigated a considerable number of cases, and Mr. Makeham in vol. x p. 241, has also treated some of the different problems that occur. In these questions, assuming the values of the different probabilities involved to be accurately known, the solution is still not an easy matter; but in practice we cannot even go this far. In determining the probabilities in a very simple case, viz., the probability that out of two given lives x and y, x shall die first, Mr. Baily very well remarks to this effect:—When x is the younger of the two lives, the fraction $\frac{1}{2}$ will exceed the chance that x dies before y, and therefore the value of the required probability thus found will be in excess of the true value in any part of their coexistence. On the other hand, when x is the older of the two lives, the chance that x dies before y is more than $\frac{1}{2}$, and consequently the value of the required probability as found will fall short of the true value in that part of their coexistence. Another difficulty, alluded to previously, arises from the fact that at present tables of the values of annuities on more than two joint lives have never been constructed, so that we have even in the case of an annuity on three joint lives to make an approximate calculation. Luckily, questions of this kind rarely, if ever, occur in practice, so that altho' theoretically of interest their practical value is inconsiderable.

In all the questions we have hitherto discussed, no matter how complicated the status on the continuance or failure of which the benefit depends, we have always considered the amount of the benefit as constant, *i.e.*, in annuities, the benefit has always been an annuity of 1, and in assurances always an assurance of 1. It is clear, however, that there may be cases where the benefit is not of constant amount, but either increasing or decreasing according to the particular circumstances of the case. These questions are known by the name of increasing and decreasing annuities and assurances. Just in the same way the annuity paid to a Company in consideration of a certain benefit may either be an increasing annuity or a decreasing annuity, *i.e.*, the premium may either be an increasing or decreasing premium. The cases of this kind that

occur in practice usually possess no great difficulty, and the employment of the Commutation Tables is very convenient. In the example alluded to just now of an endowment with returnable premiums, we have a case where part of the benefit consists of an increasing assurance, and in Mr. Gray's *Tables and Formulæ*, p. 106, you will find instances of increasing and decreasing annuities.

Let us return, now, to some other points of more immediate practical importance. There are few questions of more importance to actuaries than that of the values of policies, inasmuch as upon this hinges, as it were, the whole question of life insurance. In every Office, valuations have to be periodically made of the liability of the Office under its existing policies, and this I need scarcely tell you is simply finding the values of the different policies, and adding such values together, the total giving the estimated liability of the Office under its policies. A knowledge of the method of finding the values of policies is alone all things essential to an actuary, and in its entirety involves very considerable practical experience and skill. With the refinements of the subject we have nothing to do; but those of you who may desire hereafter to become better acquainted with the subject may refer to Mr. Tucker's paper in vol. x p. 312; Mr. Sprague's papers, vol. xi p. 90, and vol. xv p. 411, and Mr. Meikle's paper, vol. xi p. 241. Mr. Sprague's paper, vol. xi p. 90, contains nearly all that we shall require as to the theoretical part of the subject, and Mr. Manley's Prize Essay, in vol. xiv p. 249, affords us valuable information as to the result of employing particular mortality tables in an Office valuation; you will gain from the former a remarkably clear idea of the theoretical value of a policy, and the latter brings before you some of the difficulties of the subject, such for instance as the connection between the values of policies and the mortality table from which they are constructed. There are many remarkable points connected with the values of policies. As a familiar instance we very well know that the value of a policy decreases as the rate of interest increases, but a strict proof of this properly is by no means a simple matter. All these difficulties arise from the same cause, that the values of p in the expression for the value of an ordinary annuity given above, viz.,

$$a_x = p_{x,1}v + p_{x,2}v^2 + \text{\&c.},$$

do not proceed according to any known law. We cannot therefore in the above instance tell what difference in the value of the annuity will result from a given change in the value of v, *i.e.*, from a given change in the rate of interest.

Let us now return for a few moments to a point mentioned

some time ago. We have seen that the values of annuities and assurances as ordinarily calculated and tabulated are the values of annuities upon the last payment made is, that at the end of the year preceding that in which death occurs, and the values of assurances assume in like manner that the assurance is payable at the end of the year in which death occurs. In practice, however, neither of these ássumptions necessarily holds, annuities may be *complete, i.e.*, a proportionate part payable for the year in which death occurs, and in assurances the assurance may be payable immediately death occurs, or within a short time after. So, that, the values of an annuity and of an assurance as ordinarily given are less than the annuity and assurance of practice. A correction therefore has to be applied to the tabulated values, and it is desirable that you should be acquainted with the methods by which these corrections are obtained. This branch of our subject is of considerable interest and difficulty, and will afford you capital practice in the reasoning processes on which the theory of life insurance is based. You will find this point, and the kindred one of annuities and assurances, where interest and the payments of the annuity are made more than once a year, ably and fully discussed in Mr. Sprague's series of papers on Annuities in vol. xiii pp. 188, 201, 305, 358; xiv p. 36; and also in Mr. Woolhouse's papers in vol. xi pp. 61, 301, xii p. 136, and xv p. 95.

It may be said that many of such questions as those just enumerated have only a theoretical importance, and practically are of little consequence. But I would urge in reply that it is by no means a scientific method of procedure to determine *à priori* what points are of importance and what are not. The proper method should be to investigate as completely as possible the value of any corrections to be applied to the general formulas, and then to ascertain by actual calculation whether such corrections may safely be neglected. There are, however, several such investigations which must be admitted to be of great utility in the practice of life insurance. Thus, for instance, in making a valuation of the liabilities of an Office, we may require to ascertain the average present value of the current premiums at each present age. Now the next premium in individual cases will be due at periods of time ranging from those just due to those due just a year hence, and Mr. Woolhouse in his paper on Continuous Annuities and Assurances, already quoted, shows that the proper annuity value to take in calculating the value of the future premiums is $\bar{a}$.

There is one other point upon which I wish, in conclusion, to

say a few words. You will remember that when dealing with questions of compound interest, we discussed the case of a man purchasing an annuity certain for a term of years, and paying such a price for the said annuity as would pay him a certain rate of interest upon his invested capital, and likewise enable him to replace that capital at a certain other given rate of interest. Now the theory of life insurance affords us a case remarkably analogous to that just mentioned; and it will be of interest to consider in what respects the two transactions differ from one another. In the first place it is necessary that in order to reproduce his capital when the life annuity expires he must effect an insurance upon the annuitant's life. It is quite clear, too, that inasmuch as he receives no payment of the annuity at the end of the year in which the annuitant dies, while on the other hand the premium has been paid by him at the beginning of each year instead of at the end; then if S denote the amount paid for the purchase of the annuity, the capital to be reproduced by insurance at the end of the year in which the life dies is S + amount of one payment of annuity, so that we have the equation

$$(\mathrm{P}_x+d)(\mathrm{S}+m)=\text{amount of yearly payment of annuity}=m.$$

or $$\mathrm{S}=\frac{1}{\mathrm{P}_x+d}-1 \text{ if amount of yearly payment is 1,}$$

where $d=$ discount on 1 for a year

$\mathrm{P}_x=$ annual premium to insure 1 on the death of x.

We see now, then, exactly how the purchase of a life interest differs from that of an annuity certain. The term of the investment depends upon the duration of an individual life instead of a term of years certain. The amount of capital invested is something more than the purchase money of the annuity, and the life insurance premium takes the place of the surplus annuity for accumulation which is to reproduce the capital invested at the expiration of the annuity. There are other cases, such as the purchase of contingent annuities and reversions, which are more complicated than the above, but which may be shown in a similar way to bear an analogy to that we have already had to deal with; but as these questions come more properly under the subjects of the Third Year's Examination we need not consider them. At the same time, they afford a capital insight into what has become a very important branch of the subject. Mr. Jellicoe, years ago, gave a series of papers in the *Assurance Magazine*, vol. ii p. 159, vi p. 61, viii p. 310, in which he very lucidly explained the reasoning by which

the different formulas are obtained, and since then Mr. Sprague has treated the subject in a very able manner in his papers recently published in the *Journal*, vol. xiv p. 417, xv p. 126. For your present course of reading, however, you will not find it necessary to give much of your time to the study of them.

It may not be out of place to conclude this Lecture with a few general remarks. At the beginning of our studies we found that all pecuniary questions contingent upon human life, being those which may be said to fall more properly within the province of an actuary, involved two distinct elements—interest on money, and probabilities of life. The first of these is fully developed in the Doctrine of Compound Interest, and all our information about the latter is given in the form of mortality tables. Having discussed each of these elements in detail, we have now, in this, the Third Part of our course of study, seen how the values of the different pecuniary interests depending upon human life are deduced by combining the two elements of interest and mortality in the proper manner. Incidentally, too, we have discussed the theory of logarithms, and the construction of auxiliary tables, both of which are questions of great practical importance to the actuary; and both of which the Council of the Institute has very properly decided should form a part of the subjects for the Second Year's Examination. In going thro' the various subjects, I have little doubt that many points have arisen about which even now your convictions are by no means so clear as you would like them to be. You must not be dissatisfied with yourselves, altogether, on that account. In reading new subjects, and acquiring new ideas, it is almost always a work of time before we can be said to have well digested the knowledge acquired; and in this respect the theory of life insurance is no exception. Nothing, however, will do more towards assisting you towards this end than the solution of examples, and in this you will find the questions of previous years with their solutions, as given in the *Journal* from time to time, of very great service. See solutions in vol. i p. 123, by Mr. Porter, and in x p. 45, xiii p. 253, xiv p. 147, by Mr. Sprague, and the solutions for 1868 by Mr. Peter Gray and Mr. R. P. Hardy, in vol. xv p. 232.

ERRATUM.

Dr. T. N. Thiele points out the following:—

Vol. xvi p. 314, in formula (2), for $a_3{}^{-b_3{}^x}$ read $a_3{}^{b_3{}^x}$.

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END OF VOL. XVI.

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EAGLE INSURANCE COMPANY,

LONDON.

REPORT OF THE DIRECTORS

FOR THE YEAR ENDING 30TH JUNE, 1871, READ AT THE ANNUAL GENERAL MEETING OF PROPRIETORS, 11TH AUGUST, 1871, ROBERT ALEXANDER GRAY, ESQ., CHAIRMAN OF THE COMPANY, IN THE CHAIR.

In submitting to the Proprietors the Sixty-fourth Annual Report of the Company's affairs, the Directors are glad to remark that the satisfactory condition of the Company has been fully maintained during the past year.

It will be observed that the Accounts now furnished present a different aspect to those rendered on previous occasions, and are, perhaps, neither so explicit nor so comprehensive as heretofore; but they have necessarily been prepared in accordance with the provisions of the Act of Parliament passed last Session, and are in the form in which they will be deposited at the Board of Trade.

The Quinquennial Report, together with the usual full and complete Statement of the Company's Affairs, will be submitted to the Proprietors at the Meeting in 1872, and the Directors will therefore now only remark :—

That the Premiums on New Assurances amount to £12,907. 19*s.* 5*d.*, of which sum £3,428. 15*s.* 10*d.* has been devoted to re-assurance;

That the Interest from Investments is £137,730. 19*s.* 10*d.*, or £4. 8*s.* per cent. on the net amount of Assets existing in June, 1870;

That the Claims, with the additions to them, somewhat exceed the average. They amount to £313,112. 18*s.* 3*d.*, and make the annual average for the four years now terminating £304,512; and

That the Expenses of Management are again reduced, being less than those of the preceding year by upwards of £1,100.

FIRST SCHEDULE.

Revenue Account of the EAGLE INSURANCE COMPANY for the Year ending 30th Juue, 1871.

	£	s.	d.	£	s.	d.
Amount of Funds at the beginning of the year..				3,128,857	0	3
Premiums	355,515	12	7			
Less Re-assurances ..	55,663	15	4			
				299,851	17	3
Consideration for Annuities granted ..				*Nil.*		
Interest and Dividends				137,730	19	10
Other Receipts				*Nil.*		
				£3,566,439	17	4

	£	s.	d.
Claims under Policies (after deduction of Sums Re-assured)	313,112	18	3
Surrenders	24,811	4	6
Annuities	9,622	7	7
Commission	10,114	5	8
Expenses of Management..	14,599	5	3
Dividends to Shareholders	8,866	10	6
Other Payments:			
Medical Fees	628	16	0
Income Tax	2,997	14	5
Shares purchased	3,923	10	0
Cash Bonus	46	17	5
Losses on small Reversions and Agents' Balances, less Sundry small Profits	399	15	5
Amount of Funds at the end of the Year, as per Second Schedule.. ..	3,177,316	12	4
	£3,566,439	17	4

ROBERT ALEXANDER GRAY, *Chairman.*

GEORGE HUMPHREYS, *Actuary & Secretary.* } *Principal Officer.*

JOSHUA LOCKWOOD, *Deputy-Chairman,* } *Two Directors of the Company.*
JAMES MURRAY, }